A New Data-Driven Approach to Robust PID Controller Synthesis

Hossein Parastvand and Mohammad-Javad Khosrowjerdi

Department of Electrical Engineering, Sahand University of Technology
P.O.Box 51335-1996, Sahand New Town, Tabriz, Iran.
(Tel:+984113459369; h_parastvand@sut.ac.ir, khosrowjerdi@sut.ac.ir)

Abstract: Recently, a new data-driven approach has been proposed for synthesizing the family of stabilizing PID controllers where the proposed approach could not handle any type of uncertainty in the plant model. This paper extends that results to plants with single uncertainty or uncertain plants that can be approximated as plants with a dominant uncertain parameter. It is shown that if an unknown plant with an uncertain parameter satisfies the certain inequalities in terms of real and imaginary parts of the specified plants, then the family of robust stabilizing PID controllers could be obtained using the frequency responses of only two plants corresponding to extreme values of uncertain parameter. No mathematical model is needed and the resulted controller is non-fragile toward its coefficients' variations. In addition, an exact low frequency band over which the plant data must be known with desired accuracy is sufficient for the controller synthesis and beyond this band the plant data might be approximated. The simulation results on harmonic drive system (HDS) subject to loss of actuator effectiveness show the efficiency of the proposed approach.

Keywords: Robust control, PID controller, loss of Effectiveness, data-driven approach, harmonic drive system.

1. INTRODUCTION

Controller synthesis without knowing the plant mathematical model is the subject of many studies in the literature. These approaches could be classified in two categories in terms of the data used for synthesis. In the first category, the available information is time domain data and the second category is based on the frequency domain data. Most of data-driven approaches such as adaptive, neural networks and fuzzy methods use time domain information. Various types of fuzzy PID controllers and their developments in achieving auto tuning adaptive and robust capabilities have been reviewed by Malki (1999) and new advancements can be found in ACM (2012) and Zhang and Chang (2012). Tuning the PID coefficients using neural networks is another approach that has been applied by Emilia et al. (2007), Huang (2013) and Lee et al. (2003). A good review of PID tuning techniques has been presented by Bansal et al. (2012).

Data-driven controller design using only the frequency response data is one of the current approaches in control literature. The Zigler-Nichols tuning approach that is based on the critical frequency for PID synthesis usually leads to good responses to load disturbances for systems that can be approximated by a first order model with relatively small delay; but for more complicated systems the results are not satisfactory. A bunch of frequency domain approaches such as phased locked loop (PLL) by Crowe and Johnson (2001), linear quadratic control algorithm by Kammer et al. (2000) and minimization of the sum of square errors between the desired and measured specifications by Karimi et al. (2003) and Garcia et al. (2006) are all iterative methods that use the Gauss-Newton algorithm and lead to a local optimum of the criteria. Also, they need many experiments on the real system and exact data are needed on the whole range of frequency. Despite its old history, this criteria has not lost its attraction for researchers and over the years new features of frequency response have been released; for example, see Zhou and Hagiwara (2002). In fact, the controller design with minimum knowledge of plant, i.e., the bode or Nyquist diagrams, is the main motivation for control engineers to use these approaches. The frequency response data can be obtained from input-output data using Fourier analysis (Pinelton and Schokens (2001)), virtual sine sweeping (Taghirad (1997)), spectrum analyzer (Petersen and Pota (2003)), network analyzer or using audio sine-wave generators and the sine function of function generators. Sweeping function generators vary (or "sweep") their frequency linearly or exponentially (to give a log plot), and the output amplitude of the swept sine-waves traces the magnitude of the frequency response on an oscilloscope. A slower way of frequency response achievement is through using a fixed-frequency (non-sweeping) sine-wave generator and measuring the output amplitudes at several frequencies with constant input amplitude.

Recently, systematic controller design methods in the frequency domain have been developed based on loop-shaping using graphical tools such as bode or Nyquist diagrams by Halikias et al. (2007) that use the concept of quantitative feedback theory (QFT) proposed by Horowitz (2001). Another frequency domain approach based on D-decomposition has been proposed by Gryazina and Polyak

(2006). The D-decomposition principle is based on dividing a closed-loop polynomial to regions that have invariant root number. These methods are intuitive and applicable on plants that could be approximated by a low order model and relatively small delay that has been mentioned by Krajewsk et al. (2005) and Galdos et al. (2007).

Although the majority of frequency domain approaches need the plant mathematical model, recently, some data-driven approaches have been proposed. A new data-driven approach in frequency domain has been proposed using a version of Hermith-Behiler Theorem by Keel and Bhattacharyya (2008) and Silva et al. (2002). This approach is a systematic method to calculate the "family" of stabilizing PID controllers that satisfy nominal stability and performance. The required data are just the frequency response of nominal plant. This approach has also been extended to simultaneous stabilization and fixed order controllers by Parastvand et al. (2011) and Parastvand and Khosrowjerdi (2013). It is however useful to know the "family" of stabilizing controllers.

The calculations presented by Keel and Bhattacharyya (1997) show that H_∞, H_2, μ and l^1 designs can lead to fragile controllers or controllers that are sensitive to variations in their coefficients. Controller implementation is subject to the imprecision inherent in analog-digital and digital-analog conversion, finite word length, and finite resolution measuring instruments and roundoff errors in numerical computations. This means that any useful design procedure should generate a controller which also has sufficient room for readjustment of its coefficients. The problem of fragility has also been discussed by Silva et al. (2003) where they introduced an analytical approach to obtain the admissible range of stabilizing PID coefficients.

