Modelling the Cutting Process using Response Surface Methodology and Artificial Intelligence Approach: a Comparative Study for Milling

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Abstract: The paper deals with the milling force modelling concerning average and maximum forces for a set of four materials. The main purpose of the work is to obtain a function of three variables (cutting depth, feed per toot and cutting speed) using response surface methodology (RSM), and artificial intelligence approach (AI). A new method based on hybrid multiple regression (HMR) using RSM, and also a novel algorithm are proposed. In AI, determination of the optimal neural network of fuzzy neural network is an important aspect when we use these models for prediction. Differential genetic algorithms for variable length genotype are proposed to optimize simultaneously both structure and parameters for AI structures. A comparative study based on performance analysis is made also.

Keywords: cutting forces, regression, response surface methodology, neural network, novel algorithm.

1. INTRODUCTION

The analysis of cutting forces in milling process has been a subject of research for decades, as shown by (Zhigang, 2005; Li *et al.,* 1999; Wilson *et al.*, 1990; Zheng *et al.,* 1998). A large amount of work has been carried out in the area of force modelling in milling process. The measurement and prediction of forces in arbitrary work conditions is a condition for a good monitoring and optimization of the process. The objective optimizations conduct to planning operations that usually take into account the productivity, quality, cost and time. Also, a good model can estimate directly or indirectly some process parameters like tool wear and life, surface finish etc.

Vibrations due to milling process can be the source of deterioration of the machine accuracy. Accurate modelling of forces can be used to determine the prediction of machine performance along with determination of the machining parameters that affect this performance. The traditional approach predict the cutting forces using cutting coefficients identified through empirical curve fit to measured average and minimum (maximum) milling forces.

An extensive work is done by (Zhigang, 2005). After a presentation of the main methods used in high speed milling of titanium alloys, dynamic programming, geometric programming, simulated annealing and genetic algorithm, (Zhigang, 2005) proposes a parallel genetic simulated annealing (PSGA). The problem of optimization is a multi-objective one, the minimum production time and the minimum production cost. The solutions with higher fitness are selected towards the Pareto-optimal region. Genetic algorithms are attractive solutions for cutting forces modelling during milling process as shown by (Gallova, 2009; Wang *et al.,* 2004; Milfelner *et al.,* 2005). In her paper (Gallova, 2009) proposed an analytic fuzzy logic controller with fuzzy parameters optimized by genetic algorithm for cutting speed on total average cutting force fitness function. Genetic algorithm (GA) and simulated annealing (SA) have been applied to fitness objective, minimum production time being obtained by (Wang *et al.,* 2004; Milfelner *et al.,* 2005) use a new method- the genetic programming. Using principles of genetic programming, an analytic function for cutting forces has been found by (Myers and Montgomery, 1995). Genetic programming can be efficiently but in many cases, the formulas that achieve a predefined fitting error can be very long and very complex.

(Myers and Montgomery, 1995; Patwari *et al.,* 2009; Noordin *et al.,* 2004; Kumar *et al.,* 2012) showed in their works that response surface methodology (RSM) is an attractive method to give solutions for prediction of cutting forces from experimental data. Most of the models can achieve a desired performance using linear models. Quadratic models are very rarely needed in known application of force cutting predictions. RSM is connected with design of experiments and ANOVA analysis that predicts the influence between parameters, as can be found in papers by (Orhan *et al.,* 2011; Fnides *et al.,* 2011; Ginta and Amin, 2012; Rosales *et al.,* 2012) Taguchi's method is a very used one in design of experiments.

Some models from artificial intelligence have been proved to be good approximators for cutting forces in milling process. Neural networks (Szecsi, 1999; Radhakrishnan and Nandan, 2005; Raksiri and Parnichkun, 2004) and fuzzy neural networks that use Takagi-Sugeno inference rules (Hossain and Ahmad, 2005; Nandi, 2012) are good solution for approximation and prediction.

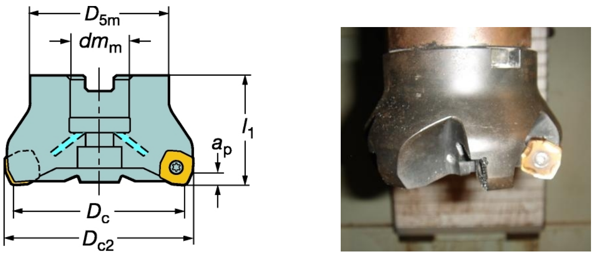
In a comparative study, some of models that have been used in literature in order to describe the cutting forces have been shortly reviewed by Fang N. and Wu Q.

2. CUTTING PROCESS AND CUTTING FORCES

The milling investigations were carried out on a CNC machining centre with three axes, FIRST MCV 300. The measurements were made with a Kistler dynamometer fixed on the machine tool table; the workpiece was mounted on the dynamometer plate. The signals received by the dynamometer were transmitted through the amplifier, Multichannel Type 5070, to the acquisition board (PCIM-DAS1602/16) installed on the PC and processed by the program used for data acquisition, DynoWare Type 2 825.

The materials used for the experimental tests were initially tested in order to establish the material composition and hardness. Two types of tests, namely a spectrometry test for establishing the chemical composition, and a test of hardness measuring were performed. Spectrometry test was performed using a spark optical emission spectrometer named SPECTROMAXx. The tests for hardness measurements were performed using a Shimadzu HSV30 device.

For achieving the cutting tests, two types of milling cutters were used, namely CoroMill R 365-080Q27-S15M having



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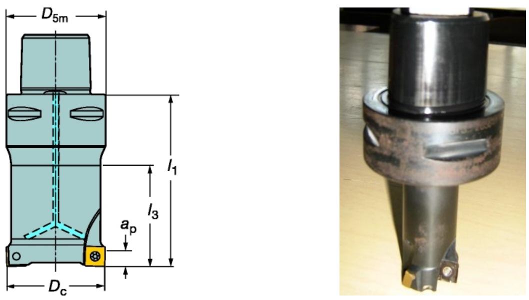

b

Fig. 1. Cutting tools used in machining tests:

a. R365−080Q27−S15M (image from SanvikCoromant catalogue and actual used in machining);

b. 490−025C5−08M ((image from SanvikCoromant catalogue and actual used in machining).

the diameter of 80 mm, and 490−025C5−08M having the diameter of 25 mm from SandvikCoromant (Figs. 1a and 1b, respectively, and Table 1). The used inserts are the following (Table 2):

- for machining improved steel – R365−1505ZNE−PM 4230;
- for machining Al 7178 – 490R-08T308M-PL 1030;
- for machining cast iron – 490R-08T308M-KM 1020;
- for machining Ti grade 3 – 490R-08T308M-PM 1030;

Table 1. Cutting tool technical data.

R365−080Q27−S15M		490−025C5−08M	
Parameter	**Value**	**Parameter**	**Value**
Weight	**1.3**	**Weight**	**0.6**
D_c	80	D_c	25
D_{c2}	86.7	κ_r	90
D_{5m}	64	D_{5m}	50
d_{mm}	27	l_1	75
l_1	50	l_3	50
$a_{p\ max}$	6	$a_{p\ max}$	5.5
Max_rpm	11 500	*Max_rpm*	28 000
κ_r	65	Z_n	3
		Z_c	3

Table 2. Cutting tool inserts technical data.

