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Adaptive Fuzzy Control of Doubly-Fed Induction Machine

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Abstract: This paper presents a new adaptive fuzzy backstepping control (AFBC) scheme for doubly-fed induction machine (DFIM). This proposed controller guarantees speed tracking and reactive power regulation at stator side. The DFIM stator windings are directly connected to the line grid, while the rotor ones are controlled by means of an inverter. Using a state-all-flux model, the stator voltage vector oriented reference frame is adopted. The control design principle is particularly based on the decomposition of the motor model in two coupled subsystems, namely: the stator flux subsystem and the speed-rotor flux subsystem. The stator flux subsystem is stabilized independently of the speed behaviour. The DFIM unity power factor control and speed tracking problem is transferred to the rotor flux control problem. The unknown load torque is estimated on-line by a suitable adaptive law, the nonlinear functions appearing in the tracking errors dynamics and uncertainties are reasonably approximated by adaptive fuzzy systems. The proposed control scheme guarantees the tracking error exponential convergence to a small residual set. The performances of the proposed control system are evaluated in comparison with a non-adaptive backstepping control (NABC) scheme by simulation tests.

Keywords: Doubly-fed induction motor, Backstepping approach, Fuzzy systems, Adaptive control, Load torque estimator, Lyapunov stability.

INTRODUCTION

In the last decade, significantly less attention has been paid to the control development of so called Double Fed Induction Machine (DFIM). The DFIM is a wound rotor asynchronous machine which can be controlled from the stator or rotor by various possible combinations. Connecting the stator windings of a DFIM directly to the line grid and the rotor windings to a controlled converter constitutes a typical connection scheme of this machine. This solution is very attractive where small speed variations around the synchronous velocity have to be imposed; the rotor power handled by the converter is a small fraction of the overall machine power. This allows the minimizing of converter size and therefore a decreased price of the whole system (Morel *et al.,* 1998).

However, DFIM usually presents the features of nonlinear and multivariable system with inevitable modeling uncertainties and hence can be considered as a challenging engineering problem. Different strategies were proposed in the literature to solve the DFIM control problem. One of the most significant developments in this area was the field-oriented control that offers the decoupled control of the active and reactive powers (Hopfensperger *et al.,* 1999; Peresada *et al.,* 2003; Drid *et al.,* 2005). Unfortunately, this control approach suffers from sensitivity to the machine parameter variations and inadequate rejection of external disturbances and load changes (Wai, 2007). To partly overcome these insufficiencies of the field oriented control approach for DFIM, some advanced vector control techniques based on the nonlinear feedback linearization and sliding mode control principles have been addressed (Drid *et al.,* 2005; Bekakra and Benattous, 2010; Ardjoun *et al.,* 2011). The resulting controllers are not only robust to model uncertainty and to parameter variations; but also they having good disturbance rejection properties. Unfortunately, such performance is obtained at price of extremely high control activity. As consequence, the chattering phenomenon always occurs in the sliding and steady state modes, and may excite unmodeled high frequency dynamics. If system uncertainties are large, the sliding mode controller would require a high sliding gain causing higher chattering effect (Wai, 2007). So, the concept of “Intelligent Control” has been suggested as an alternative approach to conventional control techniques for complex control systems. The objective is to introduce new mechanisms permitting a more flexible control, but especially more robust one, able to deal with model uncertainties and parameter variations. One of these approaches is adaptive fuzzy logic control (Li and Lau, 1989). Using fuzzy systems for approximating of the nonlinear uncertain functions, adaptive fuzzy controllers for inductions motors (IM) have been developed in (Agamy *et al.*, 2004; Youcef and Wahba, 2009). The aim of this paper is to present a new adaptive fuzzy controller for DFIM drives when the stator windings are directly connected to the line grid, while the rotor ones are controlled by means of a converter. Our control objectives are:

- Tracking of a smooth speed reference, in the presence of an unknown load torque.
- Reactive power regulation at stator side (unity power factor at stator).

For this, the machine model is decomposed in two coupled subsystems, namely: the stator flux subsystem and the speed-rotor flux subsystem. First, the stator voltage vector oriented reference frame is adopted, and the control problem of a stator unity power factor is converted into a stator flux regulation problem. In fact, the time varying stator flux vector is required to be orthogonal to line voltage, and the d-axis component of rotor flux appears as the control input for the stator flux subsystem. Then, with appropriate choice of the stator flux reference and the strict control of *d*-axis component of rotor flux to a suitable value, the stator flux error dynamics becomes linear and exponentially practically stable independently of the speed dynamics. Consequently, the DFIM stator unity power factor control and speed tracking problem is converted into a rotor flux control problem. The proposed adaptive fuzzy controller (AFC) is systematically constructed using backstepping technique. Fuzzy systems are used to reasonably approximate the unknown nonlinearities and uncertainties. While the adaptive laws, which are used to estimate on-line the load torque and the unknown fuzzy parameters, are derived in the sense of Lyapunov stability theorem.

The main contributions of this paper lie in the following

1) A novel adaptive fuzzy controller for DFIM is proposed. To the authors' best knowledge, there is no result reported in the literature on the adaptive fuzzy control design for doubly-fed induction machine. Note that the design of the adaptive control, for a DFIM being controlled by acting on the rotor winding and with a stator which is directly connected to the grid, is very challenge.
2) An adaptive estimator is designed to approximate the unknown load torque.
3) A comparative study between our proposed adaptive controller and a non-adaptive controller has been addressed.

2. DFIM MODELLING

By considering the classical simplifying assumptions, the dynamic model of the DFIM, in the synchronous *d-q* reference frame, can be described as (Drid *et al.,* 2005).

$$\begin{cases} \dfrac{d\varphi_{sd}}{dt} = -\dfrac{R_s}{L_s\sigma}\varphi_{sd} + \dfrac{R_sM}{L_sL_r\sigma}\varphi_{rd} + \omega_s\varphi_{sq} + u_{sd} \\ \dfrac{d\varphi_{sq}}{dt} = -\dfrac{R_s}{L_s\sigma}\varphi_{sq} + \dfrac{R_sM}{L_sL_r\sigma}\varphi_{rq} - \omega_s\varphi_{sd} + u_{sq} \\ \dfrac{d\varphi_{rd}}{dt} = -\dfrac{R_r}{L_r\sigma}\varphi_{rd} + \dfrac{R_rM}{L_sL_r\sigma}\varphi_{sd} + \omega_r\varphi_{rq} + u_{rd} \\ \dfrac{d\varphi_{rq}}{dt} = -\dfrac{R_r}{L_r\sigma}\varphi_{rq} + \dfrac{R_rM}{L_sL_r\sigma}\varphi_{sq} - \omega_r\varphi_{rd} + u_{rq} \end{cases} \quad (1)$$

Stator and rotor flux equations are

$$\begin{cases} \varphi_{sd} = L_s i_{sd} + M i_{rd}, \varphi_{sq} = L_s i_{sq} + M i_{rq} \\ \varphi_{rd} = L_r i_{rd} + M i_{sd}, \varphi_{rq} = L_r i_{rq} + M i_{sq} \end{cases} \quad (2)$$

The mechanical equation is given by

$$J\frac{d\Omega}{dt} = \Gamma_e - \Gamma_l - k_f\Omega \quad (3)$$

Where Γ_e is the electromagnetic torque

$$\Gamma_e = \frac{pM}{L_sL_r\sigma}\left(\varphi_{sq}\varphi_{rd} - \varphi_{sd}\varphi_{rq}\right)$$

where

s , r	Rotor and stator indices
d , q	Synchronous reference frame
α , β	Stationary reference frame
R, L, M	Resistance, inductance and mutual inductance
u, i, φ	Voltage, current and flux
θ_s, θ_r	Stator and rotor electrical angles
θ, Ω	Rotor mechanical position and speed
$\omega_s = \dot{\theta}_s, \omega_r = \dot{\theta}_r$, $\omega = \dot{\theta}$	Electrical frequencies of stator, rotor and shaft
Γ_l, Γ_e	Load and electromagnetic torque
J , p	Inertia, number of pole pairs
$\sigma = 1-(M^2/L_sL_r)$	Leakage coefficient

For all speed ranges the stator and the rotor angular frequencies are related to the shaft mechanical speed by $\omega_s = \omega_r + \omega$.

Expressions of stator and rotor active and reactive powers are respectively given by

$$\begin{cases} P_s = u_{sd}\, i_{sd} + u_{sq}\, i_{sq}, \quad Q_s = u_{sq}\, i_{sd} - u_{sd}\, i_{sq} \\ P_r = u_{rd}\, i_{rd} + u_{rq}\, i_{rq}, \quad Q_r = u_{rq}\, i_{rd} - u_{rd}\, i_{rq} \end{cases} \quad (4)$$

3. DFIM CONTROL PROBLEM

First, we suppose that the stator flux vector is aligned with *d*-axis as shown in Fig.1. In the stationary frame *abc*, the component *n* of the stator voltage equation is given by (Hopfensperger *et al.,* 1999).

$$u_{sn} = R_s i_{sn} + \frac{d\varphi_{sn}}{dt} \quad (5)$$

By neglecting the stator resistance, (5) can be rewritten as

$$u_{sn} \approx \frac{d\varphi_{sn}}{dt} \quad (6)$$

Then, the stator voltage vector is $\frac{\pi}{2}$ in advance of the stator flux. In the chosen reference frame, we can write

$$u_{sd} = 0, \; u_{sq} = u_s \quad (7)$$

Note that the stator electrical angle θ_s is calculated only with the grid voltage (Hopfensperger *et al.,* 1999).

$$\theta_s = \theta_1 - \frac{\pi}{2} \tag{8}$$

where $\theta_1 = \arctan(u_{s\beta}/u_{s\alpha})$ is the stator voltage vector angle in the stationary reference frame *abc* as shown in Fig.1.

