HYBRID CONTROL FOR SHAPE MEMORY ALLOY ACTUATORS

Pană Daniela Maria

University of Craiova,
Faculty of Automation, Computers and Electronics,
Mechatronics Department,
Bv. Decebal, nr.5

Abstract: *The intelligence exhibits self adaptively, self understanding, memory of the materials or structures. The intelligent and complex behaviour of these materials can not be often synthesized in formal way or suppose strong nonlinear characteristics (eg. the hysterezis exhibits by the functionality of shape memory alloys) becoming difficult to model and control by conventional methods. The fuzzy sets theory provides some proper tools for handling heuristic algorithms by linguistic description. Furthermore sliding mode control theory offers certain paradigms to control nonlinearities from the systems. In this paper a hybrid control method (fuzzy sliding mode control) is used to control a rotational joint with shape memory alloy actuation.*

Keywords: *SMA, fuzzy logic, Lyapunov functions, unconventional control*

1. INTRODUCTION

SMA advantages include: Simplicity of actuation mechanism. SMAs can be all-electric devices and can be used as "Direct Drive Linear Actuators" requiring little or no additional gear reduction or motion amplification hardware. These merits permit the realization of small or even miniature actuation systems in order to overcome space limitations. (b) Silent actuation. Since no acoustic signature is associated with such a propulsion system, acoustic detectability will be reduced. (c) Low driving voltages, when powered electrically. Nitinol (NiTi) SMAs can be actuated with very low voltages (10 to 20 V), thus requiring very simple power supply hardware.

However, for SMAs large currents may be required for resistive heating, depending on SMA actuator dimensions, leading to heavy power supply hardware.

On the other hand, Shape Memory Alloy actuators possess hard nonlinearities include backlash-like hysteresis and saturation due to SME, which makes difficulties in precise control. Thus, numerous approaches for SMA actuator control have been proposed [6, 9, 12]. However, some unsolved problems still remain. Ordinary PID control schemes often show steady-state errors and limit-cycle problems. A conventional control approach is proposed in [11] where authors' control system is composed of a PID feedback loop and a feed forward loop. They showed that the limit-cycle oscillations were significantly reduced and the tracking control performances were improved. A variable structure control scheme applied in [4, 5] to a pair of antagonist actuators realized smooth and robust control. In order to compensate the hysteresis of SMAs different researchers applied either neuro-fuzzy based control to SMA actuators [7, 8], but tracking was not so accurate or an inverse hysteresis model for the SMA plant [11]. However, a generalized model that accurately explains the behaviour of SMA is difficult to obtain. Further, it is also difficult to achieve a perfect compensation even though a generalized model is obtained. On the other hand, sliding mode control (SMC) does not require an exact mathematical model and also provides robustness against variations of system parameters and disturbances. Although SMC does not require an exact plant model, it is required to estimate the plant dynamics.

2. OPERATION PRINCIPLE OF SMA

The ability of SMAs to memorize their original configuration after they have been deformed in the martensitic phase, by heating the alloys above their characteristic transition temperature (Af) hence recovering large strains, is known as the shape memory effect (SME) and is utilized in most SMA actuator applications.

A qualitative schematic of the SMA phase transformation diagram is presented (fig. 1) to clarify the actuator functions.

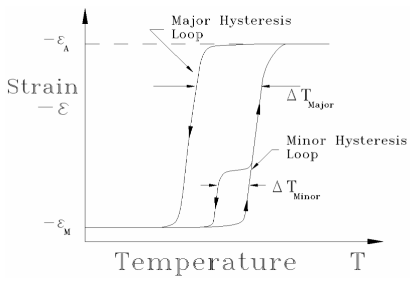

Fig. 1. Strain-temperature SMA transformation diagram.

The behaviour of SMAs is more complex than many common materials: the stress-strain relationship is nonlinear, hysteretic, exhibits large reversible strains, and is strongly temperature dependent and sensitive to number and sequence of thermo mechanical loading cycles. The key characteristics of SMAs are pseudo elasticity, one-way shape memory effect (OWSME) and two-way shape memory effect (TWSME).