Robust control which took shape in the 1980s and 1990s is still the subject of many studies. For example see the works by Guelton et al. (2012), Liu et al. (2012) and Szelitsky et al. (2011). It is worth mentioning that the data-driven approach proposed by Keel and Bhattacharyya (2008) cannot handle any kind of uncertainties. In this paper, the existing results on data-driven approach for PID controllers in Keel and Bhattacharyya (2008) are extended to uncertain systems with one dominant uncertain parameter. Although in most cases the situations are usually more complex than this, but plants with single or dominant uncertain parameter have been the subject of many studies in the literature. See, for example, Zhang et al. (2003), Achemann et al. (2005), Weijnen and Grievink (2002), Loslovich et al. (2010) and Cacuci (2003). The stability properties of linear time-invariant(LTI), single-parameter dependant systems have been assessed by Zhang et al. (2003). In real applications, there are lots of plants that have a single uncertainty or can be approximated by a plant with one dominant uncertain parameter. Two of these plants have been considered by (1), Achemann et al. (2005) to analyze and control a wastewater process where the dominant uncertain parameter is the maximum specific growth rate of biomass and (2), Reagan et al. (2005) to analyze a chemical system. Plants with one dominant uncertain parameter have also been analyzed by Weijnen and Grievink (2002) and Loslovich et al. (2010). The dominance of uncertain parameter in an uncertain plant has been analytically discussed by Cacuci (2003) using *sensitivity and uncertainty analysis*, where a complete list of related works can be found in the references therein. It should be noted that the proposed approach in this paper can handle different control problems on the systems that can be cast as a plant with single uncertainty such as (1), robustness against loss of effectiveness in actuators and sensors (for examples see Yee (2000), Xiaozheng et al. (2010) and Xiao et al. (2012); (2), robustness against load uncertainty that has been mentioned by Liu and Wang (2008) and Yang and McCalley (2000); (3), some performance achievements that can be translated to the problem of simultaneously stabilizing. It has been shown by Keel and Bhattacharyya (2008) that the problems of $H\infty$-norm attainment on sensitivity and complementary sensitivity functions and satisfying gain and phase margins can be transformed to the problem of simultaneously stabilizing. Solving the above problems illustrates the reasonable justification for the proposed approach in this paper.

In this paper, it is shown that if a plant with one dominant uncertain parameter q satisfies the certain inequalities in terms of real and imaginary parts of specified number of plants then the family of robust stabilizing PID controllers could be obtained using the frequency responses of only two plants with extreme values of uncertain parameter, i.e. $P(s, q_{min})$ and $P(s, q_{max})$. Here, the required data are just the frequency responses of the plants $P(s, q_i)$, $i = \{1, 2, ..., Q\}$, corresponding to a gridded values on the range of q and there is no need to the plant mathematical model.

The organization of this paper is as follows. In Section 2, a review is presented on the technique proposed by Keel and Bhattacharyya (2008) for calculating the family of stabilizing PID controllers for nominal plants. The main result of this paper, which extends the idea of Section 2 to robust PID controller synthesis, is presented in Section 3. The performance criteria have been addressed in Section 4. The usefulness of this approach is illustrated by simulation on harmonic drive system in Section 5. Concluding remarks, discussion and open areas for more investigation are mentioned in Section 6.

2. REVIEW OF EXISTING RESULTS

In this section, a review is presented on the problem of achieving the family of stabilizing PID controllers for nominal plant $P(s)$ proposed by Keel and Bhattacharyya (2008). First, some mathematical preliminaries are summarized. The proofs of theorems and lemma can be found in Keel and Bhattacharyya (2008). The required data are only the frequency response of the plant $P(j\omega)$ for $\omega \geq 0$. The feedback system with PID controller is shown in Fig. 1. Consider a real rational function

$$P(s) = \frac{A(s)}{B(s)}$$

where $A(s)$ and $B(s)$ are polynomials with real coefficients and of degrees m and n, respectively. Assume that $A(s)$ and $B(s)$ have no zero on $j\omega$ axis. Let z^+, p^+ (z^-,p^-) determine the number of open right half plane (RHP) (open left half plane (LHP)) zeros and poles of $P(s)$. Also let $\Delta\angle P(j\omega)$ denotes the net change in phase of $P(j\omega)$ as ω runs from 0 to $+\infty$. The (Hurwitz) signature of $P(s)$ is defined as

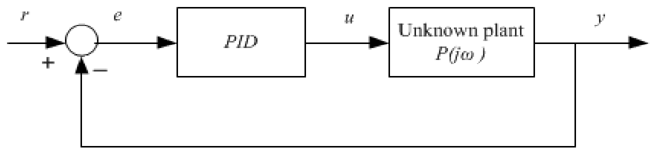

Fig. 1. The feedback structure of plant with PID

$$\sigma(P) = z^- - z^+ - (p^- - p^+) = \frac{2}{\pi}\Delta\angle P(j\omega). \quad (1)$$

Since $P(s)$ has no pole and zero on $j\omega$ axis, it can be written

$$\sigma(P) = -(n-m) - 2(z^+ - p^+) = -r_p - 2(z^+ - p^+). \quad (2)$$

The value of $z^+ - p^+$ can be calculated from Bode diagram of $P(s)$. If $P(s)$ be stable, then z^+ can be obtained from Equation (2). If $P(s)$ be unstable, the frequency response could not be achieved directly from the input-output data of plant. Let $K(s)$ be a known stabilizing controller possibly of high order. Then the frequency response of unstable plant $P(s)$ can be obtained from

$$P(j\omega) = \frac{H(j\omega)}{K(j\omega)(1 - K(j\omega))} \quad (3)$$

where $H(j\omega)$ is the frequency response of closed loop transfer function.

Theorem 2.1.

$$z^+ = \frac{1}{2}[-r_p - r_k - 2z_k^+ - \sigma(H)] \quad (4)$$

$$p^+ = \frac{1}{2}[\sigma(P) - \sigma(H) - r_k] - 2z_k^+ \quad (5)$$

where z_k^+ and r_k are the number of RHP zeros and relative degree of $K(s)$, respectively.

Let $P(j\omega) = P_r(\omega) + jP_i(\omega)$ where $P_r(\omega)$ and $P_i(\omega)$ denote the real and imaginary parts of $P(j\omega)$, respectively. Let the real, distinct, finite zeros of $P_i(\omega) = 0$ denote as $\omega_0, \omega_1, ..., \omega_{l-1}$ such that $0 = \omega_0 < \omega_1 < ... < \omega_{l-1}$. Now consider the modified PID controller as

$$C(s) = \frac{k_i + k_p s + k_d s^2}{s(1 + sT)}$$

where k_p, k_i and k_d are the proportional, integral and derivative coefficients of PID controller and T is a positive constant.