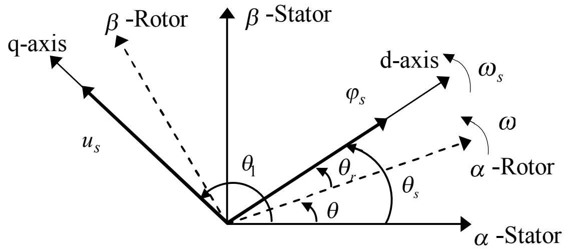

Fig. 1. Reference frames and angles for the oriented DFIM.

The control objectives are the following

- Tracking of a smooth speed reference, with unknown load torque.
- Reactive power regulation at stator side (unity power factor at stator).

It will be demonstrated that the stator-side reactive power regulation problem can be formalized as the requirement to guarantee that the line voltage vector and the stator flux vector are orthogonal.

Considering the stator equations expressed in terms of stator fluxes and currents in the line voltage reference frame.

$$\begin{cases} \dot{\varphi}_{sd} = -R_s i_{sd} + \omega_s \varphi_{sq} \\ \dot{\varphi}_{sq} = -R_s i_{sq} - \omega_s \varphi_{sd} + u_s \end{cases} \tag{9}$$

From the second equation of (4), the unity power factor objective is equivalent to $i_{sd} = 0$. In steady-state condition, all the derivatives are zero. According to the first equation of (9), $\varphi_{sq} = 0$ is necessary to ensure $i_{sd} = 0$. Then, the stator-side unity power factor control is reformulated as a stator flux orientation control objective, i.e. the stator flux vector is required to be orthogonal to line voltage vector.

The stator flux subsystem control is designed in order to achieve asymptotic alignment of the stator flux vector with the *d*-axes of the line voltage vector reference frame, consequently, the stator voltage and flux vectors become orthogonal.

Let us define stator flux tracking errors as

$$\tilde{\varphi}_{sd} = \varphi_{sd} - \varphi_{sd}^*, \tilde{\varphi}_{sq} = \varphi_{sq} \tag{10}$$

Using (7) and (10), the stator flux dynamic equations in (1) can be written in error form as

$$\begin{cases} \dot{\tilde{\varphi}}_{sd} = -a_1\tilde{\varphi}_{sd} - a_1\varphi_{sd}^* + a_2\varphi_{rd} + \omega_s\tilde{\varphi}_{sq} - \dot{\varphi}_{sd}^* \\ \dot{\tilde{\varphi}}_{sq} = -a_1\tilde{\varphi}_{sq} + a_2\varphi_{rq} - \omega_s\tilde{\varphi}_{sd} - \omega_s\varphi_{sd}^* + u_s \end{cases} \tag{11}$$

where $a_1 = R_s / L_s\sigma$, $a_2 = R_s M / L_r L_s \sigma$

To realize the required stator flux orientation, the *d*-axis component of rotor flux φ_{rd} can be considered as control input in (11), and should be

$$\varphi_{rd} = \frac{1}{a_2}\left(a_1\varphi_{sd}^* + \dot{\varphi}_{sd}^*\right) \tag{12}$$

with the *d*-axis stator flux reference computed from the second equation of (11).

$$\varphi_{sd}^* = \frac{1}{\omega_s}\left(u_s + a_2\varphi_{rq}\right) \tag{13}$$

Using (12) and (13), (11) becomes

$$\begin{cases} \dot{\tilde{\varphi}}_{sd} = -a_1\tilde{\varphi}_{sd} + \omega_s\tilde{\varphi}_{sq} \\ \dot{\tilde{\varphi}}_{sq} = -a_1\tilde{\varphi}_{sq} - \omega_s\tilde{\varphi}_{sd} \end{cases} \tag{14}$$

However, in a DFIM, The rotor flux is not available as control input and φ_{rd} in (12) can only represent the *d*-axis rotor flux reference φ_{rd}^* for the real flux φ_{rd}. The rotor voltages u_{rd} and u_{rq} are the only physical available control inputs of DFIM. From (14), one concludes that the dynamics of the stator flux are exponentially stable (i.e. $\lim_{t\to\infty} \varphi_{sd} = \varphi_{sd}^*$ and $\lim_{t\to\infty} \varphi_{sq} = 0$) provided that $\lim_{t\to\infty} \varphi_{rd} = \varphi_{rd}^*$.

Now, it is required to design a control law (u_{rd} and u_{rq}) which guarantee that $\lim_{t\to\infty} \varphi_{rd} = \varphi_{rd}^*$ and $\lim_{t\to\infty} \Omega = \Omega^*$. We consider the reduced order DFIM model represented by the rotor flux and speed equations.

$$\begin{cases} \dot{x}_1 = a_5(x_4x_3 - x_5x_2) - a_6x_1 - a_7\Gamma_l \\ \dot{x}_2 = -a_3x_2 + a_4x_4 - \omega_r x_3 + \delta_1(x) + u_1 \\ \dot{x}_3 = -a_3x_3 + a_4 x_5 + \omega_r x_2 + \delta_2(x) + u_2 \end{cases} \tag{15}$$

with $x_1 = \Omega$, $x_2 = \varphi_{rq}$, $x_3 = \varphi_{rd}$, $x_4 = \varphi_{sq}$, $x_5 = \varphi_{sd}$, $x = [x_1, x_2, x_3, x_4, x_5]^T$, $u_1 = u_{rq}$, $u_2 = u_{rd}$, $a_3 = R_r / L_r\sigma$, $a_4 = R_r M / L_r L_s \sigma$, $a_5 = pM / J L_r L_s \sigma$, $a_6 = k_f / J$ and $a_7 = 1/J$.

where $\delta_1(x)$ and $\delta_2(x)$ are the uncertainties and perturbations that can be naturally generated from the parameter variations. In the following, two nonlinear controllers are developed for the DFIM, namely

- a non-adaptive backstepping controller (NABC) and
- an adaptive fuzzy backstepping controller (AFBC).

3.1 Non-Adaptive Backstepping Control System

The following realistic assumptions are used in the control design and the stability analysis.

Assumption 1: We assume that the load torque satisfies the following relations

$$|\Gamma_l| \le \rho_0, \quad \dot{\Gamma}_l \approx 0 \tag{16}$$

where ρ_0 is a known positive constant.

Assumption 2 : The functions $\delta_1(x)$ and $\delta_2(x)$ are uncertain. However, they are bounded by known positive nonlinear functions as follows:

$$|\delta_1(x)| \le \rho_1(x) \tag{17}$$

$$|\delta_2(x)| \le \rho_2(x) \tag{18}$$

Assumption 3: Assume that

-the reference speed profile $x_{1d} = \Omega^*$ is bounded and sufficiently smooth, and

-the reference signal of x_3 i.e. $x_{3d} = \varphi_{rd}^* = \frac{1}{a_2}(a_1\varphi_{sd}^* + \dot{\varphi}_{sd}^*)$ is assumed to be derivable and bounded.

For the system (15), the backstepping design procedure (Krstic et al., 1995) is used for the construction of the control system which guarantees a practical exponential tracking of rotor speed and rotor flux reference signals.

Step 1: For a continuous bounded reference signal $x_{1d} = \Omega^*$, we define the tracking error e_1 as follows

$$e_1 = x_1 - x_{1d} \tag{19}$$

Its time-derivative $\dot{e}_1$ is given by

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} \tag{20}$$

From (15), we can write

$$\dot{e}_1 = a_5x_4x_3 - a_5x_5x_2 - a_6x_1 - a_7\Gamma_l - \dot{x}_{1d} \tag{21}$$

Choose $a_5x_5x_2$ as a virtual control to stabilize e_1 and select υ given below as a desired reference signal for $a_5x_5x_2$:

$$\upsilon = a_5x_4x_3 + \left(c_1 + \frac{\rho_0^2}{4\varepsilon_1}\right)e_1 - a_6x_1 - \dot{x}_{1d} \tag{22}$$

where $c_1 > 0$ is a free design constant and $\varepsilon_1 > 0$ is a small design constant.

This leads to the following dynamics

$$\dot{e}_1 = -e_2 - \left(c_1 + \frac{\rho_0^2}{4\varepsilon_1}\right)e_1 - a_7\Gamma_l \tag{23}$$

with e_2 is the tracking error of the variable $a_5x_5x_2$, given by

$$e_2 = a_5x_5x_2 - \upsilon \tag{24}$$

The Lyapunov function candidate for the e_1-subsystem is selected as

$$\Xi_1 = \frac{1}{2}e_1^2 \tag{25}$$

The time-derivative of (25) can be expressed as follows

$$\dot{\Xi}_1 = -e_1e_2 - c_1e_1^2 - \frac{\rho_0^2}{4\varepsilon_1}e_1^2 - a_7\Gamma_le_1 \tag{26}$$

From Young's inequality, one has

$$-a_7\Gamma_le_1 \le a_7\rho_0|e_1| \le \frac{\rho_0^2}{4\varepsilon_1}e_1^2 + \varepsilon_1a_7^2 \tag{27}$$

Using (27), (26) becomes

$$\dot{\Xi}_1 \le -e_1e_2 - c_1e_1^2 + \varepsilon_1a_7^2 \tag{28}$$

The next step consists in stabilizing the tracking error e_2.