R365-1505ZNE -PM 4230		490R-08T308M -PL 1030		490R-8T308M -KM 1020		490R-08T308M -PM 1030	
	Value		**Value**		**Value**		**Value**
Weight	0.014	Weight	0.002	Weight	0.002	Weight	0.002
Size	15	Size	0.8	Size	0.8	Size	0.8
ap_max	6	la	5.6	la	5.6	la	5.6
iC	15	s	3.3	s	3.3	s	3.3
la	6.4	bs	1.2	bs	0.85	bs	1.2
s	5.66	rε	0.8	rε	1.2	rε	0.8
			b_s	1.5			

Table 3. Experimental plan (Taguchi method).

Exp. No.	Parameter 1	Parameter 2	Parameter 3
1	1	1	1
2	1	2	2
3	1	3	3
4	1	4	4
5	2	1	2
6	2	2	1
7	2	3	4
8	2	4	3
9	3	1	3
10	3	2	4
11	3	3	1
12	3	4	2
13	4	1	4
14	4	2	3
15	4	3	2
16	4	4	1

The experimental plan was chosen according to Taguchi method (Taguchi and Konishi, 1987; Yang and Tarng, 1998; Pawade and Joshi, 2011). Taguchi method in industrial practice is a method that seeks to help researchers to obtain faster and cheaper the best results by performing the experiments. This method is based on the orthogonal factorial plan (Fig. 2 in Appendix A.). The method consists in defining the process objective, or more precisely defining a target value which measures the process performance, determining the design parameters which affect the process, and establishing the levels of variation of these parameters.

After setting the parameters and their levels, the suitable array of experiences from the orthogonal matrix can be also chosen, Fig. 2. In our case, the experimental matrix has three parameters, each parameter having 4 values, resulting 16 tests, as shown in Table 3.

The cutting parameters and average and maximum measured cutting forces for four materials (improved C 45 steel; aluminium alloy Al 7178; titanium grade 3; cast iron) are presented in Tables 4–7 (shown in Appendix A and B).

In many cases, when Response Surface Method (RSM) is used to predict the cutting forces, many authors rescaled the forces in order to avoid too much difference of magnitude order among coefficients that correspond to variables. Also, rescaling force offer the possibility to translate the modelled cutting force in a range useful RSM model.

3. RESPONSE SURFACE METHOD (RSM)

According to Sandvik Coromant, we have three main parameters of machinability assessment: (1) tool life; (2) surface finish; and (3) cutting force. In what follows we will discuss the main mathematical models used in prediction of cutting forces (Fig. 3).

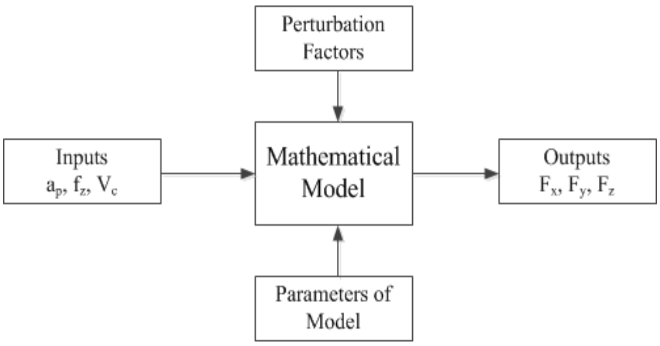

Fig. 3. Mathematical model of cutting forces prediction.

Response surface methodology (RSM) is a collection of mathematical techniques used for model construction. RSM use design of experiments (DoE) for independent input variables in order to optimize the response of the model (output variable). Most applications that use RSM involve more than one output. In our case we have three inputs $X = [a_p, f_z, V_c]$ and three outputs $Y = [F_x, F_y, F_z]$.

The fit quality model is measured by comparing the response of the model versus experimental data that is the output variable. Two measures are commonly used to measure this performance: RMSE (Root Mean Square Error) and R^2 (coefficient of determination).

In RSM, the modelling method is an approximating model between response y and input independent variables $\xi_1, \xi_2, \ldots, \xi_n$ (natural variables because they are measured in natural units of measurement):

$$y = f(\xi_1, \xi_2, \ldots, \xi_n) + \varepsilon \tag{1}$$

where ε represents the errors due to approximation of the model. In most RSM work, it is most convenient to deal with coded variables. The coded variables $X_1, X_2, \ldots, X_n$ are dimensionless with mean zero and the same standard deviation.

$$h = f(X_1, X_2, \ldots, X_n) \tag{2}$$

Cutting force model for end milling in terms of the parameters can be expressed in general terms using generalized Taylor's Equation. This model is used in most of the research papers that deal with this subject.

$$F = \alpha_0 \cdot a_p^{\alpha_1} \cdot f_z^{\alpha_2} \cdot V_c^{\alpha_{31}} \tag{3}$$

In the equation above, the variables are a_p (axial depth of cut [mm]), f_z (feed per tooth [mm/tooth]), V_c (cutting speed [m/min]), F (force (F_x, F_y, F_z)), and $\{\alpha_0, \alpha_1, \alpha_2, \alpha_3\}$ are the constants of the model that must be determined.

If we made a logarithm for both sides of the formula (3), we can transform the problem in a linear regression one. The new variables are $X_k = \log(x_k)$ and the constants of the model can be determined by one of the optimization method that minimizes the error (e.g. Least Mean Square-LMS). The outputs are given by $h_x = \log(F_x)$, $h_y = \log(F_y)$, and $h_z = \log(F_z)$.

The most common multiple regression models used in modelling are linear, quadratic and cubic. If interaction among variables is considered, the models are given by:

$$h = b_0 + \sum_{i=1}^{n} b_i \cdot x_i \tag{4}$$

$$h = b_0 + \sum_{i=1}^{n} b_i \cdot x_i + \sum_{i=1}^{n} b_{ii} \cdot x_i^2 + \sum_{\substack{i=1 \\ i<j}}^{n-1} \sum_{\substack{j=2 \\ i\neq j}}^{n} b_{ij} \cdot x_i \cdot x_j \tag{5}$$

$$h = b_0 + \sum_{i=1}^{n} b_i \cdot x_i + \sum_{i=1}^{n} b_{ii} \cdot x_i^2 + \sum_{i=1}^{n} b_{iii} \cdot x_i^3 + \sum_{\substack{i=1 \\ i<j}}^{n-1} \sum_{\substack{j=2 \\ i\neq j}}^{n} b_{ij} \cdot x_i \cdot x_j + \sum_{\substack{i=1 \\ i<j}}^{n-2} \sum_{\substack{j=2 \\ j<k}}^{n-1} \sum_{\substack{k=3 \\ i\neq j \\ j\neq k \\ i\neq k}}^{n} b_{ijk} \cdot x_i \cdot x_j \cdot x_k \tag{6}$$

In some cases, the curvature in the response surface is strong enough that linear model is inadequate, so a model of higher order is required. The model with no interactions of first order is simple enough but models of greater order become more complicated. In the equation (6), if we take into account the interactions among higher order of variables, we must append additional factors (e.g., for three input variables, we take into account the factors $x_1x_2^2$, $x_1x_3^2$, $x_2x_3^2$, and so one). We will denote this model by cube-expanded (*cube-e*). The numbers of coefficients (parameters that must be determined in multiple regressions) are {4, 10, 14, and 20} according to model: linear, quadratic, cubic, and cubic-extended respectively. In the equations (4)–(6), the terms {b_i, b_{ij}, b_{ijj},...} are the parameters of the model that must be identified using one of the optimization algorithm used to minimize the output error in fitting the model according with experimental data.