Lemma 1. The feedback control system in Fig. 1 is internally stable if and only if the following conditions hold:

(1) There are no pole-zero cancelations in $Re(s) \geq 0$ when the loop function $L(s)$ is formed.
(2) $\sigma(\bar{F}(s)) = r_p + 2z^+ + N$, where $\bar{F}(s) = F(s)P(-s)$.

write

$$\overline{F}(\omega) = \overline{F}_r(\omega, k_i, k_d) + j\omega\overline{F}_i(\omega, k_p)$$

where

$$\overline{F}_r(\omega, k_i, k_d) = (k_i - k_d\omega^2)|P(j\omega)|^2 - \omega^2 T P_r(\omega) + \omega P_i(\omega)$$

and

$$\overline{F}_i(\omega, k_p) = k_p|P(j\omega)|^2 + P_r(\omega) + \omega P_i(\omega).$$

Consider $\overline{F}_i(\omega, k_p^*) = 0$ and define

$$k_p^* := g(\omega) = -\frac{\cos\phi(\omega) + \omega T \sin\phi(\omega)}{|P(j\omega)|}$$

$$\Longrightarrow k_p^* := g(\omega) = -\frac{P_r(\omega) + \omega T P_i(\omega)}{|P(j\omega)|^2} \quad (6)$$

and $J = sgn[\overline{F}_i(\infty^-, k_p)]$ where $k_p^{min} < k_p^* < k_p^{max}$.

Theorem 2.2. Let $\omega_1 < \omega_2 < ... < \omega_{l-1}$ denote the distinct frequencies of odd multiplicities which are solutions of $\overline{F}_i(\omega, k_p) = 0$. Determine strings of integers

$$V = [v_0, v_1, v_2, ..., v_l]$$

where $v_t \in \{-1, 1\}$ such that: for $n - m$ even:

$$[v_0 - v_1 + v_2 + ... + (-1)^{l-1}2v_{l-1} + (-1)^l v_l](-1)^{l-1}J = n - m + 2z^+ + 2 \quad (7)$$

and for $n - m$ odd:

$$[v_0 - v_1 + v_2 + ... + (-1)^{l-1}2v_{l-1}](-1)^{l-1}J = n - m + 2z^+ + 2 \quad (8)$$

and $v_l = sgn(\overline{F}_r(\infty_-, k_i, k_d))$. Then for $k_p = k_p^*$, the values of (k_i, k_d) corresponding to closed loop stability are given by

$$\overline{F}_r(\omega_t, k_i, k_d)v_t > 0$$

$$\Longrightarrow (k_i - k_d{\omega_t}^2 - \frac{{\omega_t}^2 P_r - \omega_t P_i}{|P|^2})v_t > 0 \quad (9)$$

where v_t's are taken from strings satisfying Equations (7) or (8) and ω_t's are taken from the Equation (6).

From the above Theorem the admissible values of (k_i, k_d) can be determined. Next theorem shows how to calculate the admissible range of k_p.

Theorem 2.3. The necessary condition for PID stabilizing is that there exists admissible range of k_p in which the function $g(\omega)$ has at least R distinct roots of odd multiplicities such that

$$\begin{cases} R \geq \dfrac{n - m + 2z^+ + 2}{2} - 1 & : if \quad n - m \quad even \\ R \geq \dfrac{n - m + 2z^+ + 3}{2} - 1 & : if \quad n - m \quad odd. \end{cases} \quad (10)$$

The procedure for calculating the family of stabilizing PID controllers is summarized in the following algorithm.

Algorithm 1: Calculating the family of stabilizing PID controllers for nominal plant:

Required data: Frequency response or Bode diagram of the nominal plant for $\omega \geq 0$.

(1) Determine the relative degree r_p of $P(s)$ from high frequency slope of bode magnitude of $P(j\omega)$.
(2) Determine z^+ from (2) or (4).
(3) Plot the function $g(\omega)$ for $\omega \geq 0$ from Equation (6).
(4) Apply Theorem 2.3 and determine the admissible range of k_p.
(5) For $k_p = k_p^*$, solve (6) and obtain frequencies of odd multiplicities as $\omega_1 < \omega_1 < ... < \omega_{l-1}$.
(6) Let $\omega_0 = 0$ and $\omega_l = \infty$. Determine $v_0, v_1, ..., v_{l-1}$ from Equations (7) or (8).
(7) For $k_p = k_p^*$, determine the (k_i, k_d) values from Equation (9).
(8) Change the value of k_p and go to step 5 to obtain another stabilizing PID controllers.

3. ROBUST PID SYNTHESIS : A DATA-DRIVEN APPROACH

In this section, the main result of this paper is presented. Consider an uncertain real rational plant $P(s, q)$ as shown

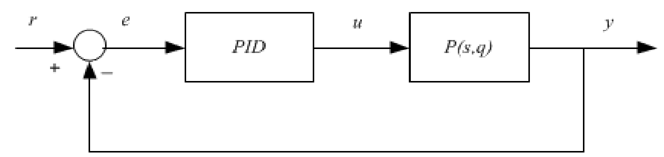

Fig. 2. The feedback structure of uncertain plant with PID

in the feedback structure of Fig. 2 where $q \in [q_{min}, q_{max}]$ is an uncertain parameter. The control objective is to calculate the family of robust stabilizing PID controllers for uncertain plant $P(s,q)$. The required data is as follows:

- For stable plants: the frequency spectrum of $P(j\omega, q_\delta)$, $\delta = 1, 2, ..., Q$ for $\omega \geq 0$ where Q is the number of samples on the gridded range of q.
- For unstable plants: the stabilizing controller $K(s)$ and frequency spectrum of $P(j\omega, q_\delta)$ for $\omega \geq 0$.

Remark 2. The number of samples, i.e. Q, must be high enough so that the set of frequency responses $P(j\omega, q_\delta)$ represents an approximation for frequency spectrum of $P(s,q)$.

The main idea of this section is based on the fact that if by increasing the uncertain parameter q, $g(\omega)$ varies monotonically, then the following three cases could be verified.

