Fault Tolerant Control for a Class of Switched Linear Systems using Generalized Switched Observer Scheme

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Abstract: This paper concerns the design of a new active fault tolerant control framework for a class of switched linear systems subject to sensor faults and unknown bounded disturbances. The framework herein proposed ensures the fault tolerance capabilities by means of the interaction between three main blocks called generalized switched observer scheme, pre-designed multiple controllers and reconfiguration block. The fault detection and isolation problem has been solved by minimization of the H_∞-norm and maximization of the H_- index. Then, a suitable trade-off between the robustness to disturbances and the sensitivity to sensor faults has been obtained. The main results are reformulated by using linear matrix inequality formulation. An example is included to illustrate the design procedure.

Keywords: Switched systems; Observer design; Fault detection and isolation; H_∞ optimization; Fault tolerant control; Linear Matrix Inequality (LMI).

1. INTRODUCTION

Nowadays, Switched Systems (SS) begin to find place everywhere in our life. They are widely used in control, communication network and biology engineering (Balluchi et al., 2000; Belkhiat et al., 2011; Jabri et al., 2012). Generally, a SS is a two-level Hybrid Dynamic Systems (HDS). The lower level is governed by a set of modes described by differential and/or difference equations, whereas the upper level is a coordinator that orchestrates the switching among the modes. The system clearly admits continuous states and discrete states which take values respectively from a vector space and from a discrete index set. The interaction between the continuous and discrete states makes, that the switched systems are widely representatives, see (Corona et al., 2014; Zhang et al., 2014).

Furthermore, the complex technological systems including the switched systems are often vulnerable to unpredictable events (faults) which can cause unwanted behaviours, and as a consequence, damage to technical parts of the plant, to personnel or to the environment. For these reasons, the Fault-Tolerant Control (FTC) is continuing to receive growing interest, see (Yang et al., 2000; Noura et al., 2000; Staroswiecki, 2005). The main objective of the FTC system is to prevent faults from being developed into serious failures, and therefore increases the availability and reliability of the system and reduces the risk of loss. Generally, FTC systems are divided into two classes: passive and active. Passive FTC systems are based on robust controller design techniques and aim at synthesizing one (robust) controller which makes the closed-loop system insensitive to certain faults. This approach does not require online fault detection. Hence, it is computationally more attractive. Quite the reverse, the active approach is based on controller redesign or selection/mixing of predesigned controllers. This technique usually requires a Fault Detection and Diagnosis (FDD) scheme which has the task to detect and localize the faults that eventually occur in the system. A look at the literature shows that various approaches for FTC have been suggested (Wen et al., 2008; Park and Cho, 2009; Yang et al., 2010; Nke and Lunze, 2010).

However, only a few results were devoted to FTC for SS. Then, the authors in (Rodrigues et al., 2006) have developed a FTC strategy for a class of discrete-time Switched Linear Systems (SLS) in order to preserve closed-loop stability in spite of multiple actuator failures. In the same context, another approach based on switched observer has been proposed in (Yang et al., 2007). This one deals with fault accommodation problem for a class of SLS with both continuous and discrete faults, and without full state measurements. Moreover, a safe-parking and safe-switching framework to handle actuator faults for a class of switched nonlinear systems, subject to input constraints, has been considered in (Du and Mhaskar, 2011). In the same way and for the case of the SLS, a new synthesis approach allowing to ensure a fault tolerant capability has been proposed in (Gao and Zhang, 2011). By means of Linear Matrix Inequality (LMI) techniques, a stabilizing state feedback controller and a class of switching signals are designed, so as to guarantee the finite-time stability of the SLS. Recently, a non-fragile H_2 reliable control problem for SLS with actuator faults and circular disk pole constraints has been investigated in (Hu et al., 2013). The state feedback and the switching law were designed in terms of LMI to guarantee that the closed-loop H_2 performance is less than a specified scalar and all closed-loop poles are located in a specified circular disk.

Furthermore, other authors (Jin et al., 2013) developed a robust FTC method for a class of uncertain switched systems with strong structural uncertainties. The proposed controller can stabilize the switched systems containing strong uncertainties with actuator faults.

Based on the statement above, we can remark that there is still much room for new synthesis approaches such that concerning the sensor faults affecting the SLS. In fact, the impact of sensor failure can vary considerably with the application domain. It can range from a nuisance (e.g. when the set point of an air conditioner is not properly read) to equipment damage (e.g., when sensors on an assembly line are malfunctioning) or even to loss of life (Sensor failures in the aircraft domain). Thus, to our knowledge, the FTC for switched linear systems with sensor faults has not been fully investigated, which motivates the present study.

Hence, the aim of this work consists in the proposition of new active-FTC framework for a class of Multiple-Input Multiple-Output (MIMO) SLS with sensor faults, unknown bounded disturbances, and without full state measurements. The proposed FTC framework is designed around an observer-based state-feedback strategy to guarantee the stability of MIMO SLS and to maintain the nominal performance of the system when sensor fault occurs. The primary contribution of this paper can be stated within the following points:

- Taking account the advantage of separation principle between estimation and control, the proposed synthesis approach is based on the separation of both state-feedback control and observer designs.
- The state-feedback control is designed such that the SLS has a robust H_∞ performance.
- For the purpose of Fault Detection and Isolation (FDI), we develop a Generalized Switched Observer Scheme (GSOS). The FDI problem is solved by minimizing H_∞-norm and maximizing H_- index, allowing a suitable trade-off between the robustness to disturbances and the sensitivity to sensor faults.
- The main results of this work are formulated in terms of LMI conditions.

The paper is organized as follows: section 2 presents the considered class of MIMO SLS. Section 3 gives an overview about the proposed FTC framework. Section 4 details the design of the observer-based state-feedback control. Finally, some numerical results illustrating the efficiency of the approach are given in section 5.

2. SYSTEM DESCRIPTION

We consider in this paper a class of MIMO SLS affected by sensor faults and unknown bounded disturbances. The SLS considered is composed of N linear continuous-time subsystems. Each linear subsystem is defined as follows:

$$\begin{cases} \dot{x}(t) = A_q x(t) + B_q u(t) + B_{d_q} d(t) \\ y(t) = Cx(t) + D_d d(t) + f(t) \end{cases} \tag{1}$$

with $x(t) \in \Re^n$ is the state vector (unmeasurable), $u(t) \in \Re^m$ is the control input vector, $y(t) \in \Re^p$ is the measurement (output) vector and $d(t) \in \Re^m$ is the L_2-norm bounded external disturbances. $f(t) \in \Re^p$ represents the L_2-norm bounded sensor fault to be detected and isolated. A_q, B_q, B_{d_q}, C, D_d are known matrices with appropriate dimensions, $q \in Q = 1, 2, \ldots, N$ is the index indicating the active mode at instant t. q is known at any time.

Note that the matrices C and D_d can be written as: $C = \begin{bmatrix} C_1^T & \cdots & C_p^T \end{bmatrix}^T$, $D_d = \begin{bmatrix} D_{d_1}^T & \cdots & D_{d_p}^T \end{bmatrix}^T$, where C_i and D_{d_i}, for $i \in \Im = 1, 2, \ldots, p$, are respectively the i^{th} row of matrices C and D_d.

