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ADAPTIVE CONTROL USING DELTA MODEL IDENTIFICATION

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***Abstract**: This paper presents the adaptive control for linear systems modelled by δ -models. The process is identified by regression (ARX) model using the recursive least-squares method with LD decomposition and applied directional forgetting. Controller synthesis is designed on the basis of a practical criterion for digital PID control loops.*

***Key words**: adaptive control; PID control; δ -model; identification.*

1. INTRODUCTION

The widely use of digital process computer as control unit of the control loop in automatic systems had imposed the improvement of the discrete-time model identification. Using the computer as controller, impose to utilize sampler and holder in combination with analogue-digital and digital analogue converters as interface between the differently operating dynamic systems. The sampler samples the continuous signal in k-multiples of sampling periods to produce an output signal as an impulse sequence in discrete time $t_k=kT_S$, where T_S is the sampling period. The height of impulses is equal to the value of the input signal over the sampling period. For technological process control the zero-order holder is used almost exclusively to hold the impulse constant over the entire sampling period. We must therefore use suitable mathematical description to express the dynamic behavior of the thus discretized members of the control loop. One such description is an expression using the Z-transformation. If G(s) is the transfer function of a continual dynamic system, then the following expression for the discrete transfer function with the zero-holder is valid:

$$G(z) = \frac{z-1}{z} Z\left\{\frac{G(s)}{s}\right\} \qquad (1)$$

This step transfer function (1) is a rational polynomial function with variable z. The simple model structure identification by using measurable data, being most good suitable for the synthesis of the discrete control loop and for the description and expression of different types of stochastic processes including disturbance modeling, are the main advantages of the Z-transformation (1).

The step z-transform function has some disadvantages when sampling period decrease:

- the Z-transformation parameters do not converge as the sampling period decreases to the Laplace – transformation continuous parameters from which the are derived;
- very small sampling periods yield very small numbers from the transfer function numerator;
- the poles transfer function approaches the unstable domain as the sampling period decreases.

These disadvantages can be avoided by introducing a more suitable discrete model ([1], [2]), a half-way between discrete models and continuous models.

2. DELTA-MODELS

For this purpose the δ - model is the most suitable, where parameter δ converges with decreased sampling period T_S to a continuous operator s

$$\lim_{T_S \to 0} \delta = s \tag{2}$$

It is possible to prove that equality

$$\delta = \frac{z-1}{\alpha T_S z + (1-\alpha) T_S} \tag{3}$$

holds for interval $0 \le \alpha \le 1$. By substituting α into equation (3) we obtain an infinite number of new δ -models. The most widely know and used δ -models in practice are:

$$\text{for } \alpha = 0,\ \delta = \frac{z-1}{T_S} \tag{4}$$

forward δ -model

$$\text{for } \alpha = 0.5,\ \delta = \frac{2}{T_S}\frac{z-1}{z+1} \tag{5}$$

Tustin δ -model

$$\text{for } \alpha = 1,\ \delta = \frac{z-1}{zT_S} \tag{6}$$

backward δ -model.

In this paper we will use only the forward δ -model (4). The δ -models will be used of process modeling adaptive control based on self-tuning controller. The main idea of the self-tuning controller is based on a recursive identification procedure and a selected control synthesis. For this reason it is necessary to apply suitable recursive identification algorithm to this model.

3. PROCESS IDENTIFICATION

For parameter estimation of the δ -model, the recursive least-squares method with LD decomposition and with directional forgetting is applied [3].

A useful model to apply this method of identification is the regression (ARX) model which is often expressed in its compact form:

$$y(k) = \theta^T(k)\varphi(k-1) + n(k) \tag{7}$$

where

$$\theta^T(k) = [a_1, a_2, ..., a_{na}, b_1, b_2, ..., b_{nb}] \tag{8}$$

is the parameter vector and

$$\begin{aligned} &\varphi^T(k-1) = [-y(k-1), -y(k-2), ..., \\ &-y(k-n), u(k-1), u(k-2), ..., u(k-n)] \end{aligned} \tag{9}$$

is the regression vector (y(k) is the process output variable, u(k) is the controller output variable). The non-measurable random component n(k) is assumed to have zero mean value E[n(k)]=0 and constant covariance (dispersion) R=E[n^2(k)].