- *Case 1: The range of admissible k_p for stabilizing the uncertain plant $P(s,q)$ is the common range of admissible k_p for two plants $P(s, q_{min})$ and $P(s, q_{max})$.*
- *Case 2: The inequalities corresponding to $P(s, q_{min})$ and $P(s, q_{max})$ have the minimum and maximum slope, not necessarily respectively.*
- *Case 3: The common space of the inequality (9) for $P(s, q_{min})$ and $P(s, q_{max})$ is a polygon larger than the convex region of admissible (k_i, k_d). A simple approach to obtain the exact range of coefficients might be vis choosing some test points and analyzing the stability correspond to. This is needed just at inner points of polygon near the vertexes.*

An intuitive proof for the above three cases is provided in the Appendix A.

Thus the key condition for calculating the PID coefficients is the monotonic varying of $g(\omega)$ with increasing the uncertain parameter q. This idea will be used in the next theorem to find what kind of systems satisfy this condition.

Theorem 3.1. Let $P(j\omega, q_\delta)$, $\delta = 1, 2, ..., Q$ be a set of frequency spectrums of an uncertain LTI plant $P(s,q)$ where $q \in [q_{min}, q_{max}]$ is an uncertain parameter. Then the family of stabilizing PID controllers for uncertain plant $P(s,q)$ is a subset of common space between two sets of stabilizing PID coefficients for $P(s, q_{min})$ and $P(s, q_{max})$ if the following inequality satisfies for $P(j\omega, q_\delta) = P_r + jP_i$ and $\omega \geq 0$

$$(P_i^2 + P_r^2 + \omega P_i + P_r)(P_r + \omega P_i) > 0\,. \tag{11}$$

Proof: From Case 2, if increasing the uncertain parameter leads to monotonic varying the function $g(\omega)$, then the family of stabilizing PID coefficients for uncertain plant is a subset of common space between two sets of stabilizing PID coefficients for $P(s, q_{min})$ and $P(s, q_{max})$. The function $g(\omega)$ is monotonic if

$$\frac{dg(\omega)}{dq} > 0 \qquad or \qquad \frac{dg(\omega)}{dq} < 0.$$

Without loss of generality, consider the case of monotonic increasing in $g(\omega)$, i.e. $\frac{dg(\omega)}{dq} > 0$. Increasing the uncertain parameter leads to increasing in $g(\omega)$ if the following inequality holds

$$dP_r\,(P_r^2 - P_i^2 + 2\omega P_r Pi) + dP_i\,(P_i^2\omega - P_r^2\omega + 2P_rP_i) > 0\,. \tag{12}$$

To obtain the solution, it is needed to apply some divisions during solving the above differential inequality. These divisions might change the real rational symbol of inequality (i.e. the symbol '>'). For the sake of simplicity, the above inequality will be solved as an equality, and at the end, the effects of divisions on the real rational symbol of the final results will be considered. Therefore, the following equation must be solved

$$dP_r\,(P_r^2 - P_i^2 + 2\omega P_r Pi) + dP_i\,(P_i^2\omega - P_r^2\omega + 2P_rP_i) = 0. \tag{13}$$

By dividing the above equation on

$$\alpha := P_i^2\omega - P_r^2\omega + 2P_rP_i, \tag{14}$$

it can be written

$$\frac{dP_i}{dP_r} = \frac{P_i^2 - P_r^2 - 2\omega P_r Pi}{P_i^2\omega - P_r^2\omega + 2P_rP_i}\,. \tag{15}$$

Through change of variable as $z = \frac{P_i}{P_r}$, the above equation transforms to

$$P_r\,dz = \frac{-z^3\omega - z^2 - z\omega - 1}{z^2\omega + 2z - \omega}\,dP_r\,. \tag{16}$$

To obtain a homogeneous differential equation, both sides of Equation (16) are devided by

$$\beta := P_r, \tag{17}$$

and

$$\gamma := \frac{-z^3\omega - z^2 - z\omega - 1}{z^2\omega - \omega + 2z}\,. \tag{18}$$

As a result, the Equation (16) could be transformed to

$$\frac{z^2\omega + 2z - w}{-z^3\omega - z^2 - z\omega - 1}\,dz = \frac{1}{P_r}\,dP_r$$

$$\Rightarrow \left(\frac{-2z}{z^2+1} + \frac{-\omega}{-z\omega - 1}\right)dz = \frac{1}{P_r}\,dP_r \;\Rightarrow ln\left(\frac{-z\omega - 1}{(z^2+1)P_r}\right) = 0,$$

$$\Rightarrow \quad \frac{-z\omega - 1}{(z^2+1)P_r} = 1, \tag{19}$$

and by substituting $z = \frac{P_i}{P_r}$, one can write

$$P_i^2 + P_r^2 + \omega P_i + P_r = 0. \tag{20}$$

Now the real rational symbol of above equation could be obtained by considering the effect of divisions on (14), (17) and (18), i.e. terms α, β and γ. Obviously the real rational symbol of the resulted equation in (20) is dependent to the sign of the multiplication of $(\alpha\beta\gamma)$. Then, it can be written

$$\begin{cases} If \quad \alpha\beta\gamma > 0 \quad then \quad P_i^2 + P_r^2 + \omega P_i + P_r < 0, \\ If \quad \alpha\beta\gamma < 0 \quad then \quad P_i^2 + P_r^2 + \omega P_i + P_r > 0\,. \end{cases} \tag{21}$$

On the other side, from the multiplication of $(\alpha\beta\gamma)$ as

$$\alpha\beta\gamma = (P_i^2\omega - P_r^2\omega + 2P_rP_i)P_r\left(\frac{-z^3\omega - z^2 - z\omega - 1}{z^2\omega - \omega + 2z}\right),$$

and by substitution $z = \frac{P_i}{P_r}$ it can be written

$$\begin{cases} for \quad \alpha\beta\gamma > 0 \quad then \quad -(P_r + \omega P_i) > 0, \\ for \quad \alpha\beta\gamma < 0 \quad then \quad -(P_r + \omega P_i) < 0\,. \end{cases} \tag{22}$$

The combination of two sets of inequalities in (21) and (22) leads to (11). Similarly, the same results can be obtained for $\frac{dg(\omega)}{dq} < 0$. □

Remark 3. Thanks to Theorem 3.1, the class of systems that proposed data-driven approach could deal with can be defined. In fact, the class of systems that can be addressed by the proposed approach of this paper must satisfy the following conditions

I. The inequality (11) should be hold at each frequency $\omega \geq 0$ for $P(j\omega, q_\delta)$,
II. There should be a common range between admissible k_p for $P(s, q_{min})$ and $P(s, q_{max})$.