Moreover, we define the following variables $\bar{y}_i(t) \in \Re^{p-1}$, $\bar{f}_i(t) \in \Re^{p-1}$, $\bar{C}_i \in \Re^{p-1\times n}$ and $\bar{D}_{d_i} \in \Re^{p-1\times m}$ such as:

for $i \in \Im = 1, 2, \ldots, p$,

$$\bar{y}_i(t) = \begin{bmatrix} y_1^T(t) & \cdots & y_{i-1}^T(t) & y_{i+1}^T(t) & \cdots & y_p^T(t) \end{bmatrix}^T,$$
$$\bar{f}_i(t) = \begin{bmatrix} f_i^T(t) & \cdots & f_{i-1}^T(t) & f_{i+1}^T(t) & \cdots & f_p^T(t) \end{bmatrix}^T,$$

$$\bar{C}_i = \begin{bmatrix} C_1^T & \cdots & C_{i-1}^T & C_{i+1}^T & \cdots & C_p^T \end{bmatrix}^T,$$
$$\bar{D}_{d_i} = \begin{bmatrix} D_{d_1}^T & \cdots & D_{d_{i-1}}^T & D_{d_{i+1}}^T & \cdots & D_{d_p}^T \end{bmatrix}^T.$$

Without loss of generality, we assume that there are only a finite number of mode changes in finite time, all the couples $(A_q, \bar{C}_i)$ are observable and all the couples (A_q, B_q) are controllable. Furthermore, we consider only the single-sensor fault.

***Notations*:** In the sequel, when there is no ambiguity, the time t in a time varying variable will be omitted for space convenience. As usual, X^T and X^{-1} are the transpose and the inverse of matrix X, respectively. The star symbol $\{*\}$ in a symmetric matrix denotes the transposed block in the symmetric position. Moreover, $I_{n.p}$ represents the identity matrix of dimension $n \times p$.

The L_2-norm is defined as $\|x(t)\|_2 = \sqrt{\int_0^\infty x^T(t)\, x(t)\, dt}$.

3. FTC FRAMEWORK

In this section, we give an overview about the proposed FTC framework for a class of SLS subject to sensor faults. The whole structure of the proposed method is shown in figure 1. It is based on the interaction between three main blocks called GSOS, pre-designed multiple controllers (PMC) and reconfiguration block (RB).

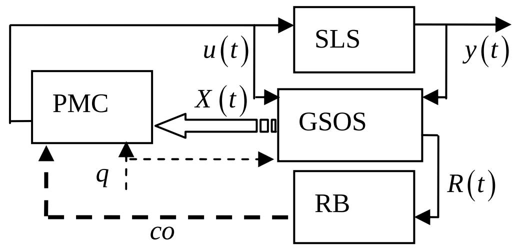

Fig. 1. FTC framework.

3.1 GSOS Block

The GSOS block, illustrated in figure 2, is composed of p Switched Robust observers (SR). They receive the system's inputs $u(t)$ and outputs $y(t)$. Their task are to generate a set of p state estimation vectors $X(t)=\{x_1(t),\ldots,x_p(t)\}$, and a set of p residual signal $R(t)=\{r_1(t),\ldots,r_p(t)\}$.

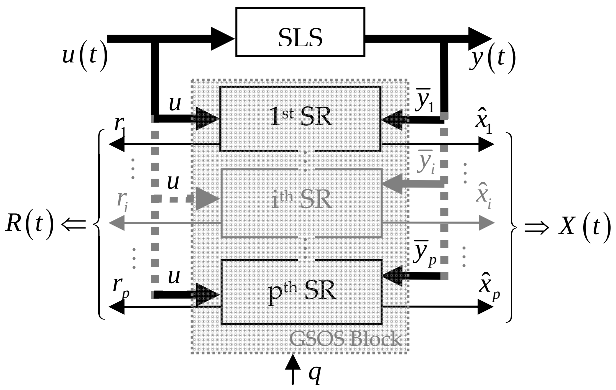

Fig. 2. Structure of GSOS block.

The working principle of the GSOS is quite simple. Assume that there exist p faults to be isolated. The fault isolation using the GSOS consists in the design of a bank of residual generator (SR observer) that fulfils the relation:

$$\begin{bmatrix} r_1(t) \\ \vdots \\ r_p(t) \end{bmatrix} = \begin{bmatrix} Q_1\left(u(t),\ \bar{y}_1(t)\right) \\ \vdots \\ Q_p\left(u(t),\bar{y}_p(t)\right) \end{bmatrix} \tag{2}$$

where $Q_i\left(u(t),\bar{y}_i(t)\right)$, for $i=1,\ldots,p$, stands for a function of inputs $u(t)$ and the i^{th} vector $\bar{y}_i(t)$. Then, an unique fault isolation is performed in accordance with the following logic:

If all $r_i(t)\neq 0$ except $r_1(t)$ then $f_1(t)\neq 0$.

…

If all $r_i(t)\neq 0$ except $r_{k_f}(t)$ then $f_{k_f}(t)\neq 0$.

This logic is resumed in a binary table, named theoretical signatures table. To design this table, when the i^{th} residual must be sensible (respectively robust) to the j^{th} fault apparition, the binary value 1 (respectively 0) in the line and the column correspond to the theoretical signatures table.

The theoretical signatures table associated to the proposed GSOS structure is given in the following table:

Table 1. Theoretical signature table corresponding to the sensor faults localization.

	f_1	f_2	f_3	……	f_p
σ_{r_1}	0	1	1	$1\cdots\cdots 1$	1
σ_{r_2}	1	0	1	$1\ldots\ldots 1$	1
$\vdots$	$\begin{matrix}1\\1\\\vdots\\1\end{matrix}$	$\begin{matrix}1\\1\\\vdots\\1\end{matrix}$	$\begin{matrix}0\\1\\\vdots\\1\end{matrix}$	$\begin{matrix}1 & 1 & \cdots & 1\\0 & 1 & \cdots & 1\\\vdots & \vdots & \ddots & \vdots\\1 & 1 & \cdots & 0\end{matrix}$	$\begin{matrix}1\\1\\\vdots\\1\end{matrix}$
σ_{r_p}	1	1	1	$1\cdots\cdots 1$	0

We note by f_i the i^{th} sensor fault affecting the component of the vector output y. For example, the theoretical fault signature f_1 is equal to $\Sigma_{f_1}=\left(\sigma_{r_1},\sigma_{r_2},\ldots,\sigma_{r_p}\right)=(0,1,\ldots,1)$.

Thus, for the case of the considered class of MIMO SLS, the i^{th} SR observer can be defined as:

$$\begin{cases} \dot{\hat{x}}_i(t)=A_q\hat{x}_i(t)+B_q u(t)+L_q^i\left(\bar{y}_i(t)-\bar{C}_i\hat{x}_i(t)\right) \\ r_i(t)=\bar{y}_i(t)-\bar{C}_i\hat{x}_i(t) \end{cases} \tag{3}$$

with $\hat{x}_i\in\Re^n$, for $i\in\Im=1,2,\ldots,p$, is the state estimation vector delivered by the i^{th} SR observer. $r_i(t)\in\Re^{p-1}$ is the residual signal delivered by the i^{th} SR observer. $L_q^i\in\Re^{n\times p-1}$ is a gain matrix.

In normal operation (without fault), the GSOS allows to have a redundancy of p state estimation vector that are correct. Hence, we have p residual that converge to zero. However, when one among the sensors is out of order, only one state estimation vector is correct (instead of p state estimation vector).

Remind that the estimation error and the residual signal are respectively defined as: For $i\in\Im=1,2,\ldots,p$

$e_i(t)=x(t)-\hat{x}_i(t)$ and $r_i(t)=\bar{y}_i(t)-\hat{\bar{y}}_i(t)$

Then, we can develop that:

$$\begin{cases} \dot{e}_i(t)=\left(A_q-L_q^i\bar{C}_i\right)e_i(t)+\left(B_{d_q}-L_q^i\bar{D}_{d_i}\right)d(t)-L_q^i\bar{f}_i(t) \\ r_i(t)=\bar{C}_i e_i(t)+\bar{D}_{d_i}d(t)+\bar{f}_i(t) \end{cases} \tag{4}$$

The design of GSOS, which is one of the main objectives of this work, can be considered as a problem of deriving matrices L_q^i (for $i\in\Im$, $q\in Q$) such that the matrices $\left(A_q-L_q^i\bar{C}_i\right)$ are asymptotically stable. Moreover, a trade-off between the sensitivity to the sensor-faults and the robustness to the disturbances should be obtained. In other words, the objective can be resumed as follows:

Problem 1. The objective is to determine the observer gains L_q^i (for $i \in \Im$, $q \in Q$), such that the generalized switched observer scheme is called H_- / H_∞ fault detection observer.