We must remark that it is not very often the case when a cubic order is necessary. There are not so many examples but they exist and must be taken into consideration. Also, the level of interaction among variable is a subject of discussion. In a practical case, if we consider all the levels of interaction among three variables, the linear, quadratic and cubic models have the following representation:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 \tag{7}$$

$$\hat{y} = linear\ model + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 \tag{8}$$

$$\hat{y} = quadratic\ model + b_{123}x_1x_2x_3 + b_{112}x_1^2x_2 + b_{113}x_1^2x_3 + + b_{122}x_1x_2^2 + b_{133}x_1x_3^2 + b_{223}x_2^2x_3 + b_{233}x_2x_3^2 + b_{111}x_1^3 + + b_{222}x_2^3 + b_{333}x_3^3 \tag{9}$$

The models reflect the interaction among variable and a useful analysis of these interactions can be made by using an appropriate statistical method (e.g. ANOVA).

The ANOVA method will give information about the contribution of each term to the output along with significant statistical values.

Different choices model of DoE have a subsequent choice of assignment levels for experimental input data. The common choice is five level assignment {–1, $-\sqrt{2}$, 0, $+\sqrt{2}$, +1}, three level assignment {–1, 0, +1} and two level assignment {–1, +1}.

$$X_k = 2\cdot\frac{\log\xi_k - \log\xi_{k,\max}}{\log\xi_{k,\max} - \log\xi_{k,\min}} + 1 \tag{10}$$

In the equation above, ξ_k is the measured value, X_k is the coded value and min/max indices represent the minimum and maximum value of ξ_k input variable. By equation (7), the input experimental values are mapped into [–1, +1] range. The classification is made using R^2 factor: (i) most favourable case, the highest R^2 value; (ii) and the most unfavourable case, the lowest R^2 value.

Other fitness measures as RMSE, maximum absolute error can be used also as performance measure. We have been chosen R^2 values as measure of performance because of requirement of fitting model with experimental data. The error between predicted values and experimental values must be minimized in each point where experimental data is available.

We can see from Table 8 that an acceptable performance can be achieved for cubic model. The linear and quadratic models are not acceptable because of low fitting in the most unfavourable case when R^2 is around 0.3.

Two algorithms have been used in order to identify the parameters of the multiple regression models: trust-region-reflective algorithm and Levenberg-Marquardt algorithm.

Table 8. Performance measures for multiple regression models.

Material	Model cutting forces	Most favourable case		Most unfavourable case	
		R^2	RMSE	R^2	RMSE
Medium-carbon steel C45	Linear	0.8400	0.1442	0.4771	0.1607
	Quadratic	0.9749	0.0572	0.6294	0.1353
	Cubic	>0.9999	$8.0771\cdot10^{-14}$	>0.9999	$8.0771\cdot10^{-14}$
Titanium grade 3	Linear	0.8264	0.1143	0.2462	0.5334
	Quadratic	0.9368	0.0651	0.5018	0.3172
	Cubic	>0.9999	$1.2739\cdot10^{-13}$	>0.9999	$1.2739\cdot10^{-13}$
Al 7178	Linear	0.5480	0.3994	0.2038	0.1912
	Quadratic	0.8557	0.2257	0.4930	0.2677
	Cubic	>0.9999	$3.3063\cdot10^{-13}$	>0.9999	$3.3063\cdot10^{-13}$
Cast iron	Linear	0.7508	0.1558	0.1910	0.0940
	Quadratic	0.8804	0.1079	0.3637	0.0834
	Cubic	>0.9999	$6.7906\cdot10^{-14}$	>0.9999	$6.7906\cdot10^{-14}$

Table 9. Coefficients for cubic model, average and maximum measured cutting forces [N] according to experimental plan for medium-carbon C45.

Coeff.	Average measured cutting forces [N]			Max. measured cutting forces [N]		
	F_x	F_y	F_z	F_x	F_y	F_z
b_0	3.3711	4.7851	3.1005	5.1151	5.3742	6.4480
b_1	1.2039	-0.2361	1.6502	1.7075	-1.9625	1.4283
b_2	-0.3749	0.5459	-1.9917	2.0243	-2.8092	-0.7032
b_3	0.7552	-0.4251	0.2475	1.7734	-2.5620	-0.1185
b_{12}	2.5475	-0.1492	-0.7371	2.7477	-4.5888	-0.8684
b_{13}	0.4041	0.0997	3.1889	3.2957	-4.2785	-1.7296
b_{23}	-0.6279	-0.9690	1.9865	2.1009	-3.8051	2.0644
b_{11}	0.8753	-0.0587	1.7556	0.6683	-0.5886	-1.7534
b_{22}	0.5603	0.3498	1.7209	0.5458	-0.7288	-1.2510
b_{33}	0.1598	-0.1995	1.9664	0.7127	-0.7863	-1.4901
b_{123}	1.2396	-0.0933	3.9689	2.0055	-1.8776	-2.8709
b_{112}	0.7069	-0.1621	1.3982	1.2363	-2.0087	-1.0219
b_{113}	2.0375	0.2808	-0.5067	1.1868	1.6016	-0.3792
b_{122}	-0.8231	-0.5017	1.0514	0.8053	-1.7270	1.1841
b_{133}	-0.2466	-0.4862	0.5552	0.6010	-1.5356	0.8512
b_{223}	1.0341	0.0509	-0.8093	1.0341	-1.9362	-0.5691
b_{233}	-0.4061	0.3414	1.6203	1.5079	-1.4949	-0.9273
b_{111}	0.2938	0.7658	-1.2194	-0.6907	1.7941	-1.2916
b_{222}	0.7202	0.6134	-1.9880	-1.3737	2.1968	0.8471
b_{333}	-1.1251	0.2718	0.3434	-1.1273	1.6922	0.1622

It is known that any finding of the global minimum (maximum) of one function using iterative optimization algorithms depends more or less on the starting point (the initial value of parameters). Thereupon, the initial value set can be a condition to find a local minimum that can be a global minimum for entire domain of definition.

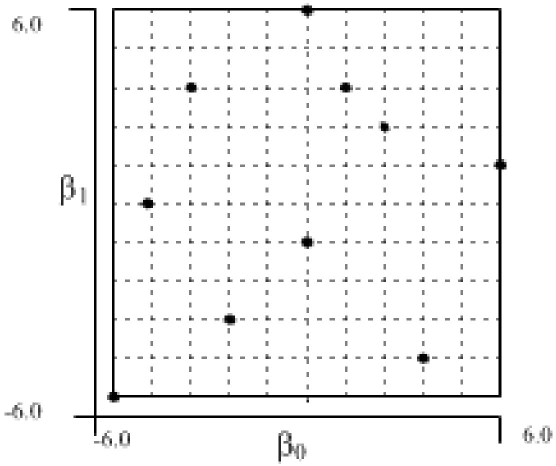

Fig. 4. Latin hypercube sampling (10 points located at random).