Remark 4. The frequency response data corresponding to $P(s, q_{min})$ and $P(s, q_{max})$ could be recognized from the set of frequency responses of uncertain plant $P(j\omega, q)$. In fact if the inequality (11) holds, then it could be deduced that the uppermost and lowermost plots of $g(\omega)$ are corresponding to $P(s, q_{min})$ and $P(s, q_{max})$, not necessarily respectively.

Remark 5. It worth mentioning that almost in every application the set of $\omega_t's$ can be found in the low frequencies. This means that only an exact data of the low frequency band is sufficient for controller synthesis and beyond this band the plant information may be rough or approximated.

The procedure for calculating the family of stabilizing PID controllers is summarized in the following algorithm.

Algorithm 2: Calculating the family of stabilizing PID controllers for uncertain plant $P(s, q)$ in which $q \in [q_{min}, q_{max}]$ is an uncertain parameter

Required data: $P(j\omega, q_\delta)$ for $\delta = 1, 2, ..., Q$ and $\omega \geq 0$.

(1) Check the inequality (11). If this inequality holds then go to the next step.
(2) Determine the extreme plots of $g(\omega)$ correspond to $P(j\omega, q_{min})$ and $P(j\omega, q_{max})$ using Remark 4.
(3) Apply Theorem 2.3 to obtain the common set of admissible k_p for two plants found in the previous step.
(4) Using Steps 5-8 of Algorithm 2 determine the set of (k_i, k_d) coefficients for $P(j\omega, q_{min})$ and $P(j\omega, q_{max})$.
(5) Calculate the common range of (k_i, k_d) coefficients between $P(j\omega, q_{min})$ and $P(j\omega, q_{max})$.
(6) Calculate the exact family of admissible (k_i, k_d) using the approach proposed in Case 3.

4. PERFORMANCE ACHIEVEMENT

Many performance achievement problems for plant $P(s)$ can be cast as the simultaneously stabilization of plant and the family of real and complex plants. Some of these performance achievement problems are listed in Keel and Bhattacharyya (2008). For example the problem of H_∞-norm achievement on the complementary sensitivity function is equivalent to simultaneously stabilizing the plant $P(s)$ and the family of real plants

$$P^C(s) = \left[1 + \frac{1}{\gamma} e^{j\theta} W(s)\right] P(s) : \theta \in [0, 2\pi]$$

where γ is an arbitrary parameter and $W(s)$ is the optional weighting function. The above function represents a plant

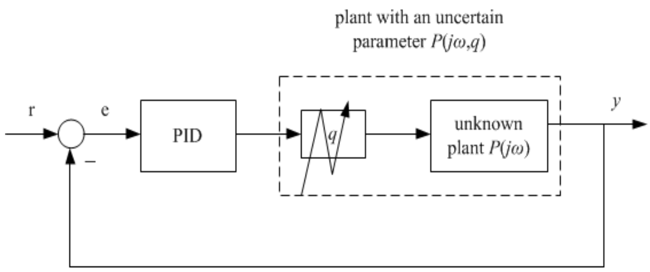

Fig. 3. The feedback structure of plant faced to loss of actuator effectiveness with PID

with an uncertain parameter θ. So the proposed approach in this paper can be effectively applied here to obtain the family of stabilizing plants that satisfy H_∞-norm on the complementary sensitivity function. The same criteria has been used for satisfying desired gain and phase margins and H_∞-norm specification on the sensitivity function by Keel and Bhattacharyya (2008).

Remark 6. Assume that $P(s, q)$ be a plant with an uncertain parameter q. Let $P^C(s, q, \theta)$ be a plant with another virtual uncertain parameter θ corresponding to above performance specification parameters. If necessary conditions of Theorem 3.1 satisfy for both $P(s, q)$ and $P^C(s, q^*, \theta)$ where q^* belongs to $\{q_{min}, q_{max}\}$, then the family of PID controllers that achieve the above performance specifications for uncertain plant $P(s, q)$ is the subset of stabilizing PID coefficients for

$$P^C(s, q, \theta) : q = \{q_{min}, q_{max}\}, \theta = \{0, 2\pi\}.$$

The approach can handle different control problems that can be evaluated as a plant with an uncertain parameter. For example, the problem of controller synthesis for different types of electrical machines, such as, DC motor, induction motor, synchronous motor, etc that are faced to load uncertainty mentioned in Liu and Wang (2008) can be cast as a plant with an uncertain parameter, in which the uncertain parameter q is the uncertain load.

5. SIMULATION RESULTS

In this section, an example is presented to show the usefulness of the proposed approach. In this example, the problem of controller synthesis for a harmonic drive system (HDS) faced to loss of actuator effectiveness has been addressed. Loss of actuator effectiveness is an important problem from practical point of view. See, for example, Yee (2000) and Xiao et al. (2012). Fig. 3 might represent the feedback structure of an uncertain plant with PID controller faced to loss of actuator effectiveness where $q = L$ is the uncertain parameter corresponding to loss of effectiveness and belongs to (0,1]. A similar case will happen when there is a loss in sensor effectiveness. Thus, using Algorithm 2, the family of robust PID controllers for plants faced to loss of actuator/sensor effectiveness can be calculated.