Step 2: The time-derivative of (24) is given by

$$\dot{e}_2 = a_5x_5\dot{x}_2 + a_5\dot{x}_5x_2 - \dot{\upsilon} \tag{29}$$

From (1), (15) and (22), we can write

$$\dot{e}_2 = h_1(z_1) + e_1 + \bar{\delta}_1(x) + a_5x_5u_1 \tag{30}$$

with

$$\begin{aligned} h_1(z_1) = {} & -e_1 + a_5x_5(-a_3x_2 + a_4x_4 - \omega_r x_3) + \\ & a_5x_2(-a_1x_5 + a_2x_3 + \omega_r x_4) - \\ & a_5x_3(-a_1x_4 + a_2x_2 - \omega_s x_5 + u_s) - \\ & a_5x_4(-a_3x_3 + a_4x_5 + \omega_r x_2 + u_2) + \\ & (c_1 + (\rho_0^2/4\varepsilon_1))(e_2 + (c_1 + (\rho_0^2/4\varepsilon_1))e_1) + \\ & \ddot{x}_{1d} + a_6(a_5(x_4x_3 - x_5x_2) - a_6x_1) \end{aligned}$$

$$\begin{aligned} \bar{\delta}_1(x) = {} & a_5x_5\delta_1(x) - a_5x_4\delta_2(x) + (c_1 + (\rho_0^2/4\varepsilon_1))a_7\Gamma_l - \\ & a_6a_7\Gamma_l \end{aligned}$$

where $z_1 = [x_1\ x_2\ x_3\ x_4\ x_5\ u_2]^T$, and e_3 is the tracking error of x_3.

$$e_3 = x_3 - x_{3d} \tag{31}$$

with x_{3d} is the desired signal of x_3.

Define a Lyapunov function candidate for the (e_1, e_2)-subsystem as

$$\Xi_2 = \Xi_1 + \frac{1}{2}e_2^2 \tag{32}$$

Its time-derivative is given by

$$\dot{\Xi}_2 \le -c_1e_1^2 + e_2\left(h_1(z_1) + \bar{\delta}_1(x) + a_5x_5u_1\right) + \varepsilon_1a_7^2 \tag{33}$$

From Young's inequality, one has

$$\begin{aligned} \bar{\delta}_1(x)e_2 \le |\bar{\delta}_1(x)||e_2| \le {} & \frac{\left(x_5^2\rho_1^2(x) + x_4^2\rho_2^2(x) + 2\rho_0^2\right)e_2^2}{4\varepsilon_2} + \\ & \left(2a_5^2 + \left(c_1 + \left(\frac{\rho_0^2}{4\varepsilon_1}\right)\right)^2 a_7^2 + a_6^2a_7^2\right)\varepsilon_2 \end{aligned} \tag{34}$$

where $\varepsilon_2 > 0$ is a small design constant.

From (33), the control input u_1 can be chosen as follows:

$$u_1 = \frac{1}{a_5 x_5}\left(-h_1(z_1) - c_2 e_2 - \frac{\left(x_5^2\rho_1^2(x) + x_4^2\rho_2^2(x) + 2\rho_0^2\right)}{4\varepsilon_2} e_2\right) \tag{35}$$

where $c_2 > 0$ is a free design constant.

Remark 1: The magnetising flux x_5 must be non-zero (Due to the remanence flux).

Using (34) and (35), (33) becomes

$$\dot{\Xi}_2 \le -c_1 e_1^2 - c_2 e_2^2 + \varepsilon_1 a_7^2 + \left(2a_5^2 + \left(c_1 + \left(\frac{\rho_0^2}{4\varepsilon_1}\right)\right)^2 a_7^2 + a_6^2 a_7^2\right)\varepsilon_2 \tag{36}$$

The next step consists in stabilizing the tracking error e_3.

***Step 3*:** At this step, we will construct the control law u_2 to stabilize the dynamics of x_3. The time-derivative of (31) is given by

$$\begin{aligned}\dot{e}_3 &= \dot{x}_3 - \dot{x}_{3d} \\ &= -\dot{x}_{3d} - a_3 x_3 + a_4 x_5 + \omega_r x_2 + \delta_2(x) + u_2\end{aligned} \tag{37}$$

Then, let's define a Lyapunov function candidate as follows

$$\Xi_3 = \Xi_2 + \frac{1}{2} e_3^2 \tag{38}$$

Using (35) and (36), the time-derivative of Ξ_3 can be bounded by

$$\dot{\Xi}_3 \le -c_1 e_1^2 - c_2 e_2^2 + \varepsilon_1 a_7^2 + e_3\left(h_2(z_2) + \delta_2(x) + u_2\right) + \left(2a_5^2 + \left(c_1 + \left(\frac{\rho_0^2}{4\varepsilon_1}\right)\right)^2 a_7^2 + a_6^2 a_7^2\right)\varepsilon_2 \tag{39}$$

with $h_2(z_2) = -a_3 x_3 + a_4 x_5 + \omega_r x_2 - \dot{x}_{3d}$, where $z_2 = [x_1 \ x_2 \ x_3 \ x_5]^T$.

To stabilize the dynamics (37), the control input u_2 can be chosen as follows

$$u_2 = -h_2(z_2) - \left(c_3 + \frac{\rho_2^2(x)}{4\varepsilon_3}\right) e_3 \tag{40}$$

where $c_3 > 0$ is a free design constant and $\varepsilon_3 >$ is a small design constant.

From Young's inequality, one has

$$\delta_2(x) e_3 \le \rho_2(x)|e_3| \le \frac{\rho_2^2(x) e_3^2}{4\varepsilon_3} + \varepsilon_3 \tag{41}$$

Using (40) and (41), (39) becomes

$$\dot{\Xi}_3 \le -c_1 e_1^2 - c_2 e_2^2 - c_3 e_3^2 + \varepsilon_1 a_7^2 + \varepsilon_3 + \left(2a_5^2 + (c_1 + (\rho_0^2/4\varepsilon_1))^2 a_7^2 + a_6^2 a_7^2\right)\varepsilon_2 \tag{42}$$

We can rewrite (42) as follows

$$\dot{\Xi}_3 \le -K\Xi_3 + \varepsilon \tag{43}$$

where $\varepsilon = \varepsilon_1 a_7^2 + \left(2a_5^2 + (c_1 + (\rho_0^2/4\varepsilon_1))^2 a_7^2 + a_6^2 a_7^2\right)\varepsilon_2 + \varepsilon_3$ and $K = \min\{2c_1, 2c_2, 2c_3\}$.

Multiplying (43) by e^{Kt} yields

$$\frac{d}{dt}\left(\Xi_3 e^{Kt}\right) \le \varepsilon e^{Kt} \tag{44}$$

Integrating (44) over [0,*t*], it follows that

$$0 \le \Xi_3(t) \le \frac{\varepsilon}{K} + \left(\Xi_3(0) - \frac{\varepsilon}{K}\right) e^{-Kt} \tag{45}$$

This results in ultimately uniformly bounded (UUB) stabilization of the tracking errors (e_1, e_2, e_3). Since ε can be chosen arbitrary and K only depends on the design parameters $(c_1, c_2$ and $c_3)$, the ultimate error bounds can be made arbitrary small.

3.2 Adaptive Fuzzy Backstepping Control System

The control objective in this section is to design an adaptive fuzzy backstepping controller for DFIM guaranteeing global system stability with improved control robustness.

The basic configuration of a fuzzy logic system consists of a fuzzifier, some fuzzy IF-THEN rules, a fuzzy inference engine and a defuzzifier, as shown in Fig. 2.

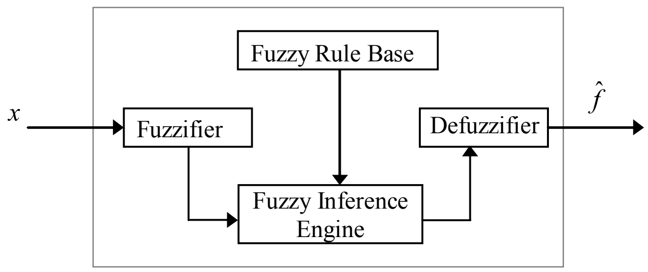

Fig. 2. The basic configuration of a fuzzy logic system.

The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input vector $x^T = [x_1 \, x_2 \ldots x_n] \in R^n$ to an output $\hat{f} \in R$. The ith fuzzy rule is written as

$$R^{(i)}: \text{if } x_1 \text{ is } A_1^i \text{ and} \ldots \text{and } x_n \text{ is } A_n^i \text{ then } \hat{f} \text{ is } f^i \tag{46}$$

where $A_1^i, A_2^i, \ldots,$ and A_n^i are fuzzy sets and f^i is the fuzzy singleton for the output in the *i*th rule. By using the singleton fuzzifier, product inference, and center-average defuzzifier, the output of the fuzzy system can be expressed as follows:

$$\hat{f}(x) = \frac{\sum_{i=1}^{m} f^i\left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)}{\sum_{i=1}^{m}\left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)} = \theta^T \psi(x) \tag{47}$$

where $\mu_{A_j^i}(x_j)$ is the degree of membership of x_j to A_j^i, m is the number of fuzzy rules, $\theta^T = [f^1, f^2, ..., f^m]$ is the adjustable parameter vector (composed of consequent parameters), and $\psi^T = [\psi^1\ \psi^2 ... \psi^m]$ with

$$\psi^i(x) = \frac{\left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)}{\sum_{i=1}^{m}\left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)}$$

being the fuzzy basis function (FBF). Throughout the paper, it is assumed that the FBFs are selected so that there is always at least one active rule (Wang, 1994), i.e. $\sum_{i=1}^{m}\left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right) > 0$.