We use for the system a δ -second-order model with transfer function:

$$G(\delta) = \frac{y(\delta)}{u(\delta)} = \frac{b_1\delta + b_2}{\delta^2 + a_1\delta + a_2} \tag{10}$$

which can be rearranged into the form:

$$\delta^2 y(\delta) = -a_1\delta y(\delta) - a_2 y(\delta) + b_1\delta u(\delta) + b_2 u(\delta) \tag{11}$$

By substituting δ from relation (4) and multiplying by z^{-2} we obtain the equation:

$$\begin{aligned} &\frac{1-2z^{-1}+z^{-2}}{T_S^2}Y(z) = -a_1\frac{z^{-1}-z^{-2}}{T_S}Y(z) - a_2 z^{-2}Y(z) \\ &+b_1\frac{z^{-1}-z^{-2}}{T_S}U(z) + b_2 z^{-2}U(z) \end{aligned} \tag{12}$$

where Y(z) and U(z) are the Z-transforms process output y(k) and controller output u(k) variables, respectively.

The stochastic discrete model for δ parameter estimates is then in the form:

$$\begin{aligned} &y_\delta(\delta) = -a_1 y_\delta(k-1) - a_2 y_\delta(k-2) + \\ &b_1 u_\delta(k-1) + b_2 u_\delta(k-2) + n(k) \end{aligned} \tag{13}$$

where

$$\begin{aligned} &y_\delta(k) = \frac{y(k) - 2y(k-1) + y(k-2)}{T_S^2} \\ &y_\delta(k-1) = \frac{y(k-1) - y(k-2)}{T_S} \\ &y_\delta(k-2) = y(k-2) \\ &u_\delta(k-1) = \frac{u(k-1) - u(k-2)}{T_S} \\ &u_\delta(k-2) = u(k-2) \end{aligned} \tag{14}$$

From equation (13) and (14) it is obvious that the parameter vector has the same form as (8),

$$\theta_\delta^T(k-1) = [a_1, a_2, b_1, b_2] \tag{15}$$

and the regression vector is

$$\begin{aligned} &\varphi_\delta^T(k-1) = \Bigg[-\frac{y(k-1) - y(k-2)}{T_0}, -y(k-2), \\ &-\frac{u(k-1) - u(k-2)}{T_0}, -u(k-2)\Bigg] \end{aligned} \tag{16}$$

One obtain the following model for the regression algorithm (ARX):

$$y_\delta(k) = \theta_\delta^T(k)\varphi_\delta(k-1) + n(k) \tag{17}$$

Directional forgetting supports the recursive least-squares method is utilized for calculating the parameter estimates and adaptation. The value of the directional forgetting factor $\gamma(k)$ depends on the level of conformity achieved between the model and the real behavior of the system. In this case, it is a minimized criterion:

$$J_k(\theta_\delta) = \sum_{i=k_0}^{k} \gamma^{2(k-i)} e_\delta^2(i) \tag{18}$$

where k_0 is the identification start and

$$e_\delta(i) = y_\delta(i) - \theta_\delta^T \varphi_\delta(i-1) \tag{19}$$

The algorithm of the recursive least-squares method with directional forgetting consists of the following steps in each sampling period:

1. Choosing the initial vector of parameter estimates $\theta_\delta(0)$ the main diagonal of the covariance matrix $C_{ii}(0)$, directional forgetting factor $\gamma(0), \lambda(0), \nu(0)$, and ρ.

2. Calculating the prediction error from the following expression:

$$e_\delta(k) = y_\delta(k) - \theta_\delta^T \gamma_\delta(k-1) \tag{20}$$

3. Calculating auxiliary variables from the following relations:

$$\begin{aligned} &\xi(k-1) = \varphi_\delta^T(k-1) C(k-1) \phi_\delta(k-1) \\ &\nu(k) = \gamma(k)[\nu(k-1)+1] \\ &\eta(k) = \frac{e^2(k)}{\lambda(k)} \\ &\lambda(k) = \gamma(k)\left[\lambda(k-1) + \frac{\delta(k-1)}{1+\xi(k-1)}\right] \end{aligned} \tag{21}$$

4. Calculating the directional forgetting factor

$$\begin{aligned} &\gamma(k) = \{1 + (1+\rho)[\ln(1+\xi(k-1))] + \\ &\left[\frac{(\nu(k-1)+1)\eta(k-1)}{1+\xi(k-1)+\eta(k-1)} - 1\right] \frac{\xi(k-1)}{1+\xi(k-1)}\Bigg\}^{-1} \end{aligned} \tag{22}$$

5. Calculating the auxiliary variable

$$\varepsilon(k) = \gamma(k) - \frac{1-\gamma(k)}{\xi(k-1)} \tag{23}$$

6. If $\xi(k-1) > 0$ the covariance matrix is actualized using the expression:

$$C(k) = C(k-1) - \frac{C(k-1)\varphi_\delta(k-1)\varphi_\delta^T(k-1)C(k-1)}{\varepsilon^{-1}(k) + \xi(k-1)} \tag{24}$$

else, if

$\xi(k-1) = 0$,

then

C(k)=C(k-1).