This variation in thermo mechanical response requires further understanding of its influence for successful implementation of SMA actuators.

3. CONSTITUTIVE MODEL OF SMA ACTUATOR

A variety of constitutive models have been, most aimed at the one-dimensional description of material behavior, and include work by Tanaka [13, 14], Brison [2, 3] and Liang and Rogers [10]. One feature of many of the constitutive laws of SMA behavior is that the models can generally separated into a mechanical law governing stress-strain behavior and kinetic law governing transformation behavior.

In this study the author has selected from the wide literarure approches the constitutive model of shape memory alloy proposed by Brison and Hwang [2, 3].They developed a cosine form based constitutive model which emphased that the material martensite transformation fraction is the sum between the material martensite transformation fraction caused by the temperature, f_{mar}^{T} and the material martensite transformation fraction caused applied force (stress), f_{mar}^{σ}.

$$f_{mar} = f_{mar}^{T} + f_{mar}^{\sigma} \quad (1)$$

The proposed stress-strain constitutive law is :

$$\begin{aligned}\sigma - \sigma_0 &= E(f_{mar}) \cdot (\varepsilon - \varepsilon_0) + \\ &+ \Omega(f_{mar}) \cdot (f_{mar}^{\sigma} - f_{mar}^{\sigma 0}) + \\ &+ \theta(f_{mar}) \cdot (T - T_0)\end{aligned} \quad (2)$$

Where: $\sigma - \sigma_0$ is the stress rate in the wire, ε is the strain, T is the temperature, E is the Young's modulus of the material, Θ is the thermal coefficient of expansion, f_{mar} is the martensitic fraction in material, and Ω is the transformation tensor:

$$E(f_{mar}) = E_A + f_{mar}(E_M - E_A) \quad (3)$$

$$\Theta(f_{mar}) = \Theta_A + f_{mar}(\Theta_M + \Theta_A) \quad (4)$$

$$\Omega(f_{mar}) = -\varepsilon_L E(f_{mar}) \quad (5)$$

$$\Theta(f_{mar}) = \alpha(f_{mar}) E(f_{mar}) \quad (6)$$

Where ε_L is the maximum residual strain for the given SMA material.

As a kinetic law in this paper the author has chossen to express the material martensitic transformation, according with Liang consine formulation [10] by type of equation (7):

For

$$\begin{cases} T > M \\ \sigma_s^{tr} + C_M(T - M_s) < \sigma < \sigma_f^{tr} + C_M(T - M_s) \end{cases}:$$

$$f_{mar} = \frac{1 - f_{mar}^0}{2} \cdot \cdot \cos\left\{ \frac{\pi}{\sigma_s^{tr} - \sigma_f^{tr}} \left(\sigma - \sigma_f^{tr} - C_M(T - M_s)\right) \right\} + \frac{1 + f_{mar0}}{2} \quad (7)$$

In this approximation is synthesized that the current direction of deforming force action transformation is depending on prior martensitic phase. This fact is emphased by the presence of f_{mar0} terms.

Finally the thermic model [1] is given by the equation (8):

$$\rho c_v \dot{\theta} + \frac{4h}{d}\theta - \rho \Delta u^* (\dot{f}_{mar}^{\sigma} + \dot{f}_{mar}^{T}) = \frac{16\rho_e}{\pi^2 d^4} u \quad (8)$$

Where u is input curent.

4. POSITION CONTROL OF SMA

As it shown in paragraph 3, the estimation of the system dynamics in the case of SMA is difficult as SMA dynamics possesses an ample of model uncertainties.

In the present paper author applied Fuzzy SMC, to a linear type SMA actuator, whose location on the mechanical system provide a rotary motion.

The dynamical model of the mechanical arm is described by equation (9):

$$J\ddot{\theta} + a\dot{\theta} + f(\theta) = M_{fir}(\sigma) \quad (9)$$

Where: J represents the inertial component,
a the term assigned viscous friction,
f gravitational and elastic terms caused by loading and elastic efects of the bias spring from SMA actuator, formulated:

$$f(\theta) = M_g(\theta) - M_a(\theta) \quad (10)$$

$M_{fir}(\sigma)$ represents SMA element moment, that can be expresed in terms of SMA stress σ using either Tanaka or Brinson-Hwang constituive laws.