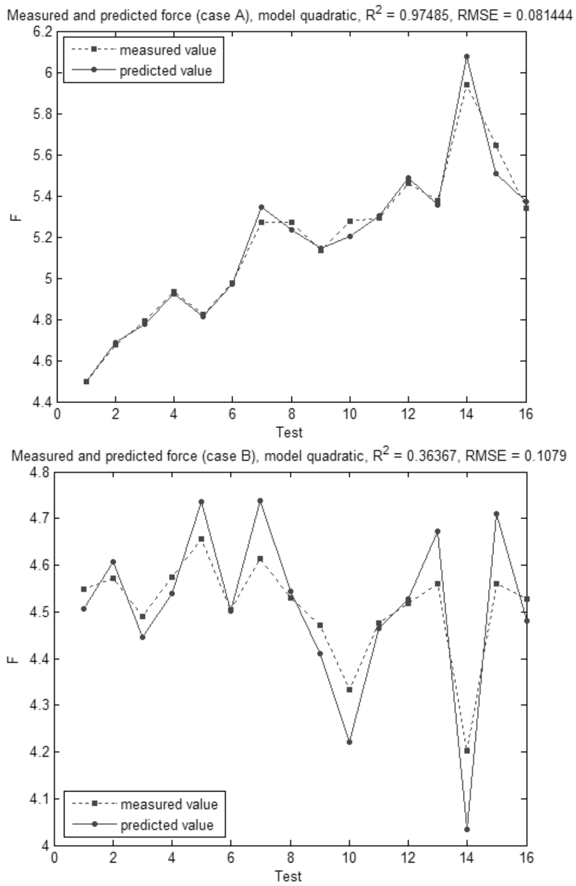

Fig. 5. Most favourable case (case A), and most unfavourable case (case B), model quadratic.

We estimate the maximum possible contribution given by one parameter to output function (pv_{max}). We set the initial set of values in the search space in a hypercube where each dimension is in the range [pv_{max}, $+pv_{max}$]. The level of granulation is set in a heuristic approach based on the analysis of experimental data (e.g. grid with 1 000 × 1 000 equidistant lines in 2D hypercube, a Latin hypercube (Fig. 4).

The selection of the points can be made in many ways. We use Audze-Eglais and random design. With 10 000 lines on each dimension, the final results have no improvements, so we can conclude with a high degree of confidence that the optimal solutions in Table 9 are the global ones.

The graphic for both (most favourable and most unfavourable) cases over all materials using quadric multiple regression are shown in Fig. 5.

A complete formula for cubic model is given below. The relation using value of coefficients is given, e.g. for steel, average measuring cutting force, F_x predicted ($\log(F_x)$) in equation (12).

$$\begin{aligned}\hat{y} &= b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + \\ &+ b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{123}x_1x_2x_3 + \\ &+ b_{112}x_1^2x_2 + b_{113}x_1^2x_3 + b_{122}x_1x_2^2 + b_{133}x_1x_3^2 + \\ &b_{223}x_2^2x_3 + b_{233}x_2x_3^2 + b_{111}x_1^3 + b_{222}x_2^3 + b_{333}x_3^3 \end{aligned} \tag{11}$$

$$\begin{aligned}\hat{F}_h &= 3.3711 + 1.2039x_1 - 0.3749x_2 + 0.7552x_3 + 2.5475x_1x_2 + \\ &+ 0.4041x_1x_3 - 0.6279x_2x_3 + 0.8753x_1^2 + 0.5603x_2^2 + 0.1598x_3^2 + \\ &+ 1.2396x_1x_2x_3 + 0.7069x_1^2x_2 + 2.0375x_1^2x_3 - 0.8231x_1x_2^2 - \\ &- 0.2466x_1x_3^2 + 1.0341x_2^2x_3 - 0.4061x_2x_3^2 - 0.2938x_1^3 + \\ &+ 0.7202x_2^3 - 1.1251x_3^3 \end{aligned} \tag{12}$$

The formulas are enough complex ones and we need for each force a formula with different coefficients.

4. HYBRID MULTIPLE REGRESSION (HMR) USING RSM

Multiple regressions are statistical techniques for estimating the relationships among several independent input variables and a dependent output predictor variable. In some cases, the model can be complex and can have many interaction terms. In our case, an adequate model can be a cubic one (14 terms) or a better model, namely cubic-extended (20 terms). Our approach proposes a trade-off between the complexity of the multiple regressions (number of terms – number of parameters) and desired performance of the model.

We propose to use a hybrid method that has two levels of prediction. First, a multiple regression is made using a multi-regression model. The second stage, given by a single regression model or a simple function of transformation corrects the error using an adaptive algorithm (e.g. LMS) in order to have an improvement of prediction of cutting forces (Fig. 6). The dimension of error is considered in this case R^2, which must be very close to 1.0.

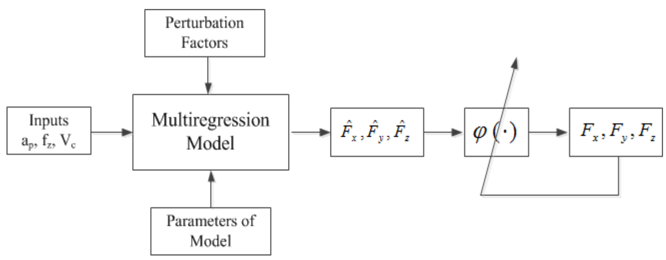

Fig. 6. The schema of Hybrid Multiple Regression (HMR) using RSM.

Table 10. Performance measures for Hybrid Multiple Regression (HMR) using RSM.

Material	Model cutting forces	Most favourable case		Most unfavourable case	
		R^2	RMSE	R^2	RMSE
Medium-carbon steel C45	Linear -HMR	0.9275	0.0970	0.9058	0.0682
	Quadratic -HMR	0.9510	0.0573	0.9423	0.0479
Titanium grade 3	Linear -HMR	0.9275	0.0970	0.9058	0.0682
	Quadratic -HMR	0.9510	0.0573	0.9423	0.0379
Al 7178	Linear -HMR	0.9275	0.0970	0.9058	0.0682
	Quadratic -HMR	0.9510	0.0573	0.9423	0.0579
Cast iron	Linear -HMR	0.9275	0.0970	0.9058	0.0682
	Quadratic -HMR	0.9510	0.0573	0.9423	0.0479

The function of correction (FC) can have several forms in the SISO sense (SISO-Single Input-Single-Output). A good choice can be sigmoidal or polynomial one. The choice of sigmoidal function gives not satisfactory results, so we chose a polynomial one. The quadratic one seems to be most suitable one, because of good trade-off between complexity (degree of the polynomial) and a good prediction at output (the value of R^2 close enough to value 1.0).

$$y = \varphi(x) = a_0 + a_1 x + a_2 x^2 \tag{13}$$

The performances of the model are shown in Table 10. We can see that a quadratic multiple regression model followed by a quadratic polynomial transform (13 terms) has $R^2 \approx 0.9423$ (in the most unfavourable case) is better than a simple multiple regression (14 terms) $R^2 \approx 0.7479$ (cast iron, in the most unfavourable case).

Moreover, if we set the acceptable R^2 threshold for performance $th_{perf} = 0.9$, the linear multiple regression followed by a quadratic polynomial transform (6 terms) has $R^2 \approx 0.9058$ (in the most unfavourable case for cast iron) is better than a simple cubic multiple regression (14 terms) $R^2 \approx 0.7479$ (cast iron, in the most unfavourable case).

In all analyzed cases, the performance of the proposed method (HRM) is better that simple multiple regression method. The graphic for both (most favourable and most unfavourable) cases over all materials using quadratic-HMR are shown in Fig. 7.