Definition 1: Given the MIMO switched linear systems (1), positive scalars $\alpha_i > 0$ and $\beta_i > 0$ (for $i \in \Im$, $q \in Q$). The generalized switched observer scheme (3) is called H_- / H_∞ fault detection observer if the following conditions are satisfied:

Residual's convergence. For $i \in \Im$, $q \in Q$, with zero disturbance input condition $d(t) \equiv 0$, with zero fault input condition $\bar{f}_i(t) \equiv 0$, the residual signal $r_i(t)$ and the estimation error $e_i(t)$ are asymptotically convergent.

Residual's robustness. With zero fault input condition $\bar{f}_i(t) \equiv 0$, for all non zero disturbance input $d(t) \in L_2$-norm, under the zero-initial condition $e_i(t_0) \equiv 0$, it holds that:

$$\int_0^\infty r_i^T(t) r_i(t)\, dt < \alpha_i^2 \int_0^\infty d^T(t) d(t)\, dt \qquad (5)$$

Residual's sensitivity. With zero disturbance input condition $d(t) \equiv 0$, for all non zero fault input $\bar{f}_i(t) \in L_2$-norm, under the zero-initial condition $e_i(t_0) \equiv 0$, it holds that:

$$\int_0^\infty r_i^T(t) r_i(t)\, dt > \beta_i^2 \int_0^\infty \bar{f}_i^T(t) \bar{f}_i(t)\, dt \qquad (6)$$

3.2 PMC Block

Regarding the PMC block, its role is to select the appropriate control vector $u(t) = -K_q \hat{x}_i(t)$ for each mode of SLS. This block consists in three modules: two activation mechanisms (Control Law activation mechanism (CL), State Vector activation mechanism (SV)) and a bank of pre-designed multiple controllers. The structure of the PMC block is shown in figure 3. The SV activation mechanism aims to establish the estimated-state feedback by choosing a state estimation vector from the p state estimation vector provided by the GSOS block. This selection is ordered by the selection signal co. In the same way, the role of the CL activation mechanism consists in activating the appropriate control vector $u(t) = -K_q \hat{x}_i(t)$ for each mode of SLS. The choice of the control vector is done using the input q (switching signal of SLS) which is known at any time. The problem of the design of PMC bank can be resumed as follows:

Problem 2. The objective is to design the switched controller $u(t) = -K_q \hat{x}_i(t)$ such that the switched linear systems (1) has a robust H_∞ state-feedback performance.

Definition 2: The switched linear systems (1) is said to have a robust H_∞ state-feedback performance, if the following conditions are satisfied:

System's stability. With zero disturbance input $d(t) \equiv 0$, the closed-loop switched linear systems is stable.

Robustness. For all non zero disturbance input $d(t) \in L_2$-norm, under zero initial condition $x(t_0) \equiv 0$, it holds that:

$$\int_0^\infty x^T(t) x(t)\, dt < \gamma^2 \int_0^\infty d^T(t) d(t)\, dt \qquad (7)$$

with γ is positive scalar.

Noting that, the synthesis of the GSOS block and the PMC block are detailed later in the paper.

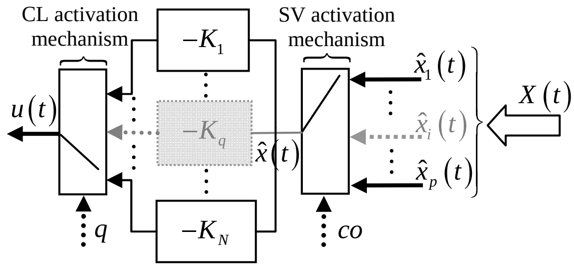

Fig. 3. Structure of pre-designed multiple controllers.

3.3 RB Block

The RB block is used to evaluate the set of p residual signals provided by the GSOS block in order to reconfigure the SV activation mechanism. This block is composed of two main functions: evaluation function and decision function. The structure of the block is illustrated in figure 4.

The evaluation function aims to assess on-line the set of residual in order to generate the fault signature. It receives as input the residual vector $R(t) = \{r_1(t), \ldots, r_p(t)\}$. In against part, it has two types of output, continuous outputs including all norms of residual $\left\{\|r_1(t)\|_{2,T}, \|r_2(t)\|_{2,T}, \ldots, \|r_p(t)\|_{2,T}\right\}$ and binary outputs which include a set of bits representing the real sensor fault signature $\Sigma_{FS} = \left(\sigma_{r_1}, \ldots, \sigma_{r_i}, \ldots, \sigma_{r_p}\right)$. This latter is generated using the so-called norm based residual evaluation approach. Thus, each bit of the real sensor fault signature is determined by comparing the norm of corresponding residual with a pre-determined threshold:

$$\begin{cases} \sigma_{r_i} = 0 \ \ if \ \ \|r_i(t)\|_{2,T} < J_i^{th} \\ \sigma_{r_i} = 1 \ \ if \ \ \|r_i(t)\|_{2,T} \geq J_i^{th} \end{cases} \quad \text{with } i \in \Im .$$

with $\|r_i(t)\|_{2,T}$ is defined as follows:

$$\|r_i(t)\|_{2,T} = \left[\int_{t_1}^{t_2} r_i^T(t) r_i(t)\, dt\right]^{\frac{1}{2}}, \quad T = t_2 - t_1 .$$

The threshold J_i^{th} is the tolerant limit for unknown inputs and uncertainties during the fault free operation. In this work, the threshold is computed as follows:

$$J_i^{th} = \sup_{d \in L_2} \left\| r_i^c(t) \right\|_{2,T} \text{ with } r_i^c(t) = r_i(t)\big|_{\bar{f}_i(t)=0} \tag{8}$$

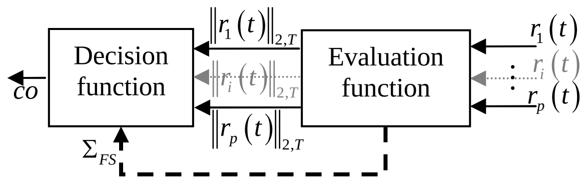

Fig. 4. Structure of RB block.

The decision function receives as input all norms of residual and the fault signature generated by the evaluation function. It generates the selection signal co and aims to designate, during both normal and faulty operations, SR observer able to give the best estimation. This estimation is then used to establish the state-feedback that ensures asymptotic stability of the SLS.

The working principle of the decision function can be stated within the following points:

Fault isolation. The first step consists in isolation of the sensor fault by decoding the real fault signature using the theoretical signature table associated to the proposed GSOS. For example, if the real fault signature Σ_{FS} is zero, this means that the MIMO SLS is free from fault. However, when a fault occurs, the fault signature is nonzero. In this case, the fault isolation task will be to seek in the theoretical signature table the signature that corresponds to the real fault signature (cf. Table .1).

Observer's selection. Objective is to select the SR observer who provides a better state estimation in order to establish the state-feedback. Two cases are possible: in the absence of fault, all the norms of residual are compared and only the SR observer who generates the residual closer to the zero will be selected to establish the state-feedback. In contrast, in the presence of fault, only one residual is closer to the zero. Thus, the SR observer corresponding to this residual is selected to establish the state-feedback.

Noting that, for reasons related to the convergence times, of the proposed SR observers, which are not same, it is most likely that we obtain a real fault signature which does not exist in the theoretical signature table. In this case, we consider that the MIMO SLS is free from fault and the SR observer establishing the state-feedback remains the same. In the following, we describe the synthesis approach used to design the observer-based state-feedback control.

