Adaptive and Robust-Adaptive Control Strategies for a Class of Fed-Batch Fermentation Processes

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Abstract: This paper presents the design and analysis of adaptive and robust-adaptive control strategies for a class of fed-batch fermentation processes. The control strategies are developed under the realistic assumption that both the bacterial growth rates and the influent flow rates are time-varying and uncertain, but some lower and upper bounds of these uncertainties are known. The adaptive control structure is achieved by combining a linearizing control law with a state asymptotic observer and a parameter estimator used for on-line estimation of bioprocess unknown kinetics. The robust-adaptive control structure is achieved by combining a linearizing control law with an interval observer able to estimates a lower and an upper bound of unmeasurable states. Also, in the proposed robust-adaptive control strategy the uncertain process parameters are replaced by their lower and upper bounds assumed known. The effectiveness of the designed algorithms is validated by several numerical simulations applied to a particular alcoholic fermentation bioprocess.

Keywords: State observers, Interval observers, Fed-batch fermentation processes, Adaptive control, Robust-adaptive control, Alcoholic fermentation bioprocess.

1. INTRODUCTION

The biotechnology, whose applications can be found in many domains, is one of the fields that over the last decades have a high development. An important issue of biotechnology is the synthesis of some products by using fermentation processes. Therefore, due to its advantages, the control of industrial bioprocesses has been an important practical problem attracting wide attention.

Fermentation processes that are carried out in perfectly stirred tank reactors are commonly described by a set of ordinary differential equations expressing mass and energy balances. A basic difficulty for the application of modern control techniques to these processes lies in the fact that, in many cases, the models include kinetic parameters, which are highly uncertain and time varying (Bastin and Dochain 1990; Dochain and Vanrolleghem 2001; Bernard and Bastin 2004; Dochain 2008; Petre 2008). Another important challenge in the monitoring and control of such living processes is finding adequate and reliable sensors to measure all the important state variables of the plant (Bastin and Dochain 1990; Dochain and Vanrolleghem 2001; Bernard and Bastin 2004). Even if several on-line sensors providing state information are available today at industrial scale, they are still expensive, especially in the field of biological processes (Bastin and Dochain 1990; Dochain and Vanrolleghem 2001; Bernard and Bastin 2004; Dochain 2008).

To overcome these difficulties, several strategies were developed, such as linearizing strategy (Bastin and Dochain 1990; Dochain 2008; Petre 2008), adaptive approach (Bastin and Dochain 1990; Dochain 2008; Petre 2008, Petre et al. 2008), optimal control (Bastin and Dochain 1990; Queinnec et al. 1991; Van Impe et al. 1994), sliding mode control (Selişteanu et al. 2007), neural strategies (Hayakawa 2008; Petre et al. 2010) and so on. Some of these approaches imposed the use of the so-called “software sensors” – combinations between hardware sensors and software estimators (Bastin and Dochain 1990; Dochain and Vanrolleghem 2001; Bernard and Bastin 2004). Note that these software sensors are used not only for the estimation of concentrations of some components but also for the estimation of kinetic parameters or even kinetic reactions (Bastin and Dochain 1990; Dochain and Vanrolleghem 2001; Dochain 2008). However, in all previously cited schemes, the knowledge of all the inputs of the process, including, for example, the substrate input concentration, is needed. Unfortunately, there are many bioprocesses for which the complete knowledge of inputs is not available. As a consequence, part of the process input vector is considered as unmeasured input disturbance and classical observer schemes cannot be used (Alcaraz-González et al. 2000, 2003, 2005; Aviles and Moreno 2009; Moisan and Bernard 2005; Rapaport and Dochain 2005). For these situations, in the last decade it was developed a special class of observers called *set-observers*, which allows the user to reconstruct a guaranteed interval on the unmeasured states instead of reconstructing their precise numerical values (Alcaraz-González et al. 2000, 2003, 2005; Aviles and Moreno 2009; Goffax et al. 2009, Mazenc et al. 2009, 2011; Moisan and Bernard 2005; Rapaport and Dochain 2005). The only requirement is to know an interval in which the unmeasured inputs of the process evolve. These robust observers are capable of coping simultaneously with the problems posed by both the uncertainties in the inputs and a full unknowledge of the nonlinearities or process kinetics (Alcaraz-González et al. 2000, 2003, 2005; Aviles and Moreno 2009; Moisan and Bernard 2005; Rapaport and Dochain 2005). Even if the design of interval observers has a high development, only few papers were dedicated to designing of robust output feedback controllers using interval observers (Alcaraz-González et al. 2000; Rapaport and Harmand 2002; Rapaport et al. 2006; Petre et al. 2012).