We can see from Table 11 that if we accept a moderate level of error (RMSE), we can obtain a simpler model if we use a HRV with linear regression in its first part and a quadratic transformation function in the second part. The model has seven coefficients and the performance is better than the performance obtained by multiple regression using a quadratic model with ten coefficients.

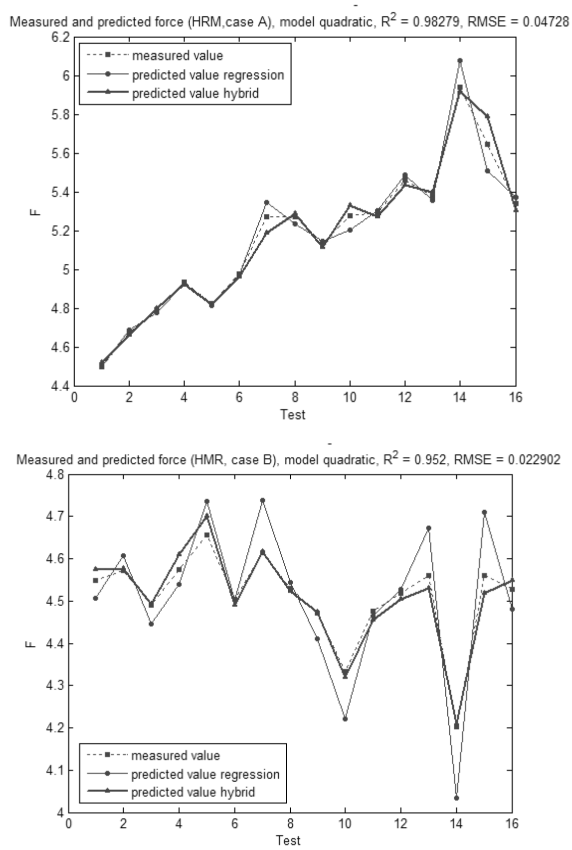

Fig. 7. Most favourable case (case A), and most unfavourable case (case B), model quadratic-HMR.

Table 11. Coefficients for model, average and maximum measured cutting forces [N] according to experimental plan for steel (HRM – linear and cubic predictor).

Coeff. Multi regre-ssion	Average measured cutting forces [N]			Max. measured cutting forces [N]		
	F_x	F_y	F_z	F_x	F_y	F_z
b_0	3.8039	4.5738	4.5740	5.2043	5.3558	4.8523
b_1	0.5962	0.5055	0.1953	0.4112	0.4263	0.2133
b_2	0.1240	0.2099	-0.0539	0.1310	0.1429	-0.0021
b_3	-0.0525	0.1333	0.0373	0.1020	0.0856	0.0052
Coefficients Quadratic polynomial						
a_0	2.1434	1.4114	2.6511	-3.5361	0.0627	0.0627
a_1	0.0819	0.6800	0.2448	2.5475	1.2247	1.2247
a_2	0.0890	0.0018	0.0380	-0.1659	-0.0441	-0.0441

5. ARTIFICIAL INTELLIGENCE (AI) APPROACHES, NEURAL NETWORKS AND FUZZY NEURAL NETWORKS

Neural networks (NN), fuzzy systems and fuzzy neural networks that belong to AI approaches proven to be universal approximators (Haykin, 1999; Jang *et al.,* 1995). In 1989, (Cybenko, 1989) demonstrated, using a Kolmogorov's older result that multilayer feed-forward network with a single hidden layer, which contains finite number of hidden neurons, is a universal approximator. The demonstration has been made for sigmoid activation function. In the most common sense, the universal approximator can approximate any non-linear function with a desired precision if its architecture is large enough. Other NNs architectures have been proved to be also universal approximators: radial basis function (RBF) (Park and Sandberg 1991), recurrent neural networks (RNN) (Schäfer and Zimmermann, 2006) and Kohonen maps. Recurrent Neural Networks (RNN) in various architectures and connection among neurons are the subject of dynamic nonlinear modelling and prediction with very good results. (Pawade and Joshi, 2011) give in their paper a proof for the universal approximation ability of RNNs in state space model form.

(Kosko, 1994) proved in his paper that fuzzy systems can be also universal approximators, even in some cases, the precision cannot however small. Other researches related to approximation and prediction subjects concentrated on fusion between neural networks and fuzzy systems, fuzzy neural networks (FNN) (Buckley and Hayashi, 1994; Castro *et al.,* 2009). FNN is a universal approximator with some precision using a set of rules and membership functions of various shapes (triangular, trapezoidal, Gaussian and bell). A known architecture of FNN is often referred as ANFIS inference system (Jang, 1993). ANFIS and its variants (MANFIS and CANFIS) are also used in many applications as approximator and predictor (Jang *et al.,* 1995; Aytek, 2009).

5.1 Neural network: predictor for cutting forces

We propose to use a NN with a single hidden layer as predictor for output forces. The inputs layer has three neurons corresponding to: a_p (axial depth of cut [mm]), f_z (feed per tooth [mm/tooth]), and V_c (cutting speed [m/min]). The outputs layer has six neurons corresponding to average cutting forces and maximum measured forces (Fig. 8).

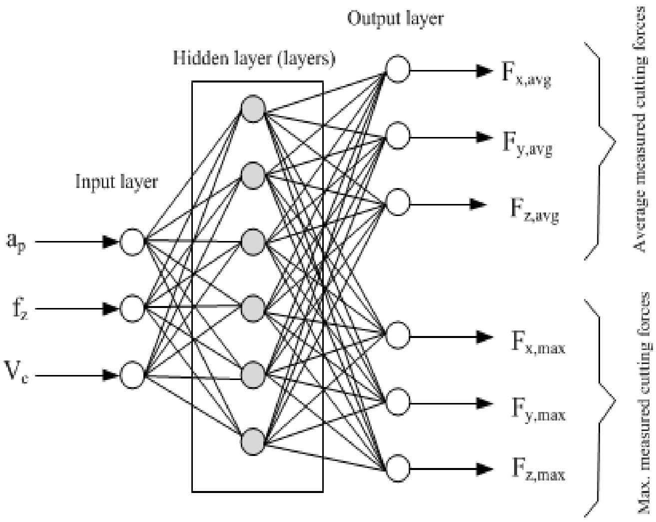

Fig. 8. Multilayer Neural Network for Predictive Forces.

The inputs and outputs are scaled to interval [–1, +1]. The scaling is made mainly to prevent the saturation of sigmoid function. Also, the scaling procedure makes comparable inputs and reduces too much difference of magnitude order among inputs.

The output of neuron in the layer k is given by an activation function that takes into account all the output of neurons of layer k – 1, weighted and biased by neuron parameters.

$$x_i^k = \varphi\left(\sum_{j=1}^{n_j} w_{ij}^k x_j^{k-1} + b_i^k\right) = \varphi(w^T x + b) \quad (14)$$

The hidden layer has sigmoidal activation function (eq. 15) meanwhile the neurons from input and input layer can have linear or sigmoidal activation function.

$$\varphi(x) = 1/(1 + e^{-x+b}) \quad (15)$$

The learning algorithms for multilayer neural networks are a large package of solutions. A most common one, used also by us is the gradient descent algorithm improved by variable learning coefficient.