4. OBSERVER-BASED STATE-FEEDBACK CONTROL DESIGN

The approach developed in this section is based on the systems theory notion of state–feedback controller that using an observer to estimate the state vector. The main goal is to propose a sufficient LMI conditions allowing to obtain the gain matrices K_q and L_q^i values such that the conditions given in definitions 1 and 2 are satisfied. Hence, the closed-loop system (SLS +GSOS) can be written as follows:

$$\begin{cases} \dot{x}(t) = \left(A_q - B_q K_q\right) x(t) + B_q K_q e_i(t) + B_{d_q} d(t) \\ \bar{y}_i(t) = \bar{C}_i x(t) + \bar{D}_{d_i} d(t) + \bar{f}_i(t) \end{cases} \tag{9}$$

where $i \in \Im$ and $q \in Q$.

Then, the equations of the closed-loop system (9) and the equations of the estimation error (4) are combined to obtain the following augmented system with $\bar{z}_i(t) = \left[x^T(t) \quad e_i^T(t)\right]^T$, $\bar{\bar{y}}_i(t) = \left[\bar{y}_i^T(t) \quad r_i^T(t)\right]^T$ are augmented variables.

$$\begin{cases} \dot{\bar{z}}_i(t) = \tilde{A}_{q,i} \bar{z}_i(t) + \tilde{B}_{d_{q,i}} d(t) + \tilde{B}_{f_{q,i}} \bar{f}_i(t) \\ \bar{\bar{y}}_i(t) = \tilde{C}_i \bar{z}_i(t) + \tilde{D}_{d_i} d(t) + \tilde{D}_{f_i} \bar{f}_i(t) \end{cases} \tag{10}$$

with $\tilde{A}_{q,i} = \begin{bmatrix} A_q - B_q K_q & B_q K_q \\ 0 & A_q - L_q^i \bar{C}_i \end{bmatrix}$, $\tilde{B}_{d_{q,i}} = \begin{bmatrix} B_{d_q} \\ B_{d_q} - L_q^i \bar{D}_{d_i} \end{bmatrix}$,

$\tilde{B}_{f_{q,i}} = \begin{bmatrix} 0 \\ -L_q^i \end{bmatrix}$, $\tilde{C}_i = \begin{bmatrix} \bar{C}_i & 0 \\ 0 & \bar{C}_i \end{bmatrix}$, $\tilde{D}_{d_i} = \begin{bmatrix} \bar{D}_{d_i} \\ \bar{D}_{d_i} \end{bmatrix}$, $\tilde{D}_{f_i} = \begin{bmatrix} I_{p-1} \\ I_{p-1} \end{bmatrix}$.

Considering the augmented system (10), we can remark that the matrices $\tilde{A}_{q,i}$, with $i \in \Im$ and $q \in Q$, are triangular. Then, the eigenvalues of these latter are just those of the matrices $A_q - B_q K_q$ together with those of the matrices $A_q - L_q^i \bar{C}_i$. Hence, the stability of the augmented system (10) can be solved by designing separately a H_- / H_∞ fault detection GSOS, which feeds into a robust H_∞ controllers. Thus the problem can be divided into two separate parts, which facilities the design. This principle is called separation principle, which is widely used in the case of the switched linear systems. In the following, we start by the design of a robust H_∞ state-feedback control.

4.1 State-feedback Control

According to the separation principle, we break the observer-based state-feedback control design into two separate parts: State-feedback control design and GSOS design. Moreover, we assume that the state vector $x(t)$ of the MIMO SLS is measurable. This assumption allows to redefine the controller $u(t) = -K_q \hat{x}_i(t)$ as $u(t) = -K_q x(t)$ and to study separately the design of a robust H_∞ state-feedback control (cf. problem 2). Then, the robust H_∞ state-feedback control can be formulated as finding a controller $u(t) = -K_q x(t)$ such that (condition 1, definition 2) the closed-loop switched system is stable when $d(t) \equiv 0$ and (condition 2, definition 2) under the zero-initial condition $x(t_0) \equiv 0$, the state vector $x(t)$ satisfies $\|x(t)\|_2^2 < \gamma^2 \|d(t)\|_2^2$ for any non-zero

$d(t) \in L_2$-norm. The main result is summarized in the following theorem.

Theorem 1. *Given positive* $\kappa > 0$, $\mu_{q,q^+} \le 1$, *for* $q, q^+ \in Q$, $q \neq q^+$, *if there exist matrices* $X_q = X_q^T > 0$ *and* Y_q *such that the following LMI hold:*

$$\begin{bmatrix} X_q A_q^T - Y_q^T B_q^T + A_q X_q - B_q Y_q + \kappa B_{d_q} B_{d_q}^T & X_q \\ * & -I_n \end{bmatrix} < 0 \qquad (11)$$

$$\begin{bmatrix} -\mu_{q,q^+} X_q & X_q \\ * & -X_{q^+} \end{bmatrix} \le 0 \qquad (12)$$

Then, the switched linear systems (1) under controllers $u(t) = -K_q x(t)$ *is stable and the robust* H_∞ *state-feedback performance is guaranteed with attenuation level* $\sqrt{\kappa^{-1}}$. *Moreover the controller gains are constructed by* $K_q = Y_q (X_q)^{-1}$.

Proof. Based on the separation principle, the closed-loop switched linear systems (9) can be simplified as follows:

$$\begin{cases} \dot{x}(t) = (A_q - B_q K_q) x(t) + B_{d_q} d(t) \\ \bar{y}_i(t) = \bar{C}_i x(t) + \bar{D}_{d_i} d(t) + \bar{f}_i(t) \end{cases} \qquad (13)$$

According to the definition 2, we have two conditions in order to ensure that the switched linear systems (1) has a robust H_∞ state-feedback performance:

Condition 1. With zero disturbance input condition $d(t) \equiv 0$, we aim to give a sufficient LMI conditions to ensure that the closed-loop switched systems (13) is stable.

Then, we consider a multiple Lyapunov-like functional candidate $V(x(t))$ composed of N quadratic Lyapunov function $\upsilon_q(x(t))$. Each one is defined as follows:

$$\vartheta_q(x(t)) = x^T(t) P_q x(t) \qquad (14)$$

with $P_q = P_q^T > 0$ and $q \in Q$.

Then, the closed-loop switched systems (13) is stable if the following inequalities (15) and (16) are satisfied:

$$\dot{\upsilon}_q(x(t)) < 0, \text{ for } q \in Q. \qquad (15)$$

and $\upsilon_{q^+}(x(t)) \le \mu_{q,q^+} \upsilon_q(x(t))$, for $q \in Q$, $q^+ \in Q$ and $q \neq q^+$. (16)

where q^+ is the successor mode of q and the decreasing rate $\mu_{q,q^+} \le 1$ is positive scalar describing the Lyapunov-like evolution at the switching time $t_{q \to q^+}$.

We develop now the inequality (14).

$$\dot{\upsilon}_q(x(t)) = \dot{x}^T(t) P_q x(t) + x^T(t) P_q \dot{x}(t) < 0$$

$$\dot{\upsilon}_q(x(t)) = x^T(t) \left[(A_q - B_q K_q)^T P_q + P_q (A_q - B_q K_q) \right] x(t) < 0 \qquad (17)$$

The inequality (17) is verified if:

$$(A_q - B_q K_q)^T P_q + P_q (A_q - B_q K_q) < 0 \qquad (18)$$

Then, left and right multiplying the latter condition (18) by P_q^{-1} and by considering the following change of variable $X_q = P_q^{-1}$, it yields:

$$X_q (A_q - B_q K_q)^T + (A_q - B_q K_q) X_q < 0 \qquad (19)$$

Now, let us focus on the stability condition (16). Their aim is to ensure the global behaviour of the Lyapunov-like function at the switching time $t_{q \to q^+}$. We assume that, we have not state jump at switching time which implicates that $x(t^-_{q \to q^+}) \cong x(t^+_{q \to q^+})$. Then, based on the condition (16), we can write:

$$P_{q^+} \le \mu_{q,q^+} P_q \qquad (20)$$

with $q, q^+ \in Q$ and $q \neq q^+$.