This paper presents the design and analysis of adaptive and robust-adaptive control schemes applied to alcoholic fermentation processes that are carried out in fed-batch reactors. In contrast with continuous stirred tank reactors, which continuously operate in steady state, fed-batch reactors are permanently in a transient regime and therefore offer challenging problems to the control engineer. Industrial fed-batch stirred tank reactors are traditionally operated in open loop using prior calculated feeding patterns and dosage schemes (Van Impe 1994). But to increase the efficiency of these processes, there has been interest in the application of modern control theories. So, for optimization of alcoholic fed-batch fermentation process there were used dynamic programming and nonlinear programming schemes (Van Impe 1994; Duvivier and Sévely 1988). Some other linear and non-linear adaptive control strategies (Dahhou et al. 1993; Petre 2005) were developed. But the knowledge of all inputs is of crucial matter for all these kind of control strategies.

In this work, the concentration of the influent substrate will be considered highly uncertain or even completely unknown, but some intervals in which this unmeasured concentration evolve are known. Also, the kinetic parameters will be considered, like in reality, highly uncertain and time varying, but some lower and upper bound of these uncertainties are assumed known. In order to design control algorithms under abovementioned conditions, a state asymptotic observer and a robust interval observer are briefly presented. Using these observers some adaptive and robust-adaptive control schemes are developed and analysed.

The adaptive control structure is achieved by combining a linearizing control law with a state asymptotic observer which plays the role of the software sensor for on-line estimation of the interest process biological states and a parameter estimator for on-line estimation of uncertain or unknown bioprocess kinetic rates.

The robust-adaptive control structure is achieved by combining a linearizing control law with an appropriately interval observer able to estimates lower and upper bounds of unmeasurable states. Furthermore, the uncertain process parameters are replaced by their lower and upper bounds assumed known.

The paper is organized as follows. A briefly description of a class of fed-batch fermentation processes and its modelling are presented in Section 2. An adaptive and a robust-adaptive control strategy for this class of bioprocesses are presented in Section 3. The performance of the proposed control algorithms applied to an alcoholic fermentation process is validated by using numerical simulations presented in Section 4. Concluding remarks in Section 5 complete this paper.

2. BIOPROCESSES DESCRIPTION AND MODELLING

Consider the class of fed-batch fermentation processes involving one limiting substrate for biomass growth and product synthesis that are carried out in fed-batch stirred tank bioreactors. A representative process from this class is the *alcoholic fermentation bioprocess* whose mathematical model obtained from mass balance considerations is given by the following set of nonlinear equations (Queinnec 1991):

$$\dot{X}(t) = \mu(\cdot)X - X(F_{in}/V), \tag{1}$$

$$\dot{S}(t) = -\nu_S(\cdot)X + (S_{in} - S)(F_{in}/V), \tag{2}$$

$$\dot{P}(t) = \nu_P(\cdot)X - P(F_{in}/V), \tag{3}$$

$$\dot{V}(t) = F_{in}, \tag{4}$$

with X - biomass concentration, S - substrate concentration, S_{in} - influent substrate concentration, P - product concentration, V - volume of the culture medium, F_{in} - volumetric feed rate, $D = F_{in}/V$ - dilution rate, μ - specific growth rate, ν_S - specific substrate consumption rate and ν_P - specific production rate. The parameters appearing in this description are complicated functions of the variables of interest. The challenge for the control engineer arises from the fact that the analytical modelling of these specific rates functions is highly uncertain and generally not reproducible from one fed-batch to the next one. After several experiments the following expressions for the bacterial growth rate have been adopted (Queinnec 1991):