It is worth noting that the fuzzy system (47) is commonly used in control applications. Following the universal approximation results (Wang, 1994; Feriyonika and Dewantoro, 2013; Husek, and Cerman, 2013; Joelianto *et al.,* 2013; Odior, 2013), the fuzzy system (47) is able to approximate any nonlinear smooth function $f(x)$ on a compact operating space to an arbitrary degree of accuracy. Of particular importance, it is assumed that the structure of the fuzzy system (i.e. the pertinent inputs, the number of membership functions for each input and the number of rules) and the membership function parameters are properly specified beforehand. The consequent parameters θ are then determined by appropriate adaptation algorithms.

Assumption 4: the functions $\delta_1(x)$ and $\delta_2(x)$ are smooth and completely unknown.

Throughout the rest paper, we will exploit the following nice property with regard to function *Tanh*(.)

$$0 \le |X| - Tanh(X/\beta_i) \le \bar{\beta}_i, \quad \text{for } i = 1,2 \tag{48}$$

where β_i is a small positive design constant and $\bar{\beta}_i = 0.2785\beta_i$.

Now, we give the procedure of the backstepping design.

Step 1: For a continuous bounded reference signal x_{1d}, the tracking error e_1 and its derivative $\dot{e}_1$ are defined respectively by (19) and (21):

$$e_1 = x_1 - x_{1d}$$

$$\dot{e}_1 = a_5x_4x_3 - a_5x_5x_2 - a_6x_1 - a_7\Gamma_l - \dot{x}_{1d}$$

Let $\hat{\Gamma}_l$ be the estimate of Γ_l and select a new virtual control υ as

$$\upsilon = a_5x_4x_{3d} + \lambda_1 e_1 - a_6x_{1d} - \dot{x}_{1d} - a_7\hat{\Gamma}_l \tag{49}$$

where $\lambda_1 > 0$ is a design constant and $\hat{\Gamma}_l$ is the estimate of Γ_l.

From (21) and (49), we can obtain the following dynamics:

$$\dot{e}_1 = a_5x_4e_3 - e_2 - (\lambda_1 + a_6)e_1 - a_7\tilde{\Gamma}_l \tag{50}$$

where $\tilde{\Gamma}_l = \Gamma_l - \hat{\Gamma}_l$ is the load torque estimation error, and e_2 is the tracking error of the variable $a_5x_5x_2$, given by

$$e_2 = a_5x_5x_2 - \upsilon \tag{51}$$

Consider the following Lyapunov function candidate for the e_1-subsystem

$$V_1 = \frac{1}{2}\left(e_1^2 + \frac{\tilde{\Gamma}_l^2}{\gamma_l}\right) \tag{52}$$

where $\gamma_l > 0$ is a design parameter.

By assuming that the load torque is slowly time-varying ($\dot{\Gamma}_l = 0$), the time-derivative of (52) along (50) is given by

$$\dot{V}_1 = -e_1e_2 + a_5x_4e_3e_1 - (\lambda_1 + a_6)e_1^2 - \tilde{\Gamma}_l\left(a_7e_1 + \frac{\dot{\hat{\Gamma}}_l}{\gamma_l}\right) \tag{53}$$

If the load torque estimator is designed as

$$\dot{\hat{\Gamma}}_l = \sigma_l\tilde{\Gamma}_l - \gamma_l a_7 e_1 \tag{54}$$

where $\sigma_l > 0$ is a design parameter.

Then, (53) can be written as

$$\dot{V}_1 = -e_1e_2 + a_5x_4e_3e_1 - (\lambda_1 + a_6)e_1^2 - (\sigma_l/\gamma_l)\tilde{\Gamma}_l^2 \tag{55}$$

The next step consists in stabilizing the tracking error e_2.

***Step 2*:** The time-derivative of (51) is given by

$$\dot{e}_2 = a_5x_5\dot{x}_2 + a_5\dot{x}_5x_2 - \dot{\upsilon} \tag{56}$$

From the second subsystem of (1), (15) and (49), we can write

$$\dot{e}_2 = f_1(\bar{z}_1) + e_1 + (a_5a_2x_2 - a_5x_5\omega_r - a_5\lambda_1x_4)e_3 - a_7(\sigma_l + \lambda_1)\hat{\Gamma}_l + a_5x_5u_1 \tag{57}$$

with

$$f_1(\bar{z}_1) = -e_1 - a_5a_3x_5x_2 + a_5a_4x_5x_4 - a_5x_5\omega_r x_{3d} - a_5a_1x_5x_2 + a_5\omega_s x_4x_2 + a_5a_1x_{3d}x_4 + a_5\omega_s x_{3d}x_5 - a_5x_{3d}u_s - a_5x_4\dot{x}_{3d} + a_6\dot{x}_{1d} + \ddot{x}_{1d} + \lambda_1\left(e_2 + (\lambda_1 + a_6)e_1\right) - a_7^2\gamma_l e_1 + a_7(\sigma_l + \lambda_1)\Gamma_l + a_5x_5\delta_1(x)$$

where $\bar{z}_1 = [x_1\ x_2\ x_4\ x_5\ \upsilon\ \Gamma_l]^T$ and e_3 is the tracking error of x_3. It is given by

$$e_3 = x_3 - x_{3d} \tag{58}$$

where $x_{3d} = \varphi_{rd}^* = (a_1\varphi_{sd}^* + \dot{\varphi}_{sd}^*)/a_2$.

The uncertain continuous function $f_1(\bar{z}_1)$ can be approximated by the fuzzy system (47) as follows

$$\hat{f}_1(\bar{z}_1,\theta_1)=\theta_1^T\psi_1(\bar{z}_1) \tag{59}$$

where $\psi_1(\bar{z}_1)$ is the FBF vector, which is fixed a priori by the designer, and θ_1 is the adjustable parameter vector of the fuzzy system. Furthermore, according to universal approximation theorem (Wang, 1994), the functions $f_1(\bar{z}_1)$ can be approximated optimally as follows

$$f_1(\bar{z}_1)=\hat{f}_1(\bar{z}_1,\theta_1^*)+\varpi_1(\bar{z}_1)=\theta_1^{*T}\psi_1(\bar{z}_1)+\varpi_1(\bar{z}_1) \tag{60}$$

where θ_1^* is the optimal parameter vector and $\varpi_1(\bar{z}_1)$ is the unavoidable fuzzy approximation error which is generally assumed to be bounded as follows (Wang, 1994; Rusu, 2002; Boulkroune et al., 2008, 2009, 2010a, 2010b; Liu *et al.*, 2011): $|\varpi_1(\bar{z}_1)|\le\bar{\varpi}_1$, $\forall\bar{z}_1\in\Omega_{\bar{z}1}$, where $\bar{\varpi}_1$ is an unknown constant.

Since the input vector $\bar{z}_1=[x_1,\,x_2,\,x_4,\,x_5,\,\bar{\upsilon},\,\Gamma_l]^T$ is not available, it must be replaced by its estimate $\hat{\bar{z}}_1=[x_1,\,x_2,\,x_4,\,x_5,\,\bar{\upsilon},\,\hat{\Gamma}_l]^T$ in (59). Thus, the fuzzy system (59) used to approximate $f_1(\bar{z}_1)$ is replaced by the following fuzzy system

$$\hat{f}_1(\hat{\bar{z}}_1,\theta_1)=\theta_1^T\psi_1(\hat{\bar{z}}_1) \tag{61}$$

From (59)-(61), we have

$$\begin{aligned}f_1(\bar{z}_1)&=f_1(\bar{z}_1)-\hat{f}_1(\bar{z}_1,\theta_1^*)+\hat{f}_1(\bar{z}_1,\theta_1^*)-\hat{f}_1(\hat{\bar{z}}_1,\theta_1^*)+\\&\quad\hat{f}_1(\hat{\bar{z}}_1,\theta_1^*)\\&=\theta_1^{*T}\psi(\hat{\bar{z}}_1)+\varpi_1(\bar{z}_1)+[\theta_1^{*T}\psi_1(\bar{z}_1)-\theta_1^{*T}\psi_1(\hat{\bar{z}}_1)]\\&=\theta_1^{*T}\psi(\hat{\bar{z}}_1)+\vartheta_1(\bar{z}_1,\hat{\bar{z}}_1)\end{aligned} \tag{62}$$

where $\vartheta_1(\bar{z}_1,\hat{\bar{z}}_1)=\varpi_1(\bar{z}_1)+[\theta_1^{*T}\psi_1(\bar{z}_1)-\theta_1^{*T}\psi_1(\hat{\bar{z}}_1)]$ is the fuzzy approximation error. Notice that $\vartheta_1(\bar{z}_1,\hat{\bar{z}}_1)$ has an upper bound, i.e. $|\vartheta_1(\bar{z}_1,\hat{\bar{z}}_1)|\le\kappa_1^*$ with κ_1^* is an unknown positive constant (Wang, 1994; Boulkroune *et al.,* 2010a).