The frequency response of the HDS is shown in Fig. 4 that is calculated by Taghirad (1997) using virtual sine sweeping (VSS). Here the control objectives are

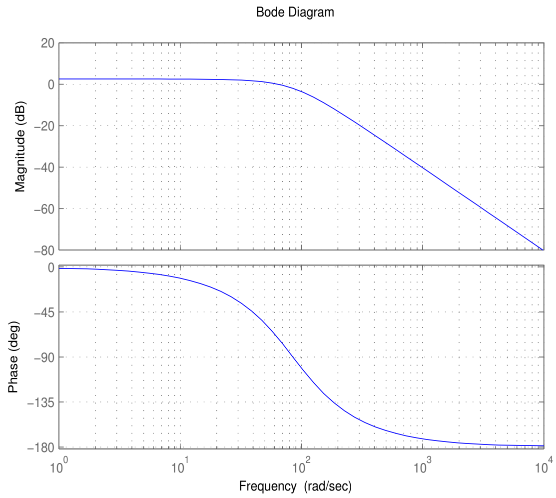

Fig. 4. The Bode diagram of HDS with $L = 1$

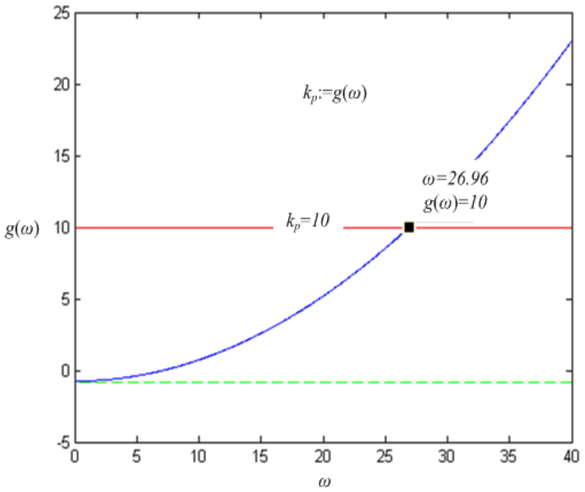

Fig. 5. The plot of function $g(\omega)$ for HDS with $L = 1$

(1) Calculating the family of stabilizing PIDs for nominal plant, i.e $P(s, L)$ where $L = 1$.
(2) Calculating the family of stabilizing PIDs that satisfy H_∞-norm on complementary sensitivity function for nominal plant.
(3) Satisfying the previous performance criterion for the HDS faced to loss of effectiveness in actuator, i.e. for the plant $P(s, L)$ where $0 < L \leq 1$.

First consider the case where there is not loss of actuator effectiveness, i.e. $L = 1$. The corresponding function $g(\omega)$ is shown in Fig. 5. So the admissible range of k_p is $[-1, +\infty)$. Calculating the (k_i, k_d) values for $k_p = 10$ is resulted to the following inequalities

$$k_i > 0$$
$$k_i < 722k_d + 497.$$

The whole range of stabilizing PID coefficients is shown in Fig. 6.

Now consider the problem of $H\infty$-norm achievement on the complementary sensitivity function. The range of (k_i, k_d) that satisfies this performance criterion for $k_p = 10$

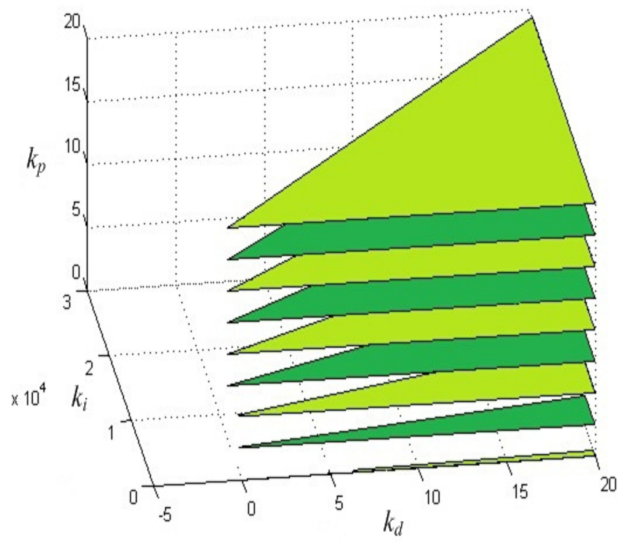

Fig. 6. The whole family of stabilizing PID controllers for HDS with $L = 1$

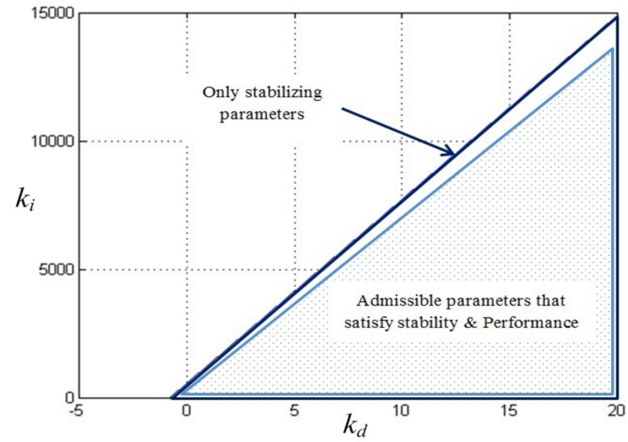

Fig. 7. Admissible ranges of $k_i - k_d$ that satisfy stability and performance for HDS with $L = 1$ and $k_p = 10$