The error of the control system is defined between the target value θ_d and the real value θ as:

$$e + \theta_d = \theta; e = \theta - \theta_d \quad (11)$$

The generalized error and the derivative of this are also introduced.

$$s = \dot{e} + \lambda e; \dot{s} = \ddot{e} + \lambda \dot{e} \quad (12)$$

The generalized error s allows defining the control problem of position by:

$$\lim_{t \to \infty} s(t) = 0 \quad (13)$$

In order to find the conditions imposed for the components of the control system equation (9) is analyzed along the target trajectory, equation (14):

$$\begin{aligned} &J(\ddot{e} + \ddot{\theta}_d) + a(\dot{e} + \ddot{\theta}_d) + \\ &+ f(e + \theta_d) = M_{fir}(\sigma) \end{aligned} \quad (14)$$

$$\ddot{e} = \dot{s} - \lambda \dot{e}; \dot{e} = s - \lambda e \quad (15)$$

This equation can be written in terms of generalized error as in (12).

$$\begin{aligned} &J\dot{s} - J\lambda\dot{e} + J\ddot{\theta}_d + a(s - \lambda e + \dot{\theta}_d) + \\ &+ f(e + \theta_d) = M_{fir}(\sigma) \end{aligned} \quad (16)$$

$$\begin{aligned} &J\dot{s} - J\lambda(s - \lambda e) + J\ddot{\theta}_d + as - a\lambda e + a\dot{\theta}_d \\ &+ f(e + \theta_d) = M_{fir}(\sigma) \end{aligned} \quad (17)$$

$$(J\dot{s} - J\lambda s + as) + J\sigma^2 e + J\ddot{\theta}_d - a\lambda e + a\dot{\theta}_d + \\ + f(e + \theta_d) = M_{fir}(\sigma) \quad (18)$$

Or as equation (19):

$$J\dot{s} - (J\lambda - a)s + H(e, \theta_d, \dot{\theta}_d, \ddot{\theta}_d) = \\ = M_{fir}(\sigma) \quad (19)$$

Where $H(e, \theta_d, \dot{\theta}_d, \ddot{\theta}_d) = J\sigma^2 e + J\ddot{\theta}_d - a\lambda e + a\dot{\theta}_d + f(e + \theta_d)$ represents the movement of the arm nonlinear term.

$$\dot{s} = J^{-1}(J\lambda - a)s + J^{-1}H + J^{-1}M_{fir}(\sigma) \quad (20)$$

To accomplish the condition $\lim_{t\to\infty} s(t) = 0$ is define a Lyapunov function as follows:

$$V = \frac{1}{2} s^T J s \quad (21)$$

Applying the usual procedure used in Lyapunov's second method is obtained equation (22):

$$\dot{V} = \dot{s}Js = \\ = (J^{-1}(J\lambda - a)s + J^{-1}H + J^{-1}M_{fir}(\sigma)) \cdot Js \quad (22)$$

$$\dot{V} = (J\lambda - a)s^2 + Hs + M_{fir}(\sigma)s \quad (23)$$

The control law:

$$M_{fir}(\sigma) = -ks - H^* + u \quad (24)$$

Where k represents a positive constant, the second component H^* assures compensation of the terms defined by the eror e and the desire position, and the last term $u = -k \operatorname{sgn} s$ represents an added comand, in order to compensate model uncertenies which forces the system evolution to alinite to the sliding surface. By introduction of control law (24) results equations (25) and (26):

$$\dot{V} = (J\lambda - a)s^2 + Hs + (-ks - H^* + u)s \quad (25)$$

$$\dot{V} = (J\lambda - a - k)s^2 + (H - H^*)s + u \cdot s \quad (26)$$

$$u = -k \operatorname{sgn} s \quad (27)$$

$$\dot{V} = -(-J\lambda + a + k)s^2 + \\ + ((H - H^*) + -k \operatorname{sgn} s) \cdot s \quad (28)$$