To stabilise the dynamics (57), the following fuzzy adaptive controller is proposed

$$\begin{aligned}u_1=\frac{1}{a_5x_5}\Big(&a_7(\sigma_l+\lambda_1)\hat{\Gamma}_l-\theta_1^T\psi_1(\hat{\bar{z}}_1)-\lambda_2e_2-\\&\kappa_1 Tanh\left(\frac{e_2}{\beta_1}\right)\Big)\end{aligned} \tag{63}$$

where $\lambda_2>0$ is a design constant, κ_1 is the estimate of the unknown bound κ_1^* and $\beta_1>0$ is a small design constant.

Replacing (63) into (57) and using (62) yield

$$\begin{aligned}\dot{e}_2&=e_1+(a_5a_2x_2-a_5x_5\omega_r-a_5\lambda_1x_4)e_3-\\&\quad\tilde{\theta}_1^T\psi_1(\hat{z}_1)+\vartheta_1(z_1,\hat{z}_1)-\lambda_2e_2-\kappa_1 Tanh\left(\frac{e_2}{\beta_1}\right)\end{aligned} \tag{64}$$

where $\tilde{\theta}_1=\theta_1-\theta_1^*$ is the parameter error vector.

Multiplying (64) by e_2 and using the inequality (48), we get

$$\begin{aligned}e_2\dot{e}_2&=e_1e_2+(a_5a_2x_2-a_5x_5\omega_r-a_5\lambda_1x_4)e_2e_3\\&\quad-e_2\tilde{\theta}_1^T\psi_1(\hat{\bar{z}}_1)+e_2\vartheta_1(\bar{z}_1,\hat{\bar{z}}_1)-\lambda_2e_2^2-\kappa_1e_2Tanh\left(\frac{e_2}{\beta_1}\right)\\&\le e_1e_2+(a_5a_2x_2-a_5x_5\omega_r-a_5\lambda_1x_4)e_2e_3-\\&\quad e_2\tilde{\theta}_1^T\psi_1(\hat{\bar{z}}_1)-\lambda_2e_2^2-\tilde{\kappa}_1e_2Tanh\left(\frac{e_2}{\beta_1}\right)+\kappa_1^*\bar{\beta}_1\end{aligned} \tag{65}$$

where $\tilde{\kappa}_1=\kappa_1-\kappa_1^*$ is the parameter error and $\bar{\beta}_1=0.2785\beta_1$.

Define a Lyapunov function candidate for the (e_1, e_2)-subsystem as follows

$$V_2=V_1+\frac{1}{2}e_2^2+\frac{1}{2\gamma_{\theta1}}\tilde{\theta}_1^T\tilde{\theta}_1+\frac{1}{2\gamma_{\kappa1}}\tilde{\kappa}_1^2 \tag{66}$$

where $\gamma_{\theta1}$ and $\gamma_{\kappa1}>0$ are design constants.

Taking the derivative of V_2 with respect to time and using (55) and (65), one can obtain

$$\begin{aligned}\dot{V}_2&=\dot{V}_1+e_2\dot{e}_2+\frac{1}{\gamma_{\theta1}}\tilde{\theta}_1^T\dot{\theta}_1+\frac{1}{\gamma_{\kappa1}}\tilde{\kappa}_1\dot{\kappa}_1\\&\le(a_5a_2x_2-a_5x_5\omega_r-a_5\lambda_1x_4)e_2e_3+a_5x_4e_1e_3-\\&\quad(\lambda_1+a_6)e_1^2-\frac{\sigma_l}{\gamma_l}\tilde{\Gamma}_l^2-\lambda_2e_2^2+\\&\quad\frac{1}{\gamma_{\theta1}}\tilde{\theta}_1^T\left[\dot{\theta}_1-\gamma_{\theta1}e_2\psi_1(\hat{\bar{z}}_1)\right]+\\&\quad\frac{1}{\gamma_{\kappa1}}\tilde{\kappa}_1\left[\dot{\kappa}_1-\gamma_{\kappa1}e_2Tanh\left(\frac{e_2}{\beta_1}\right)\right]+\kappa_1^*\bar{\beta}_1\end{aligned} \tag{67}$$

If the adaptation laws are designed as

$$\dot{\theta}_1=-\gamma_{\theta1}\sigma_{\theta1}\theta_1+\gamma_{\theta1}e_2\psi_1(\hat{\bar{z}}_1) \tag{68}$$

$$\dot{\kappa}_1=-\gamma_{\kappa1}\sigma_{\kappa1}\kappa_1+\gamma_{\kappa1}e_2Tanh\left(\frac{e_2}{\beta_1}\right) \tag{69}$$

where $\sigma_{\theta1}$ and $\sigma_{\kappa1}>0$ are small design constants.

Then, (67) can be expressed as

$$\begin{aligned}\dot{V}_2&\le(a_5a_2x_2-a_5x_5\omega_r-a_5\lambda_1x_4)e_2e_3+\\&\quad a_5x_4e_1e_3-(\lambda_1+a_6)e_1^2-\left(\frac{\sigma_l}{\gamma_l}\right)\tilde{\Gamma}_l^2-\lambda_2e_2^2-\\&\quad\sigma_{\theta1}\tilde{\theta}_1^T\theta_1-\sigma_{\kappa1}\tilde{\kappa}_1^T\kappa_1+\kappa_1^*\bar{\beta}_1\end{aligned} \tag{70}$$

In the next step, we try to stabilize the tracking error e_3.

Step 3: At this step, we will construct the control law u_2. The time-derivative of (58) is given by

$$\dot{e}_3 = -a_3 x_3 + a_4 x_5 + \omega_r x_2 + \delta_2(x) + u_2 - \dot{x}_{3d} \tag{71}$$

We can rewrite (71) as follows

$$\dot{e}_3 = -(a_5 a_2 x_2 - a_5 x_5 \omega_r - a_5 \lambda_1 x_4) e_2 - a_5 x_4 e_1 + f_2(\bar{z}_2) + u_2 \tag{72}$$

with

$$f_2(\bar{z}_2) = (a_5 a_2 x_2 - a_5 x_5 \omega_r - a_5 \lambda_1 x_4) e_2 + a_5 x_4 e_1 - a_3 x_3 + a_4 x_5 + \omega_r x_2 + \delta_2(x) - \dot{x}_{3d}$$

where $\bar{z}_2 = [x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5]^T$.

The uncertain continuous function $f_2(\bar{z}_2)$ can be approximated by the adaptive fuzzy system (47) as follows

$$\hat{f}_2(\bar{z}_2, \theta_2) = \theta_2^T \psi_2(\bar{z}_2) \tag{73}$$

According to universal approximaton theorem (Wang , 1994), the functions $f_2(\bar{z}_2)$ can be optimally approximated as follows

$$f_2(\bar{z}_2) = \hat{f}_2(\bar{z}_2, \theta_2^*) + \varpi_2(\bar{z}_2) = \theta_2^{*T} \psi_2(\bar{z}_2) + \varpi_2(\bar{z}_2) \tag{74}$$

where θ_2^* is the optimal parameter vector and $\varpi_2(\bar{z}_2)$ is the unavoidable fuzzy approximation error which is assumed to be bounded as follows, i.e.

$$|\varpi_2(\bar{z}_2)| \le \kappa_2^*, \; \forall \bar{z}_2 \in \Omega_{\bar{z}2} \tag{75}$$

where κ_2^* is an unknown constant.
To stabilise the dynamics (71), the following fuzzy adaptive controller is proposed

$$u_2 = -\theta_2^T \psi_2(\bar{z}_2) - \lambda_3 e_3 - \kappa_2 Tanh\left(\frac{e_3}{\beta_2}\right) \tag{76}$$

where λ_3 is a positive design constant, κ_2 is the estimate of the unknown bound κ_2^* and β_2 is a small positive design constant.
Replacing (76) into (71) and using (75) yield

$$\begin{aligned}\dot{e}_3 = &-(a_5 a_2 x_2 - a_5 x_5 \omega_r - a_5 \lambda_1 x_4) e_2 - a_5 x_4 e_1 - \\ &\tilde{\theta}_2^T \psi_2(\bar{z}_2) + \varpi_2(\bar{z}_2) - \lambda_3 e_3 - \kappa_2 Tanh\left(\frac{e_3}{\beta_2}\right)\end{aligned} \tag{77}$$

where $\tilde{\theta}_2 = \theta_2 - \theta_2^*$ is the parameter error vector.