7. The actualization of the parameter estimates the vector:

$$\theta_\delta(k) = \theta_\delta(k-1) + \frac{C(k-1)\varphi_\delta(k-1)}{1+\xi(k-1)} e(k-1) \tag{25}$$

The start-up conditions are chosen according to *a priori* information.

4. ADAPTIVE FEEDBACK REGULATION

4.1. Critical proportional gain

Let the system be described by the single input - single output (SISO) δ-model in the form of the discrete equation

$$\begin{aligned} y_\delta(\delta) &= -a_1 y_\delta(k-1) - a_2 y_\delta(k-2) \\ &+ b_1 u_\delta(k-1) + b_2 u_\delta(k-2) \end{aligned} \tag{26}$$

which is controlled by the proportional controller

$H_r(s)=K_P$

The command is:

$$u_\delta(k) = K_P[v_\delta(k) - y_\delta(k)] \tag{27}$$

Substituting equation (27) into equation (26) we obtain the closed control loop equation

$$\begin{aligned} &y_\delta(\delta) + (a_1 + b_1K_P)y_\delta(k-1) + (a_2 + b_2K_P)y_\delta(k-2) = \\ &b_1K_P v_\delta(k-1) + b_2K_P v_\delta(k-2) \end{aligned} \tag{28}$$

where

$$\begin{aligned} v_\delta(k-1) &= \frac{v(k-1) - v(k-2)}{T_S} \\ v_\delta(k-2) &= v(k-2) \end{aligned} \tag{29}$$

One obtain the following transfer function of the closed control loop

$$G(\delta) = \frac{y(\delta)}{v(\delta)} = \frac{b_1K_P\delta + b_2K_P}{\delta^2 + (a_1 + b_1K_P)\delta + (a_2 + b_2K_P)} \tag{30}$$

and by substituting

$a_1 + b_1K_P = b \;\; a_2 + b_2K_P = c$

one obtain the transfer function in the form

$$G(\delta) = \frac{b_1K_P\delta + b_2K_P}{\delta^2 + b\delta + c} \tag{31}$$

The denominator of the transfer function (31) is the characteristic polynomial

$$D(\delta) = \delta^2 + b\delta + c \tag{32}$$

whose poles determine the behavior of the closed control loop.

There are three possibilities for pole placement of the characteristic polynomial on the circle:

1) $D(\delta) < 0 \Rightarrow \delta_{1,2} = \alpha \pm j\beta, \Rightarrow$
$D(\delta) = \delta^2 - 2\alpha\delta + \alpha^2 + \beta^2$

$$\alpha^2 + \beta^2 = \gamma^2 = \alpha\frac{2}{T}$$

$$\delta^2 + (a_1 + b_1K_P)\delta + (a_2 + b_2K_P) = \delta^2 - 2\alpha\delta - \alpha\frac{2}{T} \tag{33}$$

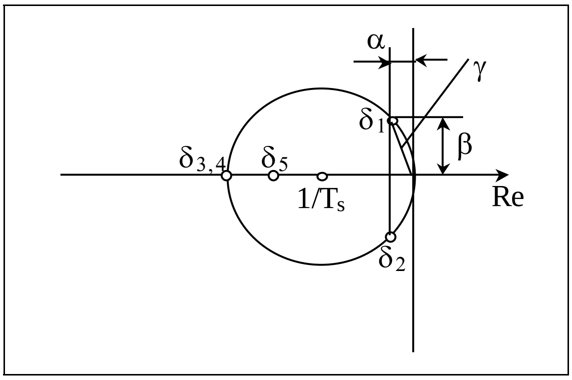

Fig.1 Pole placement of the characteristic polynomial on the circle

One obtain two equation:

$$\begin{cases} a_1 + b_1 K_P = -2\alpha \\ a_2 + b_2 K_P = -\alpha \dfrac{2}{T_S} \end{cases} \Rightarrow \begin{cases} K_P = \dfrac{a_1 - a_2 T_S}{b_2 T_S - b_1} \\ \alpha = -\dfrac{a_1 + b_1 K_P}{2} \end{cases} \qquad (34)$$