The latter inequality (20) is equivalent to the following inequality:

$$X_{q^+}^{-1} \le \mu_{q,q^+} X_q^{-1} \qquad (21)$$

Then, left and right multiplying the condition (21) by X_q, it yields:

$$X_q X_{q^+}^{-1} X_q - \mu_{q,q^+} X_q \le 0 \qquad (22)$$

Applying Schur's complement, the LMI (12) of the theorem 1 is provided.

Condition 2. In this part, under zero-initial condition $x(t_0) \equiv 0$, the objective is to provide a sufficient LMI conditions which ensure that the state vector $x(t)$ satisfies $\|x(t)\|_2^2 < \gamma^2 \|d(t)\|_2^2$ for any non-zero $d(t) \in L_2$-norm.

From the inequality condition (7), we can develop:

$$\int_0^\infty \left(x^T(t) x(t) - \gamma^2 d^T(t) d(t) \right) dt < 0 \qquad (23)$$

Let us consider the multiple Lyapunov-like functional candidate $V(x(t))$, which is defined previously. Hence, the inequality (23) can be written as follows:

$$\begin{aligned} J_1 = \int_0^\infty & \left(x^T(t) x(t) - \gamma^2 d^T(t) d(t) + \frac{dV(x(t))}{dt} \right) dt \\ & - V(x(t)) < 0 \end{aligned} \qquad (24)$$

At this stage of the study, the objective is to provide sufficient conditions that the criterion J_1 is negative. From the inequality (24), we can remark that if the condition (25) is verified then the criterion J_1 is negative.

$$x^T(t)x(t)-\gamma^2 d^T(t)d(t)+\frac{dV(x(t))}{dt}<0 \qquad (25)$$

The latter inequality (25) can be reformulated as follows:

$$\begin{bmatrix} x(t) \\ d(t) \end{bmatrix}^T E \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}<0 \qquad (26)$$

with $E=\begin{bmatrix} (A_q-B_qK_q)^T P_q+P_q(A_q-B_qK_q)+I_n & P_qB_{d_q} \\ * & -\gamma^2 I_m \end{bmatrix}$.

The inequality (26) is verified if the matrix E is negative. Applying the inverse of Schur's complement, we can write the matrix E as follows:

$$(A_q-B_qK_q)^T P_q+P_q(A_q-B_qK_q)+I_n+\kappa P_qB_{d_q}B_{d_q}^T P_q<0 \quad (27)$$

with $\kappa=(\gamma^2)^{-1}$.

Then, left and right multiplying the condition (27) by P_q^{-1} and by considering the following change of variable $X_q=P_q^{-1}$, it yields:

$$X_q(A_q-B_qK_q)^T+(A_q-B_qK_q)X_q+X_qX_q+\kappa B_{d_q}B_{d_q}^T<0 \qquad (28)$$

Remark 1. At this stage of study, a significant simplification, relating the both inequalities (28) and (19), can be considered. Regarding the inequality (19), we can remark that it is bounded by the inequality (28). Then, the inequality (19) is systematically verified when the inequality (28) is satisfied. In other words, if the inequality (28) is verified then the inequality (19) verifies that $(19)<-\left(X_qX_q+\kappa B_{d_q}B_{d_q}^T\right)<0$ because the term $\left(X_qX_q+\kappa B_{d_q}B_{d_q}^T\right)$ is positive. For this reason, the theorem 1 contains only the condition (28).

Using Schur's complement, the inequality (28) can be written as follows:

$$\begin{bmatrix} X_q(A_q-B_qK_q)^T+(A_q-B_qK_q)X_q+\kappa B_{d_q}B_{d_q}^T<0 & X_q \\ * & -I_n \end{bmatrix}<0 \qquad (29)$$

Using the following change of variable $Y_p=K_qX_q$, the LMI condition (11) is provided.

Remind that according to separation principle, the second phase of the observer-based state-feedback control design is to deal separately with the design of GSOS block. In the following text, we discuss the GSOS design problem.

4.2 GSOS Design

One of the key issues related to a FDI technique is concerned with its robustness, which involves two aspects. The first one, concerns the robustness of the residual generator to the disturbances and the second one is related with the sensitivity to the faults that should be detected and isolated. Thus, in this subsection, the GSOS block herein considered is designed for the task of robust FDI. The idea is to design each SR observer of the GSOS block so as to have a good robustness to disturbances and sensitivity to faults. In other words, SR observer design problem (cf. problem 1) can be described as designing matrices L_q^i (for $i\in\Im$, $q\in Q$) such that the residuals $r_i(t)$ is sensitive to fault $\bar{f}_i(t)$ and is robust to disturbances $d(t)$.

Then, concerning the residual robustness to disturbances, the proposed approach is inspired from the robust control techniques, specially the H_∞ model matching. This choice is justified by the capacity of the H_∞ technique to extenuate the undesirable effects of the external disturbances. The idea consists on minimizing the transfer of the external disturbances $d(t)$ on the residual signals $r_i(t)$ according to $\|r_i(t)\|_2^2<\alpha_i^2\|d(t)\|_2^2$ with $i\in\Im$ (cf. condition 2, definition 1).

Moreover, to make the residual $r_i(t)$, with $i\in\Im$, as sensitive to sensor faults $\bar{f}_i(t)$, the proposed approach is based on H_- index. The idea consists in maximising the transfer of the sensor faults $\bar{f}_i(t)$ on the residual signals $r_i(t)$ according to $\|r_i(t)\|_2^2>\beta_i^2\|\bar{f}_i(t)\|_2^2$ with $i\in\Im$ (cf. condition 3, definition 1). The main result concerning the GSOS design problem is summarized in the following theorem.

Theorem 2. *Given positive $\alpha_i>0$, $\beta_i>0$, $\mu_{q,q^+}^i\le 1$, for $i\in\Im$, $q,q^+\in Q$, $q\neq q^+$, if there exist matrices $W_{q,i}=W_{q,i}^T>0$ and Z_q^i such that the following LMI hold:*

$$A_q^TW_{q,i}-\bar{C}_i^T(Z_q^i)^T+W_{q,i}A_q-Z_q^i\bar{C}_i<0 \qquad (30)$$

$$W_{q^+,i}-\mu_{q,q^+}^iW_{q,i}\le 0 \qquad (31)$$

$$\begin{bmatrix} \Psi_q^i & W_{q,i}B_{d_q}-Z_q^i\bar{D}_{d_i} & \bar{C}_i^T \\ * & -\alpha_i^2 I_m & \bar{D}_{d_i}^T \\ * & * & -I_{p-1} \end{bmatrix}<0 \qquad (32)$$

$$\begin{bmatrix} \mathrm{T}_q^i & Z_q^i & \bar{C}_i^T \\ * & \beta_i^2 I_{p-1}-2I_{p-1} & I_{p-1} \\ * & * & -I_{p-1} \end{bmatrix}<0 \qquad (33)$$

with $\Psi_q^i = A_q^T W_{q,i} - \bar{C}_i^T \left(Z_q^i\right)^T + W_{q,i} A_q - Z_q^i \bar{C}_i$ *and* $\mathrm{T}_q^i = A_q^T W_{q,i} - \bar{C}_i^T \left(Z_q^i\right)^T + W_{q,i} A_q - Z_q^i \bar{C}_i - 2\bar{C}_i^T \bar{C}_i$.

Then, the residual signal $r_i(t)$*, with* $i \in \Im$*, is asymptotically convergent and the conditions (5) and (6) (cf. definition 1) are satisfied. Moreover, the gain matrices of the* i *th switched robust observer (3) are constructed by* $L_q^i = \left(W_{q,i}\right)^{-1} Z_q^i$.

Proof. According to the definition 1, we have three conditions if that ones are satisfied, the generalized switched observer scheme (3) can be called H_- / H_∞ fault detection observer. In the following, we will develop each condition in the form of LMI condition.