$$\mu(\cdot) = \mu_{\max}(1 - P/P_m)\cdot S/(K_S + S + S^2/K_I), \tag{5}$$

$$\nu_P(\cdot) = \nu_{\max} S/(K'_S + S + S^2/K'_I), \tag{6}$$

$$\nu_S(\cdot) = (1/Y_{X/S})\cdot\mu(\cdot) + (1/Y_{P/S})\cdot\nu_P(\cdot), \tag{7}$$

with $Y_{X/S}$ - biomass on substrate yield coefficient, $Y_{P/S}$ - product on substrate yield coefficient and P_m - alcohol inhibition factor. This model takes into account both substrate and product inhibition on the growth and the fact that growth and production interact. Fed-batch fermentation processes have been found to be most effective in overcoming such effects as substrate inhibition, catabolite repression, and glucose effects. In other words, whenever the specific rate of growth μ and/or production are non-monotonic functions of the limiting substrate concentration (as in our case), a fed-batch operation may be superior and it is then necessary to determine the optimal feed rate of substrate (for details, see Van Impe 1994).

Remark 1. It is known that temperature is an important factor that can to influence the reaction rates of the alcoholic fermentation. For example, in the case of wine fermentation, the mathematical models of reaction rates ware adapted so that to include the influence of temperature (Coleman et al. 2007; David et al. 2010). It must to note that the mathematical model (1)-(7), used in this paper, was obtained under appropriately (standard) constant ambient temperature and pressure.

Defining the state vector as $\xi = [\,X\ S\ P\,]^T$, the model (1)-(3) can be written in a compact form as:

$$\dot{\xi} = K\varphi(\xi) - D\xi + F - Q = KG(\xi)\alpha(\xi) - D\xi + F - Q\,, \qquad (8)$$

where $F = [0\ \ F_{in}S_{in}/V\ \ 0]^T$ is the vector of mass inflow rates, $Q = [0\ \ 0\ \ 0]^T$ is the vector of gaseous outflow rates, $\varphi = [\varphi_1\ \varphi_2]^T$, with $\varphi_1 = \mu X$ and $\varphi_2 = \nu_P X$, is the vector of reaction rates, which can be written as $\varphi(\xi) = G(\xi)\alpha(\xi)$, with $G(\xi)$ a diagonal matrix whose entries are products of the component concentrations involved in each reaction and $\alpha = [\,\alpha_1\ \alpha_2\,]^T$ the vector of specific reaction rates, and K is the constant yield coefficients matrix. The matrices K and G have the following structure:

$$K = \begin{bmatrix} 1 & -1/Y_{X/S} & 0 \\ 0 & -1/Y_{P/S} & 1 \end{bmatrix}^T,\ G = \begin{bmatrix} XS & 0 \\ 0 & XS \end{bmatrix}. \qquad (9)$$

For the alcoholic fermentation bioprocess, the *control objective* consists in adjusting the plant's load in order to convert the substrate (glucose) into alcohol via fermentation, to get a large production of alcohol.

From the above inhibition considerations, it follows that the alcohol production process requires regulation of the substrate concentration S inside the bioreactor at a set point S^* corresponding to a desired biomass specific growth rate by acting on the feeding substrate rate F_{in}.

An optimal value of the set point S^* can be obtained by analysing the ratio between the alcohol production and the yield of the bioreactor (see Bastin and Dochain 1990; Petre 2005). More exactly, considering that the process model (8) is incompletely known, its parameters are time varying and not all the states are available for measurements, the control goal is to maintain the substrate concentration inside the reactor at some values, which correspond to both a maximal production of alcohol and a yield of the bioreactor.

3. CONTROL STRATEGIES

In this section under the assumptions formulated in Sections 2, for the class of fed-batch fermentation bioprocess described by dynamical model (8) we will develop some adaptive and robust-adaptive control algorithms for controlling the substrate concentration S inside the bioreactor.