Multiplying (77) by e_3 and using the property (48), we get

$$\begin{aligned}e_3 \dot{e}_3 = &-(a_5 a_2 x_2 - a_5 x_5 \omega_r - a_5 \lambda_1 x_4) e_2 e_3 - a_5 x_4 e_1 e_3 - \\ &e_3 \tilde{\theta}_2^T \psi_2(\bar{z}_2) + e_3 \varpi_2(\bar{z}_2) - \lambda_3 e_3^2 - \kappa_2 e_3 Tanh\left(\frac{e_3}{\beta_2}\right) \\ \le &-(a_5 a_2 x_2 - a_5 x_5 \omega_r - a_5 \lambda_1 x_4) e_2 e_3 - a_5 x_4 e_1 e_3 - \\ &e_3 \tilde{\theta}_2^T \psi_2(\bar{z}_2) - \lambda_3 e_3^2 - \tilde{\kappa}_2 e_3 Tanh\left(\frac{e_3}{\beta_2}\right) + \kappa_2^* \bar{\beta}_2\end{aligned} \tag{78}$$

where $\tilde{\kappa}_2 = \kappa_2 - \kappa_2^*$ and $\bar{\beta}_2 = 0.2785\beta_2$.
Define a Lyapunov function candidate as follows

$$V_3 = V_2 + \frac{1}{2} e_3^2 + \frac{1}{2\gamma_{\theta 2}} \tilde{\theta}_2^T \tilde{\theta}_2 + \frac{1}{2\gamma_{\kappa 2}} \tilde{\kappa}_2^2 \tag{79}$$

where $\gamma_{\theta 2}$ and $\gamma_{\kappa 2} > 0$ are design constants.

Taking the derivative of V_3 with respect to time and using (78) and (70), one can obtain

$$\begin{aligned}\dot{V}_3 = &\dot{V}_2 + e_3 \dot{e}_3 + \frac{1}{\gamma_{\theta 2}} \tilde{\theta}_2^T \dot{\theta}_2 + \frac{1}{\gamma_{\kappa 2}} \tilde{\kappa}_2 \dot{\kappa}_2 \\ \le &-(\lambda_1 + a_6) e_1^2 - \left(\frac{\sigma_l}{\gamma_l}\right) \tilde{\Gamma}_l^2 - \lambda_2 e_2^2 - \lambda_3 e_3^2 - \\ &\sigma_{\theta 1} \tilde{\theta}_1^T \theta_1 - \sigma_{\kappa 1} \tilde{\kappa}_1^T \kappa_1 + \kappa_1^* \bar{\beta}_1 + \kappa_2^* \bar{\beta}_2 + \\ &\frac{1}{\gamma_{\theta 2}} \tilde{\theta}_2^T \left[\dot{\theta}_2 - \gamma_{\theta 2} e_3 \psi_2(\bar{z}_2)\right] + \\ &\frac{1}{\gamma_{\kappa 2}} \tilde{\kappa}_2 \left[\dot{\kappa}_2 - \gamma_{\kappa 2} e_3 Tanh\left(\frac{e_3}{\beta_2}\right)\right]\end{aligned} \tag{80}$$

The adaptation laws are designed as

$$\dot{\theta}_2 = -\gamma_{\theta 2} \sigma_{\theta 2} \theta_2 + \gamma_{\theta 2} e_3 \psi_2(\bar{z}_2) \tag{81}$$

$$\dot{\kappa}_2 = -\gamma_{\kappa 2} \sigma_{\kappa 2} \kappa_2 + \gamma_{\kappa 2} e_3 Tanh\left(\frac{e_3}{\beta_2}\right) \tag{82}$$

where $\sigma_{\theta 2}$ and $\sigma_{\kappa 2} > 0$ are small design constants.

One can henceforth easily check that

$$-\sigma_{\kappa i} \tilde{\kappa}_i \kappa_i \le -\frac{\sigma_{\kappa i}}{2} \tilde{\kappa}_i^2 + \frac{\sigma_{\kappa i}}{2} \kappa_i^{*2}, \quad \text{for } i = 1,2$$

$$-\sigma_{\theta i} \tilde{\theta}_i^T \theta_i \le -\frac{\sigma_{\theta i}}{2} \left\|\tilde{\theta}_i\right\|^2 + \frac{\sigma_{\theta i}}{2} \left\|\theta_i^*\right\|^2, \quad \text{for } i = 1,2$$

And using the previous inequalities and the adaptive laws (81)-(82), (80) becomes

$$\begin{aligned}\dot{V}_3 \le &-(\lambda_1 + a_6) e_1^2 - \left(\frac{\sigma_l}{\gamma_l}\right) \tilde{\Gamma}_l^2 - \lambda_2 e_2^2 - \lambda_3 e_3^2 - \frac{\sigma_{\theta 1}}{2} \left\|\tilde{\theta}_1\right\|^2 - \\ &\frac{\sigma_{\theta 2}}{2} \left\|\tilde{\theta}_2\right\|^2 - \frac{\sigma_{\kappa 1}}{2} \tilde{\kappa}_1^2 - \frac{\sigma_{\kappa 2}}{2} \tilde{\kappa}_2^2 + \eta\end{aligned} \tag{83}$$

where

$$\eta = \kappa_1^* \bar{\beta}_1 + \kappa_2^* \bar{\beta}_2 + \frac{\sigma_{\theta 1}}{2} \left\|\theta_1^*\right\|^2 + \frac{\sigma_{\theta 2}}{2} \left\|\theta_2^*\right\|^2 + \frac{\sigma_{\kappa 1}}{2} \kappa_1^{*2} + \frac{\sigma_{\kappa 2}}{2} \kappa_2^{*2}$$

One can rewrite (83) as follows

$$\dot{V}_3 \le -\zeta V_3 + \eta \tag{84}$$

where

$$\zeta = \min\{2(\lambda_1 + a_6), 2\sigma_l, 2\lambda_2, 2\lambda_3, \sigma_{\theta 1}\gamma_{\theta 1}, \sigma_{\theta 2}\gamma_{\theta 2}, \sigma_{\kappa 1}\gamma_{\kappa 1}, \sigma_{\kappa 2}\gamma_{\kappa 2}\}$$

Multiplying (84) by $e^{\zeta t}$ yields

$$\frac{d}{dt}\left(V_3 e^{\zeta t}\right) \le \eta e^{\zeta t} \tag{85}$$

Integrating (85) over [0 t], it follows that

$$0 \le V_3(t) \le \frac{\eta}{\zeta} + \left(V_3(0) - \frac{\eta}{\zeta}\right) e^{-\zeta t} \tag{86}$$

This results in ultimately uniformly bounded (UUB) stabilization of the tracking errors (e_1, e_2, e_3) and the parameter estimation errors ($\tilde{\theta}_1, \tilde{\theta}_2, \tilde{\kappa}_1, \tilde{\kappa}_2, \tilde{\Gamma}_l$) (Khalil, 2001). The boundedness of $\hat{\Gamma}_l, \theta_1, \kappa_1, \theta_2$ and κ_2 is respectively established from that $\tilde{\Gamma}_l, \tilde{\theta}_1, \tilde{\kappa}_1, \tilde{\theta}_2$ and $\tilde{\kappa}_2$.

Remark 2: According to the definition of ζ and η, it can be seen that the size of ζ depends on the controller design parameters $\sigma_l, \gamma_{\theta i}$, $\gamma_{\kappa i}$, $\sigma_{\theta i}, \sigma_{\kappa i}, \lambda_1$, λ_2 and λ_3, and that of η depends on the controller design parameters $\beta_i, \sigma_{\theta i}$ and $\sigma_{\kappa i}$. It is clear that if we increase $\sigma_l, \gamma_{\theta i}$, $\gamma_{\kappa i}$, λ_1, λ_2, and λ_3 and decrease $\beta_i, \sigma_{\theta i}$ and $\sigma_{\kappa i}$, it will help to reduce the term η / ζ. This implies that the tracking errors can be made arbitrary small by appropriately choosing those design parameters.

Because $e_1, e_3 \in L_\infty$ and $x_{1d}, x_{3d} \in L_\infty$, therefore $x_1, x_3 \in L_\infty$. From (11), one can write the dynamics of the tracking errors of the stator fluxes as follows

$$\dot{\tilde{\varphi}}_{sd} = -a_1 \tilde{\varphi}_{sd} + \omega_s \tilde{\varphi}_{sq} + a_2 e_3$$
$$\dot{\tilde{\varphi}}_{sq} = -a_1 \tilde{\varphi}_{sq} - \omega_s \tilde{\varphi}_{sd}$$

From those dynamics and since $e_3 \in L_\infty$, we can easily prove the boundedness of $\tilde{\varphi}_{sd}$, $\tilde{\varphi}_{sq}$ and x_4. From $(x_4, x_{3d}, e_1, \dot{x}_{1d}, x_1, \hat{\Gamma}_l) \in L_\infty$, it can be concluded that $\upsilon \in L_\infty$ based on (49). Since $x_2 = (e_2 + \upsilon) / a_5 x_5$, $e_2, \upsilon \in L_\infty$ and $x_5 > 0$, we can show that $x_2 \in L_\infty$. The boundedness of φ_{sd}^* and x_5 follows that of φ_{rq} (or x_2) and $\tilde{\varphi}_{sd}$. Due to the boundedness of x_1, x_2, x_3, x_4, x_5 and $\hat{\Gamma}_l$ and since $\theta_1, \kappa_1, \theta_2, \kappa_2 \in L_\infty$, we can conclude that the controls (u_1 and u_2) are also bounded.

Now, taking a summary, under the above stator voltage vector orientation constraint, if the DFIM system given by (1) is directly connected to the grid by the stator winding and is controlled acting on the rotor winding by the proposed AFBC described by (49), (63), (68), (69), (76), (81) and (82) with the load torque estimator given by (54), then the practical stability of the closed-loop control system can be guaranteed.