$$\dot{V} \le -(-J\lambda + a + k)s^2 + \\ + \left|H - H^*\right|\left|s\right| - k \operatorname{sgn}\frac{|s|}{s} \cdot s \quad (29)$$

$$\dot{V} \le -(-J\lambda + a + k)s^2 + \\ + (\left|H - H^*\right| - k) \cdot |s| \quad (30)$$

$$k > (\left|H - H^*\right|) \quad (31)$$

5. FUZZY SLIDING MODE CONTROL APPLIED TO SMA ACTUATOR

Assuming that the follow conditions are accomplished:

$k - J\lambda + a$: Positive definite
$u = -k \operatorname{sgn} s$
$k \ge \left|H - H^*\right|$

Relation

$$\dot{V} = -(-J\lambda + a + k)s^2 + \\ + ((H - H^*) + -k \operatorname{sgn} s) \cdot s \quad (32)$$

derived that

$$\dot{V} \le -(-J\lambda + a + k)s^2 + \\ + \left|H - H^*\right|\left|s\right| - k \operatorname{sgn}\frac{|s|}{s} \cdot s \quad (33)$$

Or

$$\dot{V} \le \Lambda_{\min}\|s\| \quad (34)$$

Condition $k - J\lambda + a$ dictates the control matrix K of conventional loop.
The relations $u = -k \operatorname{sgn} s$, $k \ge \left|H - H^*\right|$ allow using a fuzzy controller.
The control scheme is presented as following (fig.2.):

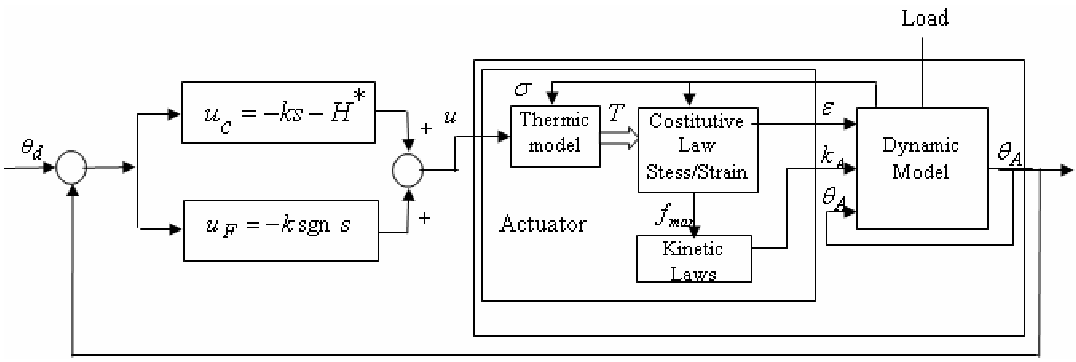

Fig. 2. Control scheme

Output (command) increase directly with the distance from the switching line

$$s = \dot{e} + \lambda e \tag{35}$$

DSCM evolution of the system contents two parts: a free movement towords the switching line and a forced movment on the switching line (fig. 3.).

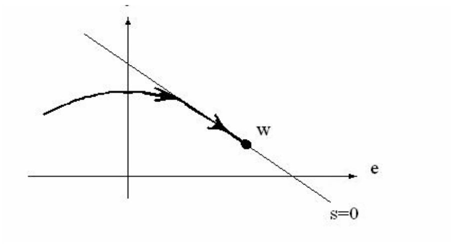

Fig. 3. Switching line

Consider a normalized space:

$e_i^N, \dot{e}_i^N \in [-1.1]$

$e_i^N = \alpha_i e_i$

$\dot{e}_i^N = \beta_i \dot{e}_i$,

α_i, β_i represent the scaling factors.

For every input $e_i, \dot{e}_i$ and output u of the controller are assigned five tags of the membership functions chosen as triangular shape (fig. 4.).