Condition 1. With zero disturbance input condition $d(t) \equiv 0$, with zero fault input condition $\bar{f}_i(t) \equiv 0$, the objective is to provide a sufficient LMI conditions which ensure that the residual signals $r_i(t)$, with $i \in \Im$, are asymptotically convergent.

Then, we consider a set of p multiple Lyapunov-like functional candidate. Each multiple Lyapunov-like functional candidate $\Omega_i\left(e_i(t)\right)$ is associated to one SR observer, with $i \in \Im$, and it is composed of N quadratic Lyapunov function $\omega_{q,i}\left(x(t)\right)$. $\omega_{q,i}\left(x(t)\right)$ is defined as follows:

$$\omega_{q,i}(t) = e_i^T(t)\, W_{q,i}\; e_i(t) \tag{34}$$

with $W_{q,i} = W_{q,i}^T > 0$, $i \in \Im$ and $q \in Q$.

Similar to the condition 1 of theorem 1 (cf. proof of theorem 1), the residual signal $r_i(t)$ and the estimation error $e_i(t)$ are asymptotically convergent if the following inequalities (35) and (36) are satisfied:

$$\dot{\omega}_{q,i}\left(e_i(t)\right) < 0 \text{, for } q \in Q,\ i \in \Im. \tag{35}$$

and
$$\omega_{q^+,i}\left(e_i(t)\right) \le \mu_{q,q^+}^i\, \omega_{q,i}\left(e_i(t)\right), \quad \text{for} \quad i \in \Im,\quad q \in Q,\quad q^+ \in Q \text{ and } q \ne q^+. \tag{36}$$

where q^+ is the successor mode of q and the decreasing rate $\mu_{q,q^+}^i \le 1$ is positive scalar describing the i th Lyapunov-like evolution at the switching time $t_{q \to q^+}$.

Then, following the same steps as the previous proof (cf. proof of theorem 1, condition 1) and by considering this change of variable $Z_q^i = W_{q,i} L_q^i$, we can obtain the LMI conditions (30) and (31) of theorem 2.

Condition 2. In this part, under zero-initial condition $e_i(t_0) \equiv 0$ ($i \in \Im$), with zero fault input condition $\bar{f}_i(t) \equiv 0$, the objective is to provide a sufficient LMI conditions which ensure that the residual vector $r_i(t)$ satisfies $\left\|r_i(t)\right\|_2^2 < \alpha_i^2 \left\|d(t)\right\|_2^2$ for any non-zero $d(t) \in L_2$-norm and for $i \in \Im$. From the inequality condition (5), we can develop:

$$\int_0^\infty \left(r_i^T(t) r_i(t) - \alpha_i^2\; d^T(t) d(t)\right) dt < 0 \tag{37}$$

Let us consider the set of p multiple Lyapunov-like functional candidate, which are defined previously. Hence, the inequality (37) can be written as follows:

$$J_2^i = \int_0^\infty \left(r_i^T(t) r_i(t) - \alpha_i^2\; d^T(t) d(t) + \frac{d\Omega_i\left(e_i(t)\right)}{dt} \right) dt - \Omega_i\left(e_i(t)\right) < 0 \tag{38}$$

with $i \in \Im$.

We can develop J_2^i as:

$$J_2^i = \int_0^\infty \begin{bmatrix} e_i \\ d \end{bmatrix}^T E_d^i \begin{bmatrix} e_i \\ d \end{bmatrix} dt - \Omega_i\left(e_i\right) < 0 \tag{39}$$

with

$$E_d^i = \left[\begin{bmatrix} \bar{C}_i^T \\ \bar{D}_{d_i}^T \end{bmatrix} \begin{bmatrix} \bar{C}_i & \bar{D}_{d_i} \end{bmatrix} + \begin{bmatrix} \Lambda_{q,i}^T W_{q,i} + W_{q,i} \Lambda_{q,i} & W_{q,i}\left(B_{d_q} - L_q^i \bar{D}_{d_i}\right) \\ * & -\alpha_i^2 I_m \end{bmatrix} \right]$$

and $\Lambda_{q,i} = \left(A_q - L_q^i \bar{C}_i\right)$, $i \in \Im$ and $q \in Q$.

At this stage of the study, the objective is to provide sufficient conditions that the criterions J_2^i, for $i \in \Im$, are negative. From the inequality (39), we can remark that if the conditions $E_d^i < 0$, for $i \in \Im$, are verified then the criterions J_2^i are negative. Using Schur's complement on the matrices $E_d^i < 0$, for $i \in \Im$, and by considering this change of variable $Z_q^i = W_{q,i} L_q^i$, the LMI conditions (32) of the theorem 2 are provided.

Condition 3. In this part, under zero-initial condition $e_i(t_0) \equiv 0$ ($i \in \Im$), with zero disturbance condition $d(t) \equiv 0$, the objective is to provide a sufficient LMI conditions which ensure that the residual vector $r_i(t)$ satisfies $\left\|r_i(t)\right\|_2^2 > \beta_i^2 \left\|\bar{f}_i(t)\right\|_2^2$ for any non-zero $\bar{f}_i(t) \in L_2$-norm and for $i \in \Im$. From the inequality condition (6), we can develop:

$$\int_0^\infty \left(r_i^T(t)\; r_i(t) - \beta_i^2\; \bar{f}_i^T(t)\; \bar{f}_i(t)\right) dt > 0 \tag{40}$$

In the same way, we consider the set of p multiple Lyapunov-like functional candidate, which are defined previously. Hence, the inequality (40) can be written as follows:

$$J_3^i = \int_0^\infty \left(r_i^T(t) r_i(t) - \beta_i^2\; \bar{f}_i^T(t)\; \bar{f}_i(t) - \frac{d\Omega_i\left(e_i(t)\right)}{dt} \right) dt + \Omega_i\left(e_i(t)\right) > 0 \tag{41}$$

with $i \in \Im$.

$$J_3^i = \int_0^\infty \begin{bmatrix} e_i \\ f_i \end{bmatrix}^T E_f^i \begin{bmatrix} e_i \\ f_i \end{bmatrix} dt + \Omega_i\left(e_i\right) > 0 \tag{42}$$

with

$$E_f^i = \left[\begin{bmatrix} \overline{C}_i^T \\ I_{p-1} \end{bmatrix}\begin{bmatrix} \overline{C}_i & I_{p-1} \end{bmatrix} - \begin{bmatrix} \Lambda_{q,i}^T W_{q,i} + W_{q,i}\Lambda_{q,i} & -W_{q,i} L_q^i \\ * & \beta_i^2 I_{p-1} \end{bmatrix}\right] \text{ and}$$

$\Lambda_{q,i} = \left(A_q - L_q^i \overline{C}_i\right)$, $i \in \Im$ and $q \in Q$.

The objective now is to provide sufficient conditions that the criterions J_3^i, for $i \in \Im$, are positive. From the inequality (42), we can remark that if the conditions $E_f^i > 0$, for $i \in \Im$, are verified then the criterions J_3^i are positive. Then, the matrices $E_f^i > 0$, for $i \in \Im$, can be developed as follows:

$$\begin{bmatrix} \overline{C}_i^T \overline{C}_i - \Lambda_{q,i}^T W_{q,i} - W_{q,i}\Lambda_{q,i} & \overline{C}_i^T + W_{q,i} L_q^i \\ * & I_{p-1} - \beta_i^2 I_{p-1} \end{bmatrix} > 0 \tag{43}$$

The latter inequality (43) is equivalent to:

$$\begin{bmatrix} \Lambda_{q,i}^T W_{q,i} + W_{q,i}\Lambda_{q,i} - \overline{C}_i^T \overline{C}_i & \overline{C}_i^T + W_{q,i} L_q^i \\ * & \beta_i^2 I_{p-1} - I_{p-1} \end{bmatrix} < 0 \tag{44}$$

We can develop the inequality (44) as follows:

$$\begin{bmatrix} \Lambda_{q,i}^T W_{q,i} + W_{q,i}\Lambda_{q,i} - 2\overline{C}_i^T \overline{C}_i & W_{q,i} L_q^i \\ * & \beta_i^2 I_{p-1} - 2 I_{p-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \overline{C}_i^T \overline{C}_i & \overline{C}_i^T \\ * & I_{p-1} \end{bmatrix}}_{\Downarrow \atop \begin{bmatrix} \overline{C}_i^T \\ I_{p-1} \end{bmatrix}\begin{bmatrix} \overline{C}_i & I_{p-1} \end{bmatrix}} < 0 \tag{45}$$

By using Schur's complement on the inequality (45) and by considering this change of variable $Z_q^i = W_{q,i} L_q^i$, the LMI conditions (33) of the theorem 2 are provided.