In Table 12 we present performance measures for multilayer NNs, one hidden layer and 20 neurons in the hidden layer (Average measured cutting forces (AMCF); Max. measured cutting forces (MMCF); All cutting forces (ACF), both AMCF and MMCF). The performance measure is considered overall specifies forces according to column notation. For 20 neurons in the hidden layer, in all the cases, for all cutting forces R^2 > 9.9916618336, a very close value to that show the excellent fitting performance. Moreover, if we increase the numbers of neurons in the hidden layer toward 30 neurons, the difference between R^2 =1.0 (exact fitting) and R2 for NN model is less than 10–30, a very good result.

Table 12. Performance measures (RMSE) for multilayer NNs (one hidden layer).

Material	AMCF	MMCF	ACF
Medium-carbon C45	$2.6\cdot10^{-14}$	$4.0\cdot10^{-14}$	$3.4\cdot10^{-8}$
Titanium grade 3	$8.4\cdot10^{-13}$	$1.4\cdot10^{-11}$	$4.5\cdot10^{-14}$
Aluminium Al 7178	$4.1\cdot10^{-12}$	$1.8\cdot10^{-14}$	$4.9\cdot10^{-13}$
Cast iron	$6.4\cdot10^{-13}$	$4.0\cdot10^{-10}$	$9.1\cdot10^{-12}$

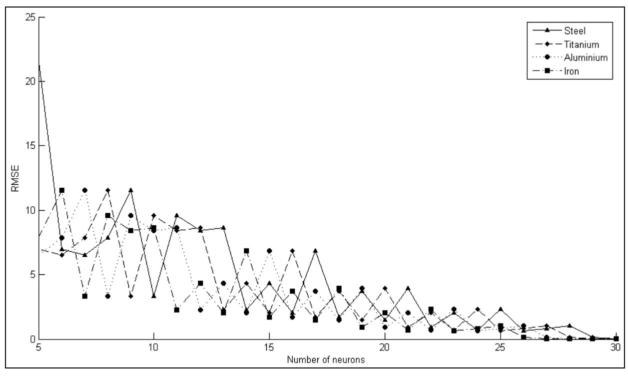

Fig. 9. RMSE vs. number of neurons in the hidden layer (all cutting forces).

The optimization of NN usually refers to a minimal NN architecture that accomplishes the desired requirements. The optimization target in our case is the variable number of neurons in the hidden layer (layers). A plot that shows the performance of the NN (RMSE) vs. number of neurons is presented in Fig. 9. We can see that if the number of neurons in the hidden layer is around 30–35, the performance is excellent for all the materials (steel, titanium, aluminium and cast iron).

We can use genetic algorithm (GA) or differential evolutionary algorithm (DEA) with variable-length genotype, but in our case it is clearly that heuristic approach is more efficient. We can try the optimum performance for 1 to 40 neurons that is a maximum 40 cases. If we use GA or DEA, a minimum population of 20 is a normal approach. After n generations, we must run the learning algorithm for 20 + 20*n individuals, that is for 3 generation we must have at least 80 runs, twice that in the case of heuristic approach.

5.2 Fuzzy Neural network (ANFIS): predictor for cutting forces

In modelling of cutting forces, we have to construct a system that has three inputs and predict at output six values of six variables. ANFIS (Jang, 1993) can have many inputs but it can have only one output. We are in the case of multiple-input, multiple-output ANFIS with nonlinear fuzzy rules. Two solutions are proposed in (Jang, 1993): MANFIS and CANFIS (Figs. 10 and 11).

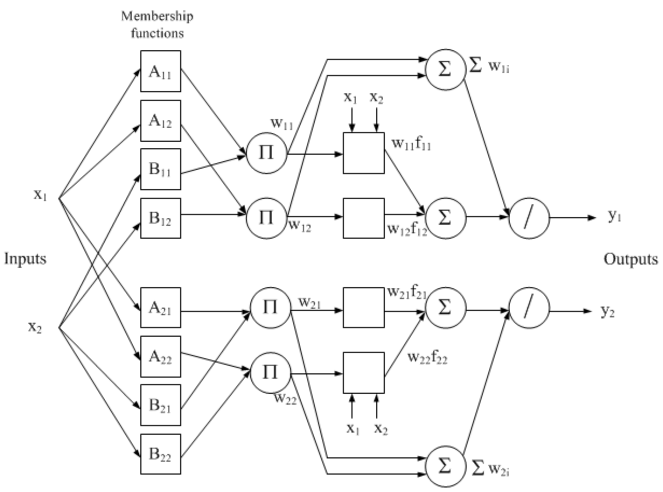

Fig. 10. Two-output MANFIS Architecture.

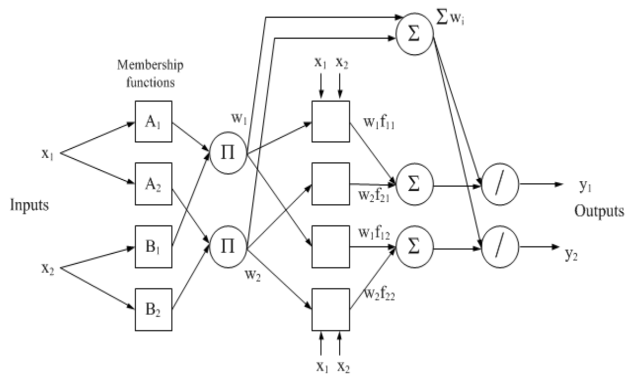

Fig. 11. Two-output CANFIS (Co-Active ANFIS) Architecture.

In the MAFIS case, the architecture has an ANFIS for each output (Fig. 10). Each ANFIS has an independent set of rules. The common part is the input layer shared by all ANFIS architecture. The learning algorithm can be a gradient based on antecedent rules and LMS (Least Mean Squares) for consequent rules.

CANFIS extend the ANFIS in order to produce multiple outputs. In this architecture, fuzzy rules are built with shared membership values. In this approach, different from MANFIS, a correlation between outputs is made via fuzzy rules. In CANFIS, two membership functions are commonly used: general bell and Gaussian. The fuzzy input axon in CANFIS applies MFs to modular network outputs. The learning algorithm can be a gradient based on antecedent rules and LMS (Least Mean Squares) for consequent rules, but also GA can be used to optimize the parameters: membership functions and coefficient for consequent rules.

We propose to use MANFIS architecture in this stage of research. If we have m = 3 inputs, the implemented rules have the form:

$$R_q : \textit{If } x_1 \textit{ is } A_{1i} \textit{ and } x_2 \textit{ is } A_{2j} \textit{ and } \\ x_3 \textit{ is } A_{3k} \textit{ then } y = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 \quad (16)$$

The performance for all the materials, for different shape of membership functions and different number of membership functions are presented in Tables 13–16 (in Appendix C and D). We can conclude from Tables 11–14 (in Appendix C) that MANFIS can approximate each cutting force with a precision of at least 10–13 in all the cases, which represent a very good approximation and a very good result.

6. CONCLUSIONS

A set of solution for approximation of cutting forces generated in the milling process have been proposed to be used for four materials. Experimental results and comparative analysis are performed.

A novel hybrid method is proposed. The method is based on RSM, multi-regression model and nonlinear transformation. The proposed method produces acceptable results with a simpler model in comparison with multi-regression model.

ANOVA analysis can reveal the contribution of each factor and level of interaction for each term. In our case however, we us a simple linear model (with no interaction between input variables) followed by a nonlinear transform that act as a corrector. One of disadvantages of our proposed model is that in the linear regression usage the model does not reveals the interaction among input variables. However, an improved transform that uses also interaction among variables, at different levels using ANOVA analysis can be constructed and this is one of the objectives of the further research.