2) $D(\delta) = 0 \Rightarrow \delta_{3,4} = -\dfrac{2}{T_S}$

$$\delta^2 + (a_1 + b_1 K_P)\delta + (a_2 + b_2 K_P) = \delta^2 - \delta \frac{4}{T_S} - \frac{4}{T_S^2}$$

One obtain two equation:

$$\begin{cases} a_1 + b_1 K_P = \dfrac{4}{T_S} \\ a_2 + b_2 K_P = \dfrac{4}{T_S^2} \end{cases} \qquad (35)$$

3) $D(\delta) > 0 \Rightarrow \delta_5 = -\dfrac{2}{T_S}, \delta_6 = \lambda$

One obtain two equation:

$$\begin{cases} a_1 + b_1 K_P = \dfrac{2}{T_S} - \lambda \\ a_2 + b_2 K_P = -\dfrac{2\lambda}{T_S} \end{cases} \Rightarrow K_P = \frac{4 - 2T_S a_1 + T_S^2 a_2}{2T_S b_1 - T_S^2 b_2} \qquad (36)$$

4.2. Critical period of oscillation

The denominator can be expressed in the following form :

$$D = \delta^2 + \frac{2(1-\cos\omega T_S)}{T_S}\delta + \frac{2(1-\cos\omega T_S)}{T_S^2} \qquad (37)$$

where

$$\frac{2(1-\cos\omega T_S)}{T_S} = b, \; \frac{2(1-\cos\omega T_S)}{T_S^2} = c \qquad (38)$$

We can derive the relation to calculate the critical frequency from the first or second equation (38). By substituting b and c into (38) one obtain the following expressions:

$$\omega_c = \frac{1}{T_S}\arccos\left(\frac{2 - a_1 T_S - b_1 K_P T_S}{2}\right),$$

$$\omega_c = \frac{1}{T_S}\arccos\left(\frac{2 - a_2 T_S^2 - b_2 K_P T_S^2}{2}\right) \qquad (39)$$

and for the critical period of oscillations T_c holds

$$T_c = \frac{2\pi}{\omega_c} \qquad (40)$$

4.3. Adaptive scheme

For practical use the recurrent control algorithms which compute the actual value of the controller output u(k) from the previous value u(k-1) and from compensation increment seem to be suitable:

$$u(k)=u(k-1)+b_0e(k)+ b_1e(k-1)+ b_2e(k-2)$$

where controller parameters are:

$$b_0, b_1, b_2=f(K_P, T_d, T_i, T).$$

The PID controller designed by Takashi et al. has been modified because the amplitude changes of the controller output u(k) are further reduced if the reference variable v(k) is only present in the integration form. The change of the process output variable y(k) on the reference value is then mainly controlled through the integral component

$$u(k) = K_P\{-y(k) + y(k-1) + \frac{T_S}{T_i}(v(k) - y(k)) + \frac{T_d}{T_S}[2y(k-1) - y(k) - y(k-2)]\} + u(k-1) \qquad (41)$$

where

$$K_P = 0.6K_{Pc}\left(1 - \frac{T_S}{T_C}\right), T_i = \frac{K_P T_C}{1.2K_{Pc}}, T_d = \frac{3K_{Pc}T_C}{40K_P}$$

In the following figure is presented the adaptation scheme:

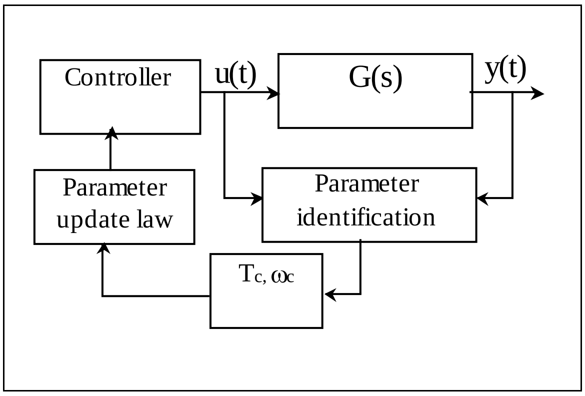

Fig. 2 Adaptation scheme

5. SIMULATION EXAMPLE

As examples of verification by computer simulation, two proportional second-order systems were used:

$$H_1(s) = \frac{1}{s^2 + 3s + 2} \quad \text{and} \quad H_2(s) = \frac{1}{s^2 + 0.5s + 2}$$