5. SIMULATION AND RESULTS

In this section, a numerical example is provided to illustrate the effectiveness of the proposed FTC method. Let us consider the following MIMO SLS with three discrete modes.

Mode 1: $A_1 = \begin{bmatrix} 0.863 & -0.120 & 12 & 0 \\ -1 & 1.160 & 1.430 & 0 \\ -1.488 & 1 & -2.590 & 0 \\ -0.893 & -1 & -0.815 & 0.160 \end{bmatrix}$, $B_{d_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$C = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0.1 \\ 0 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0 & 0.1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0.083 & 0 \\ 0 & 0 \\ 0 & 0.333 \\ 0 & 0 \end{bmatrix}$, $D_d = \begin{bmatrix} 0.1 & -0.01 \\ -0.1 & -0.1 \\ 0.1 & 0.012 \\ -0.1 & -0.02 \end{bmatrix}$

.

Mode 2: $A_2 = \begin{bmatrix} 1.863 & 0 & 1 & -1 \\ 0 & 1.160 & 0.430 & 0 \\ -0.88 & 120 & -0.590 & 1 \\ -0.893 & 0 & 0.815 & -1.160 \end{bmatrix}$, $B_{d_2} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 10 \end{bmatrix}$,

$B_2 = B_1$.

Mode 3: $A_3 = \begin{bmatrix} 0.597 & -1.034 & -1.240 & -0.5 \\ -0.244 & -0.050 & 1 & 0.5 \\ -0.042 & -3.013 & -1.160 & 0 \\ 0 & 0 & 0 & 1.160 \end{bmatrix}$, $B_{d_3} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \\ 0 & -1 \\ 10 & 0 \end{bmatrix}$,

$B_3 = \begin{bmatrix} 1.143 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 9.500 \end{bmatrix}$.

The discrete switching sequence of the MIMO SLS is as follows:

$\text{mod}\,e\ 1 \to \text{mod}\,e\ 2 \to \text{mod}\,e\ 3 \to \text{mod}\,e\ 1 \ldots\ldots \text{mod}\,e\ 2 \to \text{mod}\,e\ 3$

Before starting the synthesis of the proposed FTC method, we remind that all the subsystems and the SLS are unstable. Moreover, we can easily verify that all the couples $\left(A_q, \overline{C}_i\right)$ are observable and all the couples $\left(A_q, B_q\right)$ are controllable, for $q \in \{1, 2, 3\}$ and $i \in \{1, \ldots, 4\}$.

As mentioned above, the design of the proposed FTC method has been broken into two separate parts: State feedback control design and GSOS design. Concerning the synthesis of the state-feedback control, we apply the theorem 1 and we can obtain the following results:

$\gamma = 1.80$, $\mu_{1,2} = \mu_{2,3} = \mu_{3,1} = 0.9$,

$K_1 = 10^{+3} * \begin{bmatrix} 0.1632 & -5.5324 & -0.1043 & 0.3632 \\ 0.0466 & 2.0215 & 0.0738 & -0.2565 \end{bmatrix}$,

$K_2 = \begin{bmatrix} 196.965 & 201.378 & 3.6769 & -185.017 \\ 1.077 & 800.937 & 47.075 & 1.071 \end{bmatrix}$,

$K_3 = \begin{bmatrix} 12.939 & -6.689 & -2.016 & -0.116 \\ -0.074 & 1.615 & 0.153 & 1.686 \end{bmatrix}$.

Now, for FDI task, we use the results provided by theorem 2, and we can obtain the following gain matrices:

1st SR observer: $\alpha_1 = 0.1$, $\beta_1 = 2$, $\mu_{1,2}^1 = \mu_{2,3}^1 = \mu_{3,1}^1 = 0.9$,

$L_1^1 = \begin{bmatrix} -12.294 & 2.422 & 27.497 \\ -3.118 & 9.0163 & 31.755 \\ -0.937 & 2.713 & 6.519 \\ 12.085 & 3.0602 & -37.8049 \end{bmatrix}$,

$$L_2^1 = \begin{bmatrix} -4.4508 & -39.381 & 52.038 \\ -25.061 & 33.395 & 66.720 \\ -59.933 & 245.407 & 235.065 \\ 57.253 & 6.693 & -80.537 \end{bmatrix},$$

$$L_3^1 = 10^{+3} * \begin{bmatrix} 0.6775 & -0.203 & 0.052 \\ -0.208 & 0.0727 & 0.059 \\ -1.367 & 1.805 & 1.899 \\ 1.151 & -0.330 & -0.321 \end{bmatrix}.$$

2nd SR observer: $\alpha_2 = 0.2$, $\beta_2 = 1.68$, $\mu_{1,2}^2 = \mu_{2,3}^2 = \mu_{3,1}^2 = 0.9$,

$$L_1^2 = \begin{bmatrix} 14.607 & -0.288 & 18.584 \\ 5.618 & 7.221 & 27.825 \\ 1.679 & 1.813 & 5.990 \\ -11.909 & 6.396 & -32.174 \end{bmatrix},$$

$$L_2^2 = \begin{bmatrix} 26.253 & 1.099 & 32.592 \\ 4.731 & 28.611 & 52.153 \\ -42.609 & 193.924 & 217.010 \\ -10.434 & 17.312 & -72.777 \end{bmatrix},$$

$$L_3^2 = \begin{bmatrix} 54.017 & 32.510 & 104.152 \\ -0.203 & -3.284 & 20.856 \\ -186.852 & 456.210 & 634.388 \\ 17.3 & 135.706 & -34.787 \end{bmatrix}.$$

3rd SR observer: $\alpha_3 = 0.2$, $\beta_3 = 2$, $\mu_{1,2}^3 = \mu_{2,3}^3 = \mu_{3,1}^3 = 0.85$,

$$L_1^3 = \begin{bmatrix} 14.480 & -2.593 & 19.557 \\ 9.983 & 3.009 & 29.355 \\ 2.425 & 0.345 & 6.098 \\ -3.434 & 13.532 & -31.925 \end{bmatrix} \quad L_2^3 = \begin{bmatrix} 31.865 & -19.890 & 22.977 \\ -2.998 & -10.42 & 78.542 \\ -92.869 & 14.339 & 399.239 \\ -11.516 & 95.196 & -14.863 \end{bmatrix}$$

$$, \; L_3^3 = 10^{+3} * \begin{bmatrix} 0.073 & -0.036 & 0.141 \\ -0.003 & 0.018 & -0.048 \\ -0.306 & 0.195 & 1.296 \\ -0.055 & 0.289 & 0.255 \end{bmatrix}.$$

4th SR observer: $\alpha_4 = 0.3$, $\beta_4 = 1.45$, $\mu_{1,2}^4 = \mu_{2,3}^4 = \mu_{3,1}^4 = 0.95$,

$$L_1^4 = \begin{bmatrix} 37.582 & 9.334 & -14.746 \\ 55.503 & 26.522 & -22.651 \\ 7.355 & 3.225 & -1.984 \\ 55.982 & 19.7225 & 34.409 \end{bmatrix} \quad L_2^4 = \begin{bmatrix} 82.334 & -8.585 & 41.253 \\ 38.805 & 2.599 & 79.994 \\ 69.103 & 26.373 & 505.202 \\ -67.077 & 29.897 & -66.315 \end{bmatrix}$$

$$, \; L_3^4 = 10^{+3} * \begin{bmatrix} 0.111 & -0.075 & 0.082 \\ 0.012 & -0.118 & 0.036 \\ -0.198 & -0.463 & 1.115 \\ -0.004 & 0.313 & 0.144 \end{bmatrix}.$$