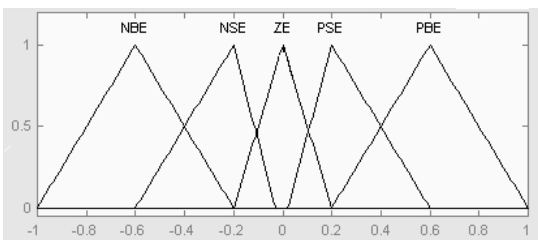

Fig. 4. Membership functions

In table 1 the controller rule base is presented:

DE \ E	NBE	NSE	ZE	PSE	PBE
PBDE	ZC	NBC	NBC	NBC	NBC
PSDE	PBC	ZC	NSC	NSC	NBC
ZDE	PBC	PSC	ZC	NSC	NBC
NSDE	PBC	PSC	PSC	ZC	NBC
NBDE	PBC	PBC	PBC	PBC	ZC

Table 1. Rule base of the controller

Note here the presence of tree regions:

- Negative values of the command u are located above the first diagonal and the positive ones below.
- Fuzzy values of the command increase directly with the distance from diagonal
- Zero values of the command are located on first diagonal
-

In order to argument the hybrid control proposed, a set of numerical tests are perforemed. These tests analyze the experimental response of SMA actuator for different exploatation conditions.

Test 1. Experimental response of the actuator by apllying a step signal with the emphase of heating times is presented in fig. 5. As is seen here the experimental response when using the approch presented in the paper is better comparing with the conventional methods.

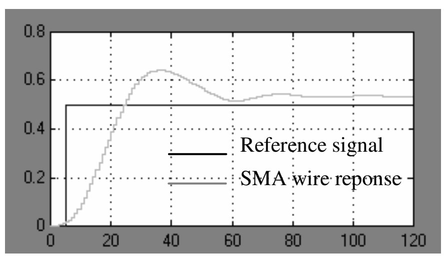

Fig. 5. SMA response

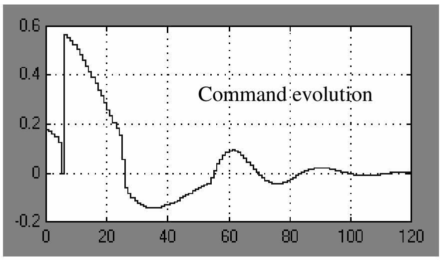

Fig. 6. Command evolution

Figure 7 presents the phase portrait system evolution.

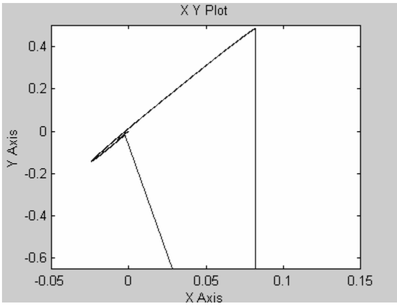

Fig. 7. Phase portrait

Test 2. Figure 8 presents system response by apllying diferent values of step signal, and the figure presents comand evolution.

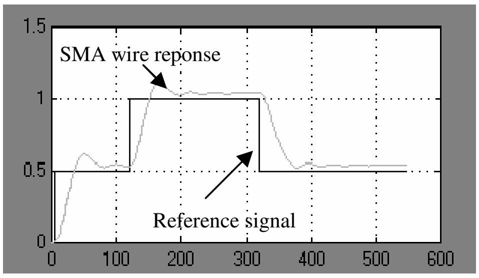

Fig. 8. SMA response

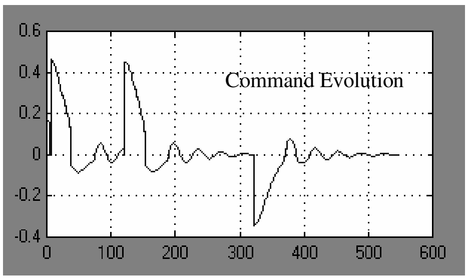

Fig. 9. Command evolution

Test 3. Figure 10 presents system response for the case when a pulse generetor reference signal is apllied and figure presents comande evolution.