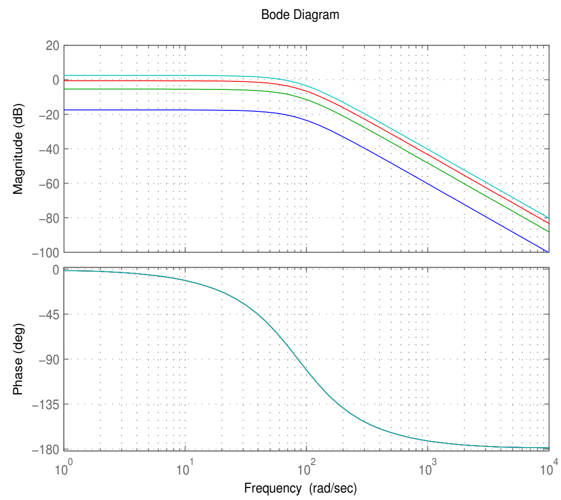

Fig. 8. bode diagram of HDS with $L = \{0.01, 0.4, 0.7, 1\}$

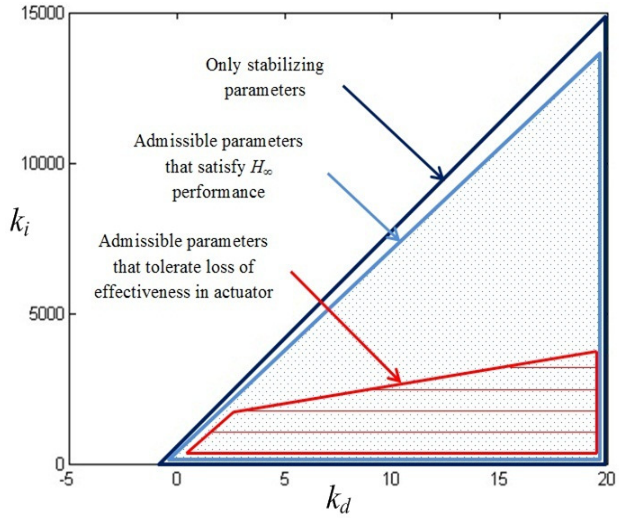

Fig. 9. (k_i, k_d) values for $k_p = 10$ of only stabilizing controllers, controllers satisfy H_∞ performance and controllers satisfy H_∞ performance at the presence of loss of actuator effectiveness

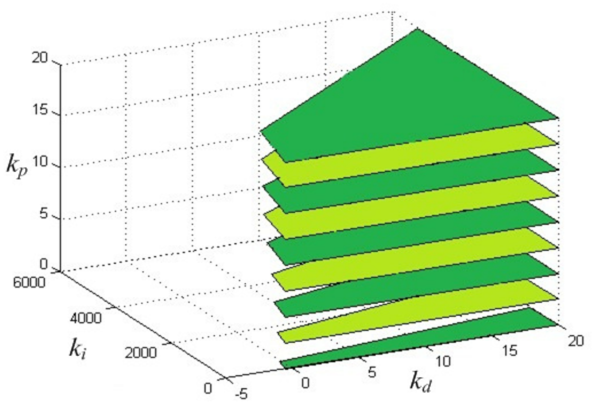

Fig. 10. The family of robust PID controllers for HDS

is shown in Fig. 7. The whole range of (k_i, k_d) could be obtained similarly.

Now, if the loss of actuator effectiveness happens, it can be inferred from Section 3 that the HDS with this phenomenon is an special case of a plant with an uncertain parameter. Therefore, the proposed approach could be applied for such systems. Figure 8 shows the effect of loss of actuator effectiveness for $L = \{0.01, 0.4, 0.7, 1\}$ on the magnitude of Bode diagrams of HDS. Simulation results show that the conditions of Remark 6 satisfy; thus the robust PID controllers can be calculated by simultaneously stabilizing of two plants $P(s, L_{min}) = P(s, 0.01)$ and $P(s, L_{max}) = P(s, 1)$.

The range of (k_i, k_d) values for $k_p = 10$ that achieve to stability and performance in the presence of loss of actuator effectiveness is shown in Fig. 9 and the whole range is obtained in Fig. 10 by sweeping on k_p. The step responses of plant is shown in Fig. 11 when the value of loss of actuator effectiveness is set to $L = 0.1$. As shown in Fig. 11, the step response of robust PID controller applied to HDS has the best transient response in comparison with controllers that satisfies nominal stability and nominal performance. The corresponding control inputs is shown in Fig. 12.

Although the example provided here considers the gain uncertainty which is rather a trivial case, but some arguments and examples are provided by Parastvand (2010) that imply this condition can be satisfied satisfied for a large class of industrial processes.

6. CONCLUSION AND DISCUSSION

This paper provided an extension of a data driven controller synthesis from nominal stability to robust stability and performance. It is shown that for an unknown plant with a single or dominant uncertain parameter, if some smooth conditions hold, then the family of stabilizing PID controllers corresponding to two plants with extreme values of uncertain parameter can guarantee robust stability for all perturbed plants with any value of uncertainty between those two extremes. The required data are the specific number of frequency responses of plants corresponding to the gridded values of uncertain parameter. There is no need to exact frequency responses on the whole frequencies, and only accurate data on the low frequency band is sufficient in almost any application. Also it is illustrated that some control problems such as achieving the performance specifications, robustness against loss of effectiveness in actuator and load uncertainty can be characterized as the problem of robust stabilizing of a plant with an uncertain parameter. Another feature of the proposed technique is the non-fragility of the obtained controller or controller robustness because of existing a reliable band for admissible PID coefficients. The proposed approach is successfully simulated on harmonic drive system faced to loss of effectiveness.

The presented method can easily be extended to model-based synthesis. Some plants that are faced to parameters variations can be approximated by plants with a dominant uncertain parameter. From this point of view, the proposed approach can be used for model-based uncertain systems to calculate the family of robust stabilizing PID controllers. Also, the proposed approach could be easily extended to time-delay systems using some modifications on the approach proposed in Keel and Bhattacharyya (2008). The open areas of research under study are (a), the design of such controllers for multivariable and nonlinear plants, and (b), developing the proposed approach to other structured and unstructured uncertainties.