The following figures show the simulation results:

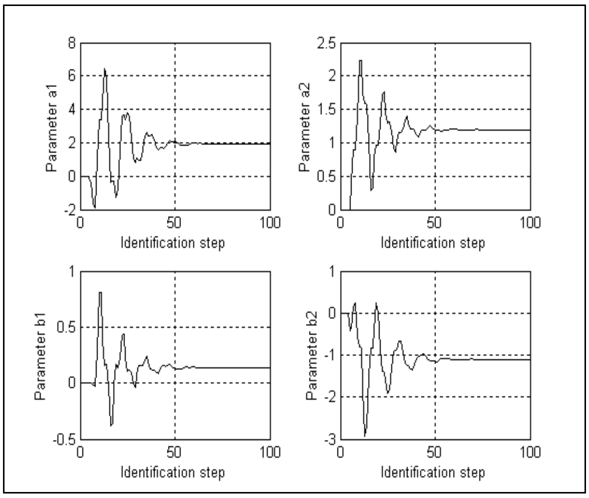

Fig. 3 Convergence of parameter estimates for $H_1(s)$

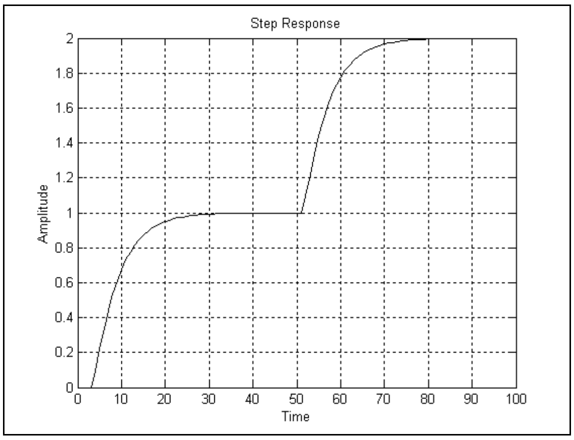

Fig. 4 Step response for $H_1(s)$

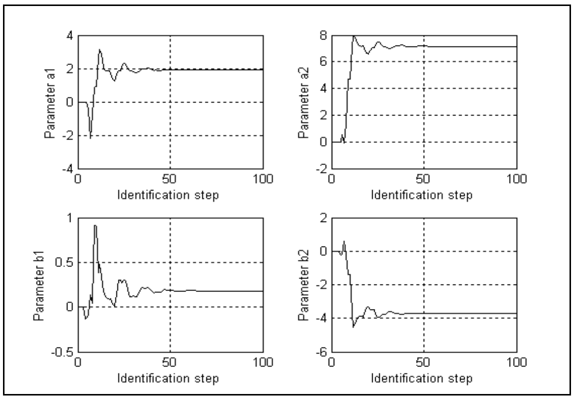

Fig. 5 Convergence of parameter estimates for $H_2(s)$

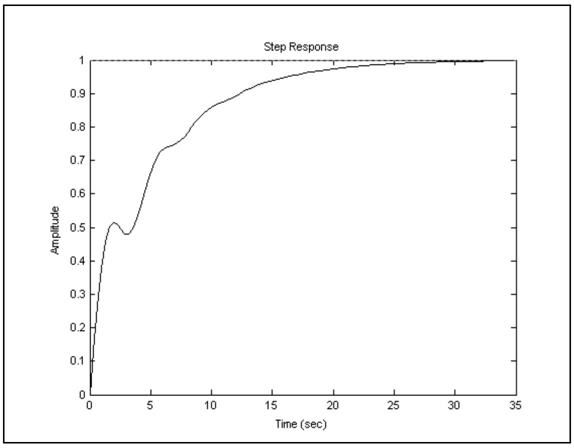

Fig. 4 Step response for $H_2(s)$

REFERENCES

[1] Middleton R.H., Goodwin G.C., *Digital Control and Estimation – A Unified Approach*. Prentice-Hall, NJ, 1990.

Ogata, K. *Modern Control Engineering*, 3rd edition, Prentice Hall, 1997

[2] Feuer A., Goodwin G.C., *Sampling in Digital Signal Processing and Control.* Birkhäuser: Boston, 1996.

[3] Kulhavy R., *Restricted exponential in real time identification.* Automatica 1987, 23, 586-600.

[4] Isermann R. *Digital Control Systems*. Springer: Berlin, 1989.

[5] Ogata, K. *Modern Control Engineering*, 3rd edition, Prentice Hall, 1997