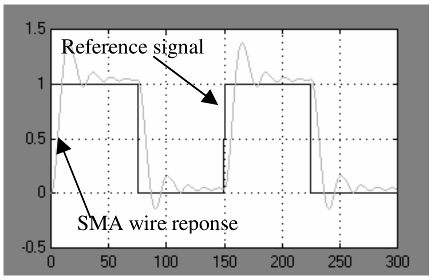

Fig. 10. SMA response

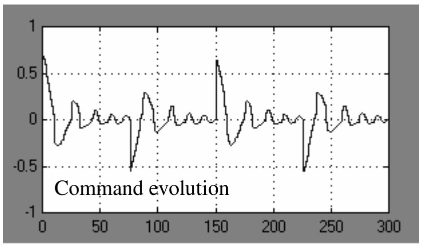

Fig.11. Command evolution

6. CONCLUSION

In the present paper author applied Fuzzy SMC, to a linear type SMA actuator whose location on the mechanical system provide a rotary motion.
The fuzzy sets theory provides some proper tools for handling euristic algorithms by lingvistic description. Furthermore sliding mode control theory offers certain paradigmas to control neliniartities from the systems.

REFERENCES

[1] A. Bhattacharyya, DC. Lagoudas, Y Wang, VK Kinra, "On the role of thermoelectric heat transfer in the design of SMA actuators: theoretical modelling and experiment", Smart Mater. Struct. 4, p. 252 (1995)

[2] Brinson C., "One dimensional constitutive behavior of shape memory alloys: thermomechanical derivation with non-constant functions and redefined martensite internal variable", *J. Intelligent mater. Syst. structures*,4, pp. 163, 1993.

[3] Brinson L. C., Huang M. S., "Simplification and Comparison of Shape Memory Alloy Constitutive Models", *Journal of Intelligent Material Systems and Structures*, **7**, pp. 108 – 114, 1996.

[4] D. Grant and V. Hayward. "Design of shape memory alloy actuator with high strain and variable structure control". In Proceedings of 1995 IEEE International Conference on

Robotics and Automation, volume 3, pages 344 - 350, Nagoya, Aichi, Japan, May 21-27 1995.
[5] D. Grant and V. Hayward. "Variable structure control of shape memory alloy actuators." IEEE Systems and Control Magazine, 17(3):80-88, 1997.
[6] M Hashimoto, M Taheda, H Sagawa, I Chiba, K Sato, "Application of shape memory alloy to robotic actuators", Journal of Robotic Systems 2(1), p. 3 (1995)
[7] K. Kuribayashi, "A new actuator of a joint mechanism using TiNi alloy wire". Int.J. Robot. res. 4(4), p. 47 (1986)
[8] K. Kuribayashi, "Improvement of the response of an SMA actuator using temperature sensor", the International Journal of Robotics Research 10, 1, p. 13 (1991)
[9] Leclercq, S., Lexcellent, C., "A general macroscopic description of the thermo mechanical behaviour of shape memory alloys", J. Mech. Phys. Solids, 44, 6, p. 953 (1996)
[10] Liang C., Rogers C. A., "One dimensional thermomechanical constitutive relations for shape memory materials", *J. Intell. Material Systems and Structures*, 1, pp. 207 - 234, 1990.
[11] D.R. Madill. "Modelling and stability of a shape memory alloy position control system." Master's thesis, Applied science, University of Waterloo (1993)
[12] R. A. Russell, R. B. Gorbet, "Improving the response of SMA actuators", Proceeding of the IEEE int. Conference on Robotics and Automation, pp 2299-2304 (1995)
[13] Tanaka K., "A thermomechanical sketch of shape memory effect one dimensional tensile behaviour", *Res. Mechanica*, 18, pp. 251 - 263, 1986.
[14]Tanaka K., Kobayashi S., Sato Y., "Thermomechanics of Transformation Pseudoelasticity and shape memory effect in alloys", *Int. J. Plast.*, 2, pp. 59 - 72, 1986.