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Appendix A

Here are the proof of three cases presented in Section 3. The proof for Case 1 is obvious. For Case 2, we know that the slope of inequalities of Equation (9) is equal to ω_t^2. The values of ω_t for every q can be obtained from $g(\omega)$ for

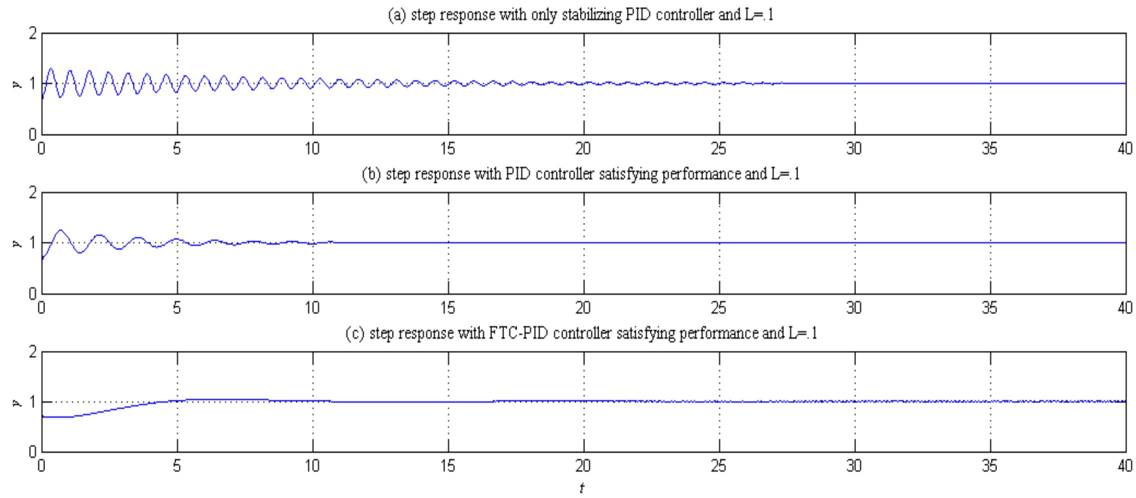

Fig. 11. Step responses of HDS system with PID

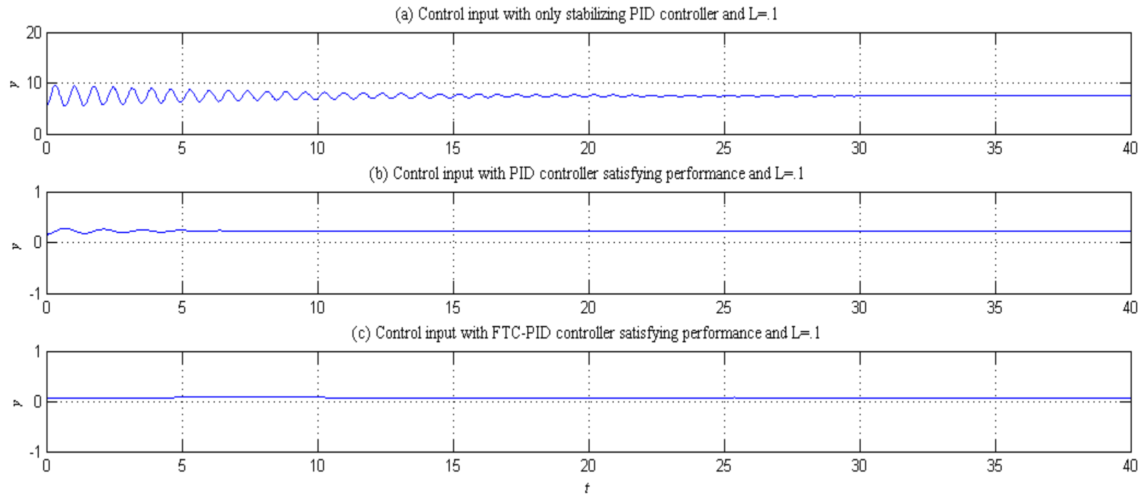

Fig. 12. Control inputs of HDS system with PID

k_p^*. Now if by changing the q, $g(\omega)$ varies monotonically then the maximum and minimum values of ω_t^2 would be obtained for q_{min} and q_{max} in $P(s, q_{min})$ and $P(s, q_{max})$. Then, the maximum and minimum slope of inequalities in (9) correspond to $P(s, q_{min})$ and $P(s, q_{max})$. In Fig. A.1, the plot of inequalities corresponding to Equation (9) is shown for some values of uncertain parameter q for a typical example. As shown in Fig. A.1, the admissible values for (k_i, k_d) are a subset of inequalities obtained from Equation (9) for $P(s, q_{min})$ and $P(s, q_{max})$. For Case 3, let by monotonic varying in q, $g(\omega)$ varies monotonically. This leads to monotonic varying in the slope of inequalities (9). Now, if the values of y-intercepts vary in the same manner, then the common space of inequalities (9) for $P(s, q_\delta)$, $\delta = 1, 2, , Q$, is exactly equal to the common space of just two plants $P(s, q_{min})$ and $P(s, q_{max})$. But this is the case that can not be guaranteed. In the other word, since the values of y-intercepts of the lines obtained from (9) are different, the common space of the inequalities (9) for $P(s, q_\delta)$ is smaller than the common space of just two plants $P(s, q_{min})$ and $P(s, q_{max})$. This can be seen from Fig. A.1. In spite of the fact that two green triangles belong to admissible range of (k_i, k_d) for $P(s, q_{min})$ and $P(s, q_{max})$, but they do not belong to admissible range of (k_i, k_d) for $P(s, q)$. To obtain the exact family of stabilizing PID coefficients, the shadowed region in Fig. A.1 must be excluded. One simple approach to exclude the shadowed regions is to choose some test points in these regions and analyze the stability correspond to.

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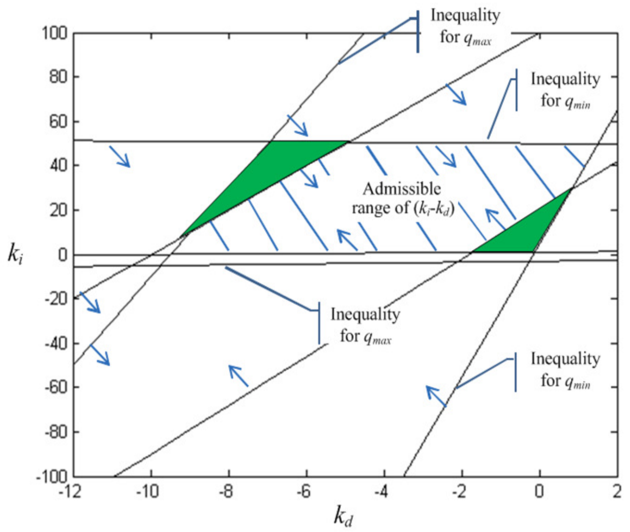

Fig. A.1. The plot of inequalities obtained from Equation (9) for k_p^* for a typical example

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