3.1 Exactly linearizing feedback control

Consider the ideal case where maximum prior knowledge concerning the process is available, that is in (8) the specific reaction rates α_1 and α_2 are assumed completely known, while all the state variables and all the inflow and outflow rates are available for on-line measurements. Then, the following exactly linearizing feedback control law (Petre, 2005):

$$F_{in} = \frac{V}{S_{in} - S}\left[\lambda_1(S^* - S) + (1/Y_{X/S})\mu X\ + (1/Y_{P/S})\ \nu_P X\right], \qquad (10)$$

leads to a dynamical behaviour of closed-loop system described by the following first order linear stable differential equation:

$$(\dot{S}^* - \dot{S}) + \lambda_1(S^* - S) = 0\,,\ \lambda_1 > 0\,. \qquad (11)$$

The control law (10) leads to a linear error model $\dot{e} = -\lambda_1 e$, where $e = y^* - y$ represents the tracking error, which for $\lambda_1 > 0$ has an exponential stable point at $e = 0$.

The controller (10) will be used as a benchmark in order to compare its behaviour with the behaviour of the indirect adaptive controller developed in subsection 3.2 as well as with the behaviour of a robust-adaptive controller developed in subsection 3.3.

3.2 An indirect adaptive control strategy

Since the prior knowledge concerning the process assumed in the previous subsection is not realistic, in this subsection we will consider that the process kinetics are incompletely known and time varying and some state variables are not accessible. So, let's assume that the only on-line available measurements are S, S_{in} and P and that reaction rates μ and ν_P and obviously ν_S are time varying and incompletely known. The implementation of the control law (10) requires the knowledge of the state X, and of the reaction rates μ and ν_P. Since X is not measured and μ and ν_P are incompletely known, the control law (10) becomes an adaptive control law by replacing the true unknown values of X, μ and ν_P by their corresponding on-line estimates provided by a suitable state observer and a parameter estimator, respectively.

For the estimation of state X, independent of the unknown reaction rates μ and ν_P, we use an asymptotic observer (Bastin and Dochain 1990; Petre 2005), which can be derived as follows. Let's define the auxiliary state $\hat{w}$ as (Petre 2005):

$$\hat{w} = (1/Y_{X/S})X + (1/Y_{P/S})P + S\,, \qquad (12)$$

whose dynamics, deduced from the model (1)-(3), is expressed by the following linear stable equation:

$$\dot{\hat{w}}(t) = -(F_{in}/V)\hat{w} + (F_{in}/V)\,S_{in}\,. \qquad (13)$$

Then, the on-line estimate $\hat{X}$ of X is calculated from values of $\hat{w}$ via dynamical equation (13), as:

$$\hat{X} = Y_{X/S}\left(\hat{w} - (1/Y_{P/S})P - S\right). \qquad (14)$$

The unknown kinetic terms μ and ν_P in (10) can be written as:

$$\mu(S, P) = S \cdot \alpha_1\,,\quad \nu_P(S) = S \cdot \alpha_2\,, \qquad (15)$$

where the specific reaction rates α_1 and α_2 are considered completely unknown and time varying. This simply expresses that S is a limiting substrate of the reactions and that, in consequence $\mu = 0$ and $\nu_P = 0$ if $S = 0$. For our example:

$$\alpha_1 = \mu_{\max}\left(1 - P/P_m\right)/\cdot(K_S + S + S^2/K_I),$$
$$\alpha_2 = \nu_{\max}/(K'_S + S + S^2/K'_I), \qquad (16)$$

which are positive functions of *S*.

The estimation of α_1 and α_2 can be performed by using an appropriately parameter estimator applied only the dynamics of *S* and *P* given by (2) and (3), respectively, which under the above assumptions can be written as follows (Petre 2005):

$$\frac{d}{dt}\begin{bmatrix} S \\ P \end{bmatrix} = \begin{bmatrix} -1/Y_{X/S} & -1/Y_{P/S} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} S\hat{X} & 0 \\ 0 & S\hat{X} \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} - \begin{bmatrix} S \\ P \end{bmatrix}\frac{F_{in}}{V} + \begin{bmatrix} F_{in}S_{in}/V \\ 0 \end{bmatrix}. \qquad (17)$$