Modeling of cutting forces is an important issue that is subject of many papers. The modeling method presented in these papers is focused on two directions: mathematical method and algorithms that give the parameters of the model. One of the main directions is to find a mathematical formula that predicts the force as function of input variables. (Milfelner *et al.,* 2005) proposed genetic programming that gives a maximum performance for average percentage deviation 3.83%. Our method is has some superiority because in the best case a value of 3.79% is given but in our case the experimental values suggest that the model is more complex that in (Milfelner *et al.,* 2005) and the usage of their method, after 800 restarts and 400 generations give the best results 7.96%.

Moreover, if we take into account the most unfavourable case our method is a substantially improvement in comparison with power of variables form proposed in (Campatellia and Scippa, 2012) using the Altintas's model (Altintas and Spence, 1991) that give for worst case an error of 34.12% meanwhile for our values are around 7%. A polynomial model of third order has been proposed in [x4]. Even value of errors are not provided numerically by authors, from graphics we can deduce that the maximum of error is around 50% for z direction meanwhile our method is substantially better.

RSM (Response surface methodology) is a method used in many papers. The various classic methods are used to calculate the parameters (LMS, Levenberg-Marquardt, multiple regression, genetic algorithms, etc.) The performance of this method in terms of complexity and performance (R^2 values) has been discussed in this paper. Our method offers a simpler method with a comparable precision (Noordin *et al.,* 2004), e.g. error is between [0.26%, 1.78%]. The same method extended to cubic RSM applied to our experimental data give the error smaller than $1.2739 \cdot 10^{-13}$, a better result that RSM in quadratic form (Wu Baohai *et al.,* 2013). The proposed second method gives an error between [4.6%, 9.7%]. This error is sometime acceptable if the experimental data are dispersed and the simplicity of model prevails to a high level of accuracy.

Also, in the further research we will investigate other suitable transform in order to extend and improve the hybrid model.

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APPENDIX A. FIRST APPENDIX

Number of Levels	Number of Parameters (P)																													
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
2	L4	L4	L8	L8	L8	L8	L12	L12	L12	L12	L16	L16	L16	L16	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32	L32
3	L9	L9	L9	L18	L18	L18	L18	L27	L27	L27	L27	L27	L36	L36	L36	L36	L36	L36	L36	L36	L36	L36								
4	L'16	L'16	L'16	L'16	L'32	L'32	L'32	L'32	L'32																					
5	L25	L25	L25	L25	L25	L50	L50	L50	L50	L50	L50																			

Fig. 2. Orthogonal matrix, Taguchi method.

Table 4. Average and maximum measured cutting forces [N] according to experimental plan for improved steel C 45.

Test	a_p	f_z	v_c	Average measured cutting forces [N]			Max. measured cutting forces [N]		
				F_x	F_y	F_z	F_x	F_y	F_z
1	0.5	0.08	150	-17.4991	-21.8793	107.604	89.6301	83.9539	125.061
2	0.5	0.092	165	-26.5493	-72.7621	64.06962	107.208	140.259	88.1195
3	0.5	0.105	181.5	-32.1633	-57.7599	83.2857	120.438	138.428	109.497
4	0.5	0.121	199.65	-38.6642	-84.2418	78.94135	138.748	166.901	99.9756
5	0.63	0.08	165	-33.92627	-70.5911	78.74032	124.146	159.073	102.539
6	0.63	0.092	150	-37.5438	-89.3923	74.46291	144.699	182.327	108.078
7	0.63	0.105	199.65	-13.64	-93.4657	97.43377	194.55	199.036	116.547
8	0.63	0.121	181.5	-33.2112	-89.5799	83.90361	194.183	219.589	124.741
9	0.78	0.08	181.5	-45.5093	-80.3299	120.2731	170.151	187.134	144.47
10	0.78	0.092	199.65	-54.3224	-108.322	86.64886	195.282	220.505	123.688
11	0.78	0.105	150	-52.7918	-116.676	103.8513	199.036	240.509	130.892
12	0.78	0.121	165	-73.9527	-129.45	117.5149	234.65	279.465	149.918
13	0.97	0.08	199.65	-60.7862	-117.021	122.0013	216.751	254.929	144.241
14	0.97	0.092	181.5	-108.773	-226.366	153.2461	379.623	456.116	197.205
15	0.97	0.105	165	-88.9186	-165.489	115.5525	282.852	333.298	162.094
16	0.97	0.121	150	-72.4686	-121.55	85.3426	208.220	245.587	133.295

Table 5. Average and maximum measured cutting forces [N] according to experimental plan for titanium, grade 3.

Test	a_p	f_z	v_c	Average measured cutting forces [N]			Max. measured cutting forces [N]		
				F_x	F_y	F_z	F_x	F_y	F_z
1	0.5	0.08	115	-27.4031	-47.5447	94.41121	57.9071	36.2091	155.136
2	0.5	0.092	120.5	-28.3029	-69.931	96.67097	70.9534	59.967	130.829
3	0.5	0.105	127	-20.534	-63.7054	89.18986	74.4324	61.7981	117.554
4	0.5	0.121	133	-18.1231	-65.2335	96.84743	73.3337	43.9453	123.367
5	0.63	0.08	120.5	-32.665	-78.5217	105.1152	58.1818	6.82068	154.678
6	0.63	0.092	115	-31.1623	-67.9245	90.3816	73.7	58.0902	136.414
7	0.63	0.105	133	-21.2236	-63.4835	100.8848	94.4824	94.4824	138.245
8	0.63	0.121	127	-31.5312	-79.8318	92.8302	79.1931	83.4045	113.297
9	0.78	0.08	127	-28.0099	-47.7201	87.49449	65.1398	43.6249	127.35
10	0.78	0.092	133	-42.7809	-85.3557	76.24919	47.5159	8.92639	125.29
11	0.78	0.105	115	-34.9478	-72.6504	87.9483	82.7179	81.8481	116.043
12	0.78	0.121	120.5	-48.3246	-101.303	91.64858	88.028	81.5277	115.173
13	0.97	0.08	133	-45.7331	-87.6667	95.56656	83.3588	81.8939	129.959
14	0.97	0.092	127	-38.1995	-73.2111	66.90491	78.3234	83.9539	100.708
15	0.97	0.105	120.5	-53.3428	-90.0497	95.59439	90.5914	153.58	159.805
16	0.97	0.121	115	-46.3203	-111.365	92.59085	102.768	78.8727	126.755

APPENDIX B. SECOND APPENDIX

Table 6. Average and maximum measured cutting forces [N] according to experimental plan for aluminium Al 7178.