Let us consider the load torque adaptation law (54) that can be written in the following form

$$\dot{\hat{\Gamma}}_l = \sigma_l \Gamma_l - \sigma_l \hat{\Gamma}_l - \gamma_l a_7 e_1 \tag{87}$$

As the actual load torque Γ_l is unknown, the first equation in (15) will be used to compute its value. Consequently, Γ_l is given by

$$\Gamma_l = -\frac{1}{a_7}\left(\dot{x}_1 + a_5 x_5 x_2 - a_5 x_4 x_3 + a_6 x_1\right) \tag{88}$$

which leads to

$$\dot{\hat{\Gamma}}_l = -\frac{\sigma_l}{a_7}\left(\dot{x}_1 + a_5 x_5 x_2 - a_5 x_4 x_3 + a_6 x_1\right) - \sigma_l \hat{\Gamma}_l - \gamma_l a_7 e_1 \tag{89}$$

Because of the integral structure of the adaptation law (89), this updating law is implementable despite the presence of $\dot{x}_1$. In fact, it is can be rewritten as

$$\hat{\Gamma}_l = \hat{\Gamma}_l(0) - \frac{\sigma_l}{a_7}\left(x_1(t) - x_1(0)\right) + \int_0^t h(\tau) d\tau \tag{90}$$

where

$$h = -\left(\sigma_l \hat{\Gamma}_l + \gamma_l a_7 e_1 + \frac{\sigma_l}{a_7}\left(a_5 x_5 x_2 - a_5 x_4 x_3 + a_6 x_1\right)\right) \tag{91}$$

The overall scheme of the controlled DFIM is depicted in Fig.3 in which the stator is directly connected to the grid, and the DIFM is controlled acting on the rotor windings.

Remark 3 : From (87), we can rewrite $\dot{\tilde{\Gamma}}_l = -\sigma_l \tilde{\Gamma}_l + \gamma_l a_7 e_1$, this equation can be seen as a standard disturbance observer. In fact, if e_1 converges to zero, then $\tilde{\Gamma}_l$ also converges to zero. Consequently, $\hat{\Gamma}_l$ converges to Γ_l.

Remark 4 : Based on the universal approximation theorem and by incorporating fuzzy logic systems into adaptive control schemes, many adaptive controllers have been proposed in the literature, among them (Boulkroune et al., 2008, 2009) for single-input single-output (SISO) nonlinear systems, and (Boulkroune et al., 2010a, 2010b) for multiple-input multiple-output (MIMO) nonlinear systems. Generally, these adaptive fuzzy control approaches can have nice performance. However, they have been applied only to a relatively simple class of nonlinear systems. The key requirement is that the unknown nonlinearities appear on the same equation as the control input in a state space representation. Such restrictions on the location of the uncertain nonlinear functions are generally referred to as matching conditions. If practical systems are subject to some unknown nonlinear functions which do not satisfy the matching conditions (as in the case of the doubly-fed induction motor considered here), these adaptive fuzzy control approaches mentioned above cannot be implemented. In this paper, a fuzzy adaptive backstepping controller is developed for a DFIM. Compared to the above works (Boulkroune et al., 2008, 2009, 2010a, 2010b), the main contributions of this paper lie in the following

- Because a part of the DFIM model is subject to some unknown nonlinear functions which do not satisfy the matching conditions, the backstepping approach has been used in the controller design.

- An adaptive estimator has been designed to approximate the unknown load torque.
- The comparative study between our proposed fuzzy adaptive backstepping controller and a non-adaptive backstepping controller has been addressed.
- The control design of the considered configuration (i.e. a DFIM being controlled by acting on the rotor winding and with a stator which is directly connected to the grid) is very challenge. To our best knowledge, in the literature, there are little works dealing with this control problem.

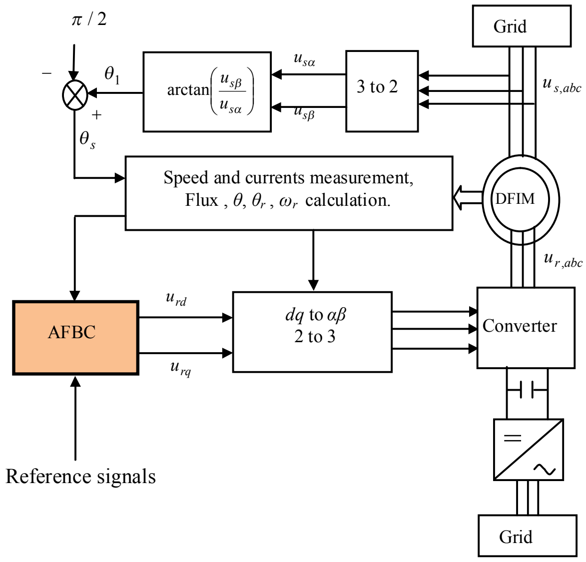

Fig. 3. The overall control scheme of the DFIM.

4. SIMULATION RESULTS

The parameters of the tested DFIM are summarized in Table.1 (Drid *et al.,* 2005). The controller parameters are set to the following values:

For NABC system: $c_1 = 200$, $c_2 = 10000$, $c_3 = 50000$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.1$ and $\rho_0 = 5$.

For AFBC system: $\gamma_l = 0.001$, $\sigma_l = 200$, $\lambda_1 = \lambda_2 = \lambda_3 = 200$, $\beta_1 = \beta_2 = 0.05$, $\gamma_{\theta 1} = 100$, $\gamma_{\kappa 1} = 0.05$, $\gamma_{\theta 2} = 1000$, $\gamma_{\kappa 2} = 0.1$, $\sigma_{\theta 1} = \sigma_{\theta 2} = 10^{-3}$, $\sigma_{\kappa 1} = \sigma_{\kappa 2} = 10^{-5}$. The initial conditions are chosen as: $\kappa_1(0) = \kappa_2(0) = 0.2$, and $\theta_{1i}(0) = \theta_{2i}(0) = 0$.

The fuzzy system $\theta_1^T \psi_1(\hat{\bar{z}}_1)$ has the vector $[x_1\, x_2\, x_4\, x_5\, \bar{\upsilon}\, \hat{\Gamma}_l]^T$ as input, while the fuzzy system $\theta_2^T \psi_2(\bar{z}_2)$ has the state vector $[x_1\, x_2\, x_3\, x_4\, x_5]^T$ as input. For each variable of the entries of these fuzzy systems, as in [19], we define three (one triangular and two trapezoidal) membership functions uniformly distributed on the intervals $[-0.5, 1.5]$ for x_2, x_3, x_4 and x_5, $[-150, 200]$ for x_1, $[-50, 50]$ for $\bar{\upsilon}$, and $[-5, 8]$ for $\hat{\Gamma}_l$.

Table1. Parameters of DFIM.

Parameter	Value
Rated power	P_n =4 kW
voltage	U =220/380 V
Current	I = 15/8.6 A
Synchronous speed	$\omega_{sn} = 2\pi 50\ Hz$
Stator resistance	R_s =1.2 Ω
Rotor resistance	R_r =1.8 Ω
Stator inductance	L_s =1.1554H
Rotor inductance	L_r =1.1568 H
Mutual inductance	M =0.15 H
Inertia	J=0.2 kg.m^2
Friction coefficient	k_f=0.014 Nm.s/rad
Pole pairs	p=2

The simulation is carried out under the followings adverse conditions

-***Model uncertainties:*** $\delta_1(x) = 3x_2$ and $\delta_2(x) = 4x_2 + 2x_3$ and introduced at $t = 0.8s$.

-***Parameter variations:*** At t=0.4s and t=0.5 s, the stator and the rotor resistances are respectively increased at 50% of theirs ratted values.

-***External load torque:*** A load torque disturbance is applied as follows

$$\Gamma_l = \begin{cases} 0\ Nm & t \le 0.3s \\ 5\ Nm & 0.3s \le t \le 0.7s \\ 0\ Nm & 0.7s \le t \le 1s \\ 5\ Nm & 1s \le t \le 1.4s \\ 0\ Nm & t \ge 1.4s \end{cases}$$

Fig. 4, Fig. 5 and Fig. 6 show the simulation results of the NABC system. As it can be seen, the system control performances degrade in the presence of external disturbances and unstructured dynamical uncertainties.

The simulation results of the proposed AFBC system are depicted in Fig.7, Fig.8 and Fig.9. From these simulation results, we can clearly see that a satisfactory behavior of the mechanical speed with regard to the imposed speed profile is achieved. Moreover, the load torque estimator gives a correct estimation for the actual load torque. The controller copes easily with the sudden external load disturbance, the parametric variations and the model uncertainties, and provides a fast tracking responses. We can observe clearly that the flux responses respect the imposed constraints. So, after transient, the stator and the rotor fluxes recover respectively their imposed values. Consequently, the flux orientation objective is guaranteed, and the stator reactive power converges to zero in steady-state operation.

The AFBC approach is compared in similar operating conditions to the NABC approach. Fig. 10 shows the speed and flux tracking performances under the two control methods. It is evident that the proposed AFBC schema yields superior control performances than the NABC scheme. In fact, this new control scheme can achieve high accuracy in speed and flux tracking and shows very strong robustness to external load disturbance and the system uncertainty. Moreover, the control effort of the AFBC is smaller than that of the NABC.

Table 2. Numerical comparison between the NABC and AFBC systems.