In order to illustrate the efficiency of the proposed FTC method when a sensor fault occurs, let us consider the bounded external disturbance $d(t)$ as a band-limited white noise with power 0.01. The fault signal is simulated as a pulse of amplitude 0.3, from 1.5 to 1.6 sec, affecting the 1^{st} output of MIMO SLS when this latter evolves in the 2^{nd} mode. The evolution of residual signals ($r_1(t) \in \Re^3$, $r_2(t) \in \Re^3$, $r_3(t) \in \Re^3$, $r_4(t) \in \Re^3$), generated by the four SR observers, is illustrated in the figure 5. This figure shows that the residual signals converge to zero which implies that the SR observers closely match the MIMO SLS. After that, only the residual signals ($r_2(t), r_3(t), r_4(t)$), sensitive to the fault affecting the 1^{st} output of MIMO SLS, depart significantly from zero when the sensor fault occurs.

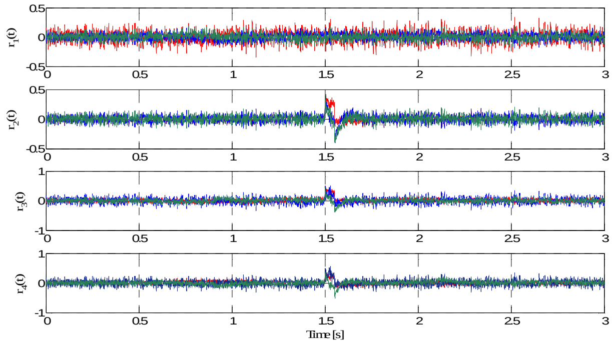

Fig. 5. Evolution of the residual signals.

The figure 6 depicts the evolution of the residual signal's norms. Note that, the solid lines represent the value of the thresholds, calculated in accordance with equation (8) ($J_1^{th} = 0.769$, $J_2^{th} = 0.578$, $J_3^{th} = 0.748$, $J_4^{th} = 0.742$, $T = 0.01$).

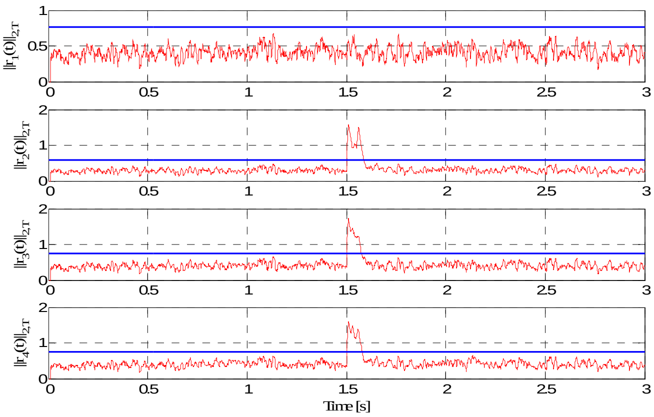

Fig. 6. Evolution of the residual signal's norms.

Moreover, the evolution of the real sensor fault signature $\Sigma_{FS} = (\sigma_{r_1}, \sigma_{r_2}, \sigma_{r_3}, \sigma_{r_4})$ is illustrated in Figure 7. By comparing the evolution of the real sensor fault signature with those given in theoretical signature table (cf. table. 1.), we can see that: Between 0 and 1.5 sec, the real fault signature is equal to zero $\Sigma_{FS} = (0,0,0,0)$, meaning that the MIMO SLS is free from fault. After that, between 1.5 and 1.7 sec, the real fault signature is equal to $\Sigma_{FS} = (0,1,1,1)$, which implies that the first output of SLS is affected by the fault. Finally, the real fault signature returns to its initial value, whereas the fault is disappeared.

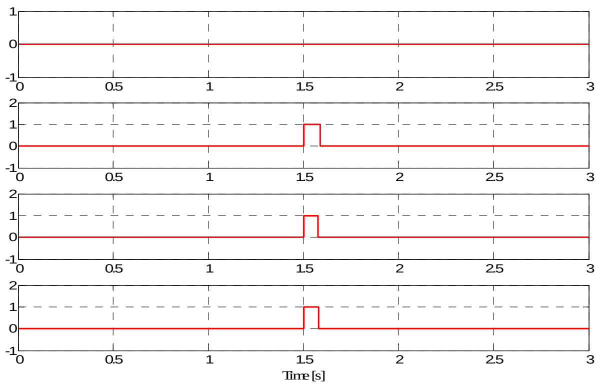

Fig. 7. Evolution of the real sensor fault signature.

Then, by decoding the real fault signature, the reconfiguration block is now able to detect and to isolate the fault. Hence, the next step consists to choose one among the four SR observers which establishes the state-feedback control.

Furthermore, the figure 8 presents the evolution of the selection signal co and the switching signal q. During the fault apparition, we remark that the RB block chooses the first SR observer to establish the state-feedback control.

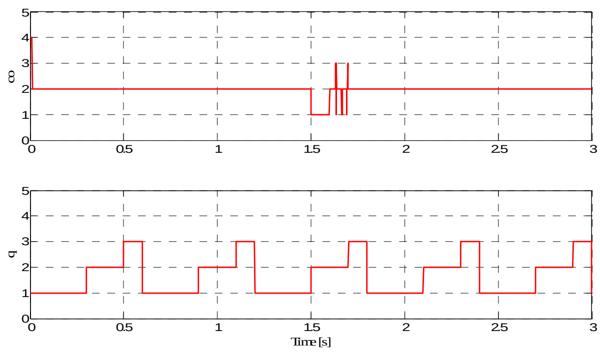

Fig. 8. Evolution of the selection signal co and of the switching signal q.

Finally, the stat vector evolution $x(t)$ and the control input evolution $u(t)$ are illustrated in the figure 9. As expected, these latter are not affected by the sensor fault.

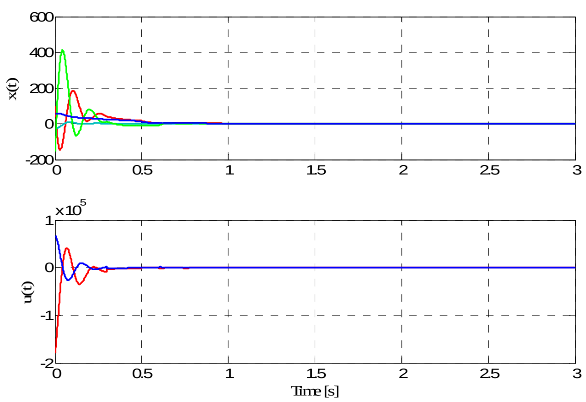

Fig. 9. Evolution of the state vector $x(t)$ and of the control input $u(t)$.

5. CONCLUSIONS

In this paper, a new observer-based fault tolerant control approach is designed for a large class of switched linear systems subject to sensor faults and unknown bounded disturbances. The proposed approach aims to preserve the system stability in the presence of sensor faults. The synthesis of the observer-based state-feedback control takes into account the information provided by a FDI scheme. Beside, the FDI problem has been solved by minimization of the H_∞-norm and maximization of the H_- index. Then, a suitable trade-off between the robustness to disturbances and the sensitivity to sensor faults has been obtained. LMI conditions have been provided to guarantee both the robustness and the convergence of the proposed method. An illustrative example with switched system has been presented to demonstrate the effectiveness of the proposed approach.

Moreover, in this work, we assumed that the MIMO SLS modes are known at any time. Further relaxation of this assumption and extension of the proposed approach to more general hybrid systems will be the focus of future work.

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