Test	a_p	f_z	v_c	Average measured cutting forces [N]			Max. measured cutting forces [N]		
				F_x	F_y	F_z	F_x	F_y	F_z
1	0.5	0.05	420	-9.124751	-15.8437	30.90414	34.10340	22.38460	64.77360
2	0.5	0.07	454	-8.21227	-18.0222	28.64228	26.04680	16.02170	68.84770
3	0.5	0.09	490	-11.1745	-18.102	28.88998	37.17040	11.03210	72.32670
4	0.5	0.12	530	-7.46612	-13.2477	27.07672	27.69470	21.78960	63.58340
5	0.63	0.05	454	-14.3433	-21.2555	33.46762	31.17370	17.80700	70.49560
6	0.63	0.07	420	-9.6639	-14.8163	30.32431	19.91270	9.70459	69.16810
7	0.63	0.09	530	-8.77888	-19.4906	36.41255	19.77540	7.32422	77.08740
8	0.63	0.12	490	-12.085	-15.1863	16.00457	13.91600	9.75037	66.51310
9	0.78	0.05	490	-12.619	-22.5779	28.21857	23.48330	21.46910	71.31960
10	0.78	0.07	530	-29.5003	-41.8598	34.27634	48.43140	18.26480	84.50320
11	0.78	0.09	420	-26.7435	-45.7916	38.09099	54.10770	21.10290	100.25000
12	0.78	0.12	454	-22.0591	-23.5341	22.65422	32.50120	26.82500	63.99540
13	0.97	0.05	530	-17.0059	-19.5923	28.27147	26.87070	56.48800	80.33750
14	0.97	0.07	490	-28.3457	-33.4982	27.49125	26.73340	51.36110	82.53480
15	0.97	0.09	454	-30.0954	-56.0964	41.64632	50.85750	28.38130	101.53200
16	0.97	0.12	420	-4.56931	-11.9227	27.7821	51.81880	53.92460	78.73540

Table 7. Average and maximum measured cutting forces [N] according to experimental plan for cast iron.

Test	a_p	f_z	v_c	Average measured cutting forces [N]			Max. measured cutting forces [N]		
				F_x	F_y	F_z	F_x	F_y	F_z
1	0.5	0.1	220	-23.1829	-78.5693	141.7407	153.717	45.0897	184.021
2	0.5	0.125	237	-21.1778	-107.762	123.671	202.286	42.6178	192.764
3	0.5	0.15	256	-13.7843	-78.4635	161.8423	132.523	30.5786	234.238
4	0.5	0.19	277	-26.3275	-89.8774	185.5072	138.565	48.3398	242.569
5	0.63	0.1	237	-34.1529	-69.5191	201.9195	140.259	138.702	248.566
6	0.63	0.125	220	-37.3887	-89.5351	192.1092	157.242	105.927	47.009
7	0.63	0.15	277	-47.173	-63.763	178.4504	132.385	97.7325	332.794
8	0.63	0.19	256	-50.34	-118.589	187.4753	150.375	25.2228	279.785
9	0.78	0.1	256	-47.4948	-75.6719	180.4223	108.81	87.3871	243.576
10	0.78	0.125	277	-40.8451	-61.9521	138.2238	104.416	120.3	218.674
11	0.78	0.15	220	-42.3398	-114.598	211.379	197.617	100.708	307.297
12	0.78	0.19	237	-47.9138	-124.825	207.469	222.336	88.2111	308.807
13	0.97	0.1	277	-47.9736	-101.807	186.6927	87.1124	23.3002	249.344
14	0.97	0.125	256	-40.2515	-115.853	175.6017	102.859	61.0199	296.86
15	0.97	0.15	237	-58.9753	-141.792	194.2749	193.863	94.9402	290.131
16	0.97	0.19	220	-58.0231	-134.662	208.5296	209.015	203.201	337.601

APPENDIX C. THIRD APPENDIX

Table 13. Performance measure (RMSE) for average measured cutting forces using ANFIS, various shapes and number of membership function, for medium-carbon steel C45.

MFs type	Number of MFs	Average measured cutting forces			Max. measured cutting forces		
		F_x	F_y	F_z	F_x	F_y	F_z
Triangular	2	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	3	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	4	$0.0291\cdot10^{-13}$	$0.0647\cdot10^{-13}$	$0.0356\cdot10^{-13}$	$0.1479\cdot10^{-13}$	$0.2114\cdot10^{-13}$	$0.0617\cdot10^{-13}$
	5	$0.0298\cdot10^{-13}$	$0.0646\cdot10^{-13}$	$0.0366\cdot10^{-13}$	$0.1439\cdot10^{-13}$	$0.2014\cdot10^{-13}$	$0.0607\cdot10^{-13}$
Trapezoidal	2	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	3	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	4	$0.0291\cdot10^{-13}$	$0.0647\cdot10^{-13}$	$0.0356\cdot10^{-13}$	$0.1479\cdot10^{-13}$	$0.2114\cdot10^{-13}$	$0.0617\cdot10^{-13}$
	5	$0.0298\cdot10^{-13}$	$0.0646\cdot10^{-13}$	$0.0366\cdot10^{-13}$	$0.1439\cdot10^{-13}$	$0.2014\cdot10^{-13}$	$0.0607\cdot10^{-13}$
Gaussian	2	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	3	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	4	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	5	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
Bell	2	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	3	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	4	$0.0281\cdot10^{-13}$	$0.0640\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2010\cdot10^{-13}$	$0.0615\cdot10^{-13}$
	5	$0.0281\cdot10^{-13}$	$0.0643\cdot10^{-13}$	$0.0355\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$0.2011\cdot10^{-13}$	$0.0615\cdot10^{-13}$

Table 14. Performance measure (RMSE) for average measured cutting forces using ANFIS, various shapes and number of membership function, for titanium, grade 3.

MFs type	Number of MFs	Average measured cutting forces			Max. measured cutting forces		
		F_x	F_y	F_z	F_x	F_y	F_z
Triangular	2	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	3	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	4	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	5	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.3\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
Trapezoidal	2	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	3	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	4	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	5	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
Gaussian	2	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	3	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	4	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	5	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
Bell	2	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	3	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.2\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	4	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$
	5	$0.0366\cdot10^{-13}$	$0.0533\cdot10^{-13}$	$0.1465\cdot10^{-13}$	$<0.1\cdot10^{-29}$	$0.0366\cdot10^{-13}$	$0.0732\cdot10^{-13}$

APPENDIX D. FOURTH APPENDIX

Table 15. Performance measure (RMSE) for average measured cutting forces using ANFIS, various shapes and number of membership function, for aluminium Al 7178.

MFs type	Number of MFs	Average measured cutting forces			Max. measured cutting forces		
		F_x	F_y	F_z	F_x	F_y	F_z
Triangular	2	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	3	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	4	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2499 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	5	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
Trapezoidal	2	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	3	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	4	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	5	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
Gaussian	2	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	3	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	4	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	5	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
Bell	2	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.231 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	3	$0.0906 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	4	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$
	5	$0.0916 \cdot 10^{-14}$	$0.1538 \cdot 10^{-14}$	$0.0888 \cdot 10^{-14}$	$0.222 \cdot 10^{-14}$	$0.2551 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$

Table 16. Performance measure (RMSE) for average measured cutting forces using ANFIS, various shapes and number of membership function, for cast iron.

MFs type	Number of MFs	Average measured cutting forces			Max. measured cutting forces		
		F_x	F_y	F_z	F_x	F_y	F_z
Triangular	2	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	3	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	4	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	5	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
Trapezoidal	2	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	3	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	4	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	5	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
Gaussian	2	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	3	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	4	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	5	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
Bell	2	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	3	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	4	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$
	5	$0.2512 \cdot 10^{-14}$	$<0.4 \cdot 10^{-27}$	$0.6153 \cdot 10^{-14}$	$0.1776 \cdot 10^{-14}$	$0.3241 \cdot 10^{-14}$	$0.6153 \cdot 10^{-14}$