Control Strategy	*MSE (%)*			
	Speed Ω	*Flux φ_{sd}*	*Flux φ_{sq}*	*Flux φ_{rd}*
NABC	40×10^{-2}	$4{,}1 \times 10^{-4}$	$0{,}4 \times 10^{-3}$	$3{,}6 \times 10^{-4}$
AFBC	$9{,}3 \times 10^{-2}$	$8{,}4 \times 10^{-5}$	$0{,}7 \times 10^{-4}$	$5{,}4 \times 10^{-4}$

The superiority of the AFBC over NABC is clearly shown in these results. Based on the simulation results, numerical comparison of the two control methods developed in this work is shown in Table. 2.

5. CONCLUSION

A new adaptive fuzzy backstepping control scheme has been proposed for high-performance DFIM drive. The backstepping technique has been applied to systematically construct our controller which guarantees uniform ultimate boundedness of all signals in the closed-loop system. Lyapunov approach has been adopted to derive the parameter adaptation laws. The tracking error dynamics have been proved to exponentially converge to a residual adjustable set. The obtained results confirmed the effectiveness of the proposed AFBC scheme for control of a DFIM. It has been shown that the proposed controller allows good tracking performances and stator reactive regulation to zero in steady state, and can deal with the unavoidable parameters variations, external disturbance and model uncertainties. Comparison results against a NABC system show better robustness to parameter variations and system uncertainty. It is worth noting that the control methodology proposed here can be easily extended to other electric drives. Our future work will address the experimental implementation of this proposed AFBC scheme and the design of a speed sensorless controller.

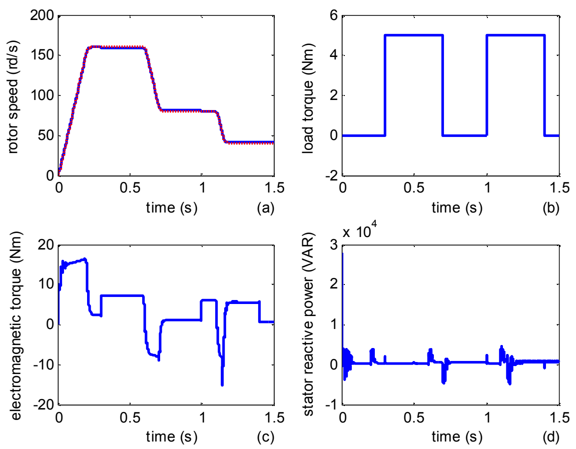

Fig.4. NABC scheme: (a) Rotor speed (Ω solid line, Ω^* dotted line). (b) actual load torque. (c) Torque. (d) Stator reactive power.

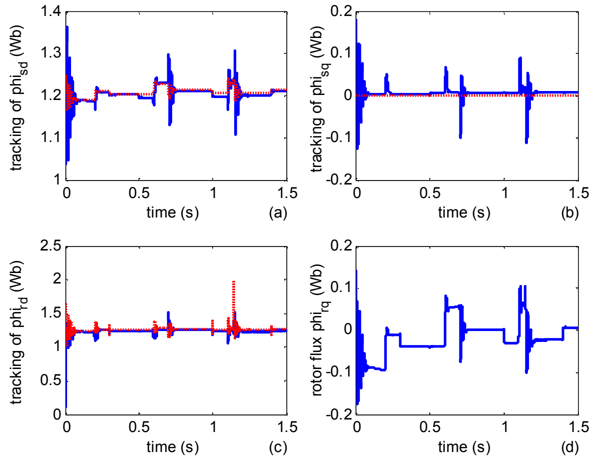

Fig. 5. NABC scheme: (a) φ_{sd} (solid line) and φ^*_{sd} (dotted line). (b) φ_{sq} (solid line) and φ^*_{sq} (dotted line). (c) φ_{rd} (solid line) and φ^*_{rd} (dotted line). (d) φ_{rq} .

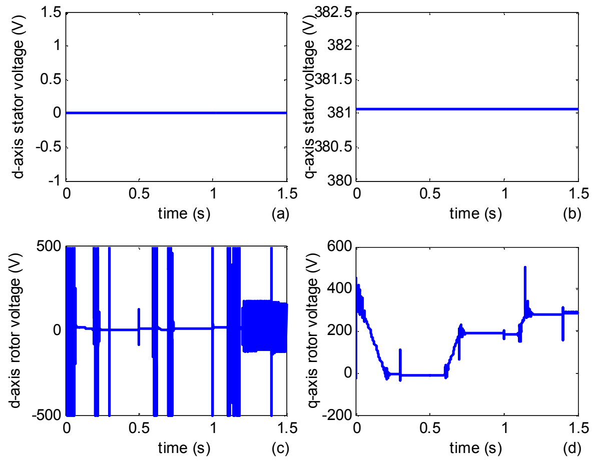

Fig.6. NABC scheme: (a) Stator voltage u_{sd} . (b) Stator voltage u_{sq} . (c) Rotor voltage u_{rd} . (d) Rotor voltage u_{rq} .

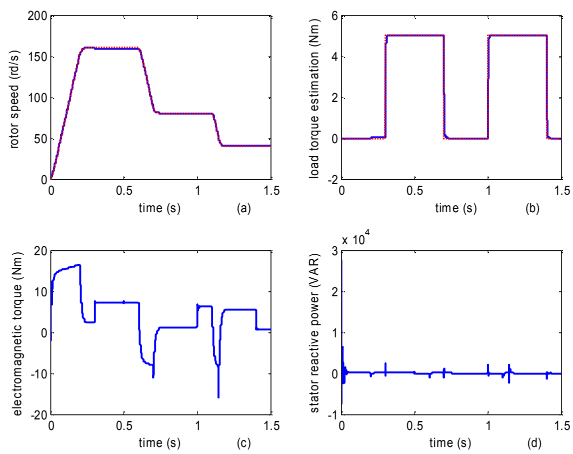

Fig.7. AFBC scheme: (a) Rotor speed: Ω (solid line) and Ω^* (dotted line). (b) Load torque: Γ_e (solid line) and $\hat{\Gamma}_e$ (dotted line). (c) Torque. (d) Stator reactive power.

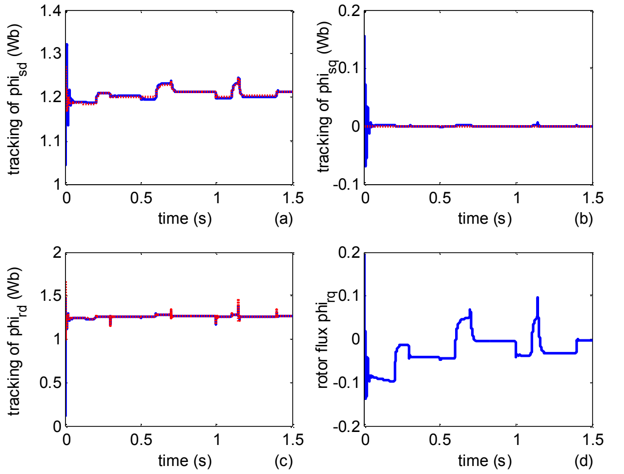

Fig.8. AFBC scheme: (a) φ_{sd} (solid line) and φ_{sd}^* (dotted line). (b) φ_{sq} (solid line) and φ_{sq}^* (dotted line). (c) φ_{rd} (solid line) and φ_{rd}^* (dotted line). (d) φ_{rq} .

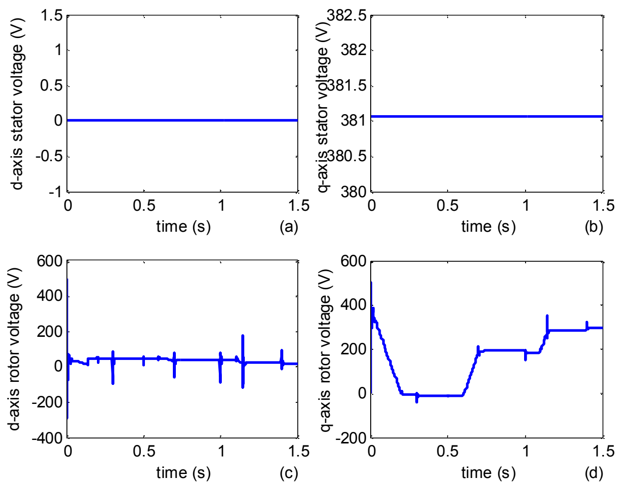

Fig.9. AFBC scheme: (a) Stator voltage u_{sd} . (b) Stator voltage u_{sq} . (c) Rotor voltage u_{rd} . (d) Rotor voltage u_{rq} .

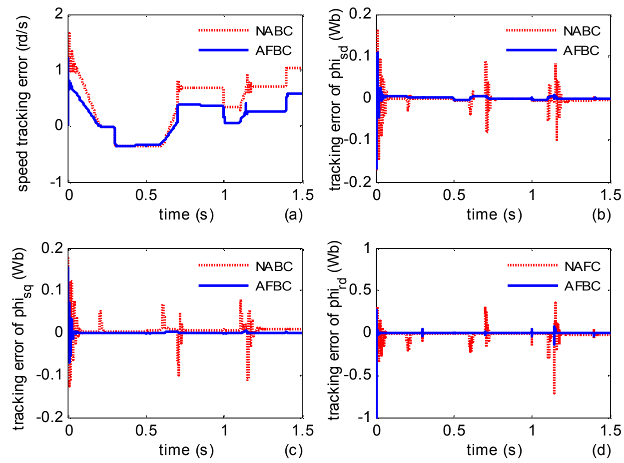

Fig.10. Tracking performances for both controllers

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