The model (17) can be written in a compact matrix form as:

$$\dot{\xi}_s = K_s G(\xi_s)\alpha - D\xi_s + F, \qquad (18)$$

with

$$\xi_s = \begin{bmatrix} S \\ P \end{bmatrix};\ K_s = \begin{bmatrix} -1/Y_{X/S} & -1/Y_{P/S} \\ 0 & 1 \end{bmatrix};\ G(\xi_s) = \begin{bmatrix} S\hat{X} & 0 \\ 0 & S\hat{X} \end{bmatrix};$$
$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix};\ D = \frac{F_{in}}{V};\ F = \begin{bmatrix} F_{in}S_{in}/V \\ 0 \end{bmatrix}. \qquad (19)$$

To obtain the on-line estimates $\hat{\alpha}_1$ and $\hat{\alpha}_2$ of α_1 and α_2 we will use a recursive least-square parameter estimator (Bastin and Dochain 1990; Petre 2005, 2008) that, using the submodel (18) and the notations from (19), is particularized as follows:

$$\dot{\Psi}^T = -\omega\Psi^T + KG(\xi),\ \dot{\Psi}_0 = -\omega\Psi_0 + (\omega - D)\xi + F, \qquad (20)$$

$$\dot{\hat{\alpha}} = \Gamma\Psi\left(\xi_s - \Psi_0 - \Psi^T\hat{\alpha}\right),\ \dot{\Gamma} = -\Gamma\Psi\Psi^T\Gamma + \lambda\Gamma,\ \Gamma(0) > 0, \qquad (21)$$

where Ψ^T is the regressor matrix, Γ is the gain matrix of the updating law (21), and $\omega > 0$ and λ (forgetting factor) are design parameters at the user's disposal to control the stability and the tracking properties of the estimator (Petre 2005, 2008). In our case the regressor matrix is given by:

$$\Psi^T = \begin{bmatrix} -1/Y_{X/S} & -1/Y_{P/S} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{bmatrix}. \qquad (22)$$

Finally, the full indirect adaptive controller is made up by combination of (13), (14) and (20)-(22) with the control law (10) rewritten as (23):

$$F_{in} = \frac{V}{S_{in} - S}\left[\lambda_1\left(S^* - S\right) + \frac{1}{Y_{X/S}}S\hat{X}\,\hat{\alpha}_1 + \frac{1}{Y_{P/S}}S\hat{X}\,\hat{\alpha}_2\right] \qquad (23)$$

and is schematized in Fig. 1.

3.3 A robust-adaptive control strategy

Now, for the class of analysed bioprocesses, we consider a more realistic case when the influent concentration S_{in} is not measurable, but some lower and upper bounds, possible time-varying, are given, and the reaction rates μ and ν_P are highly uncertain and time varying, that is the kinetic coefficients $\mu_{\max}$ and $\nu_{\max}$ are two uncertain and time-varying parameters, but some lower and upper bounds of them are known, and the state *X* is unmeasurable. The *control objective* is the same as it was formulated in Section 2.

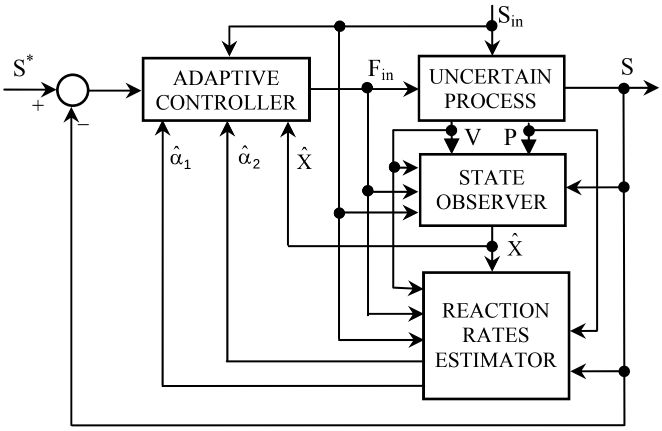

Fig. 1. The structure of the adaptive controlled system.

A robust interval observer

Since S_{in} is unknown, to estimate the unmeasurable state *X*, we cannot use the state observer (13), (14). But, since we assumed that some lower and upper bounds of S_{in} are known, then, depending on these known bounds, we can estimate a lower and an upper bound of *X* by using an appropriately observer interval.

The design of interval observers is based on properties of monotone dynamical systems or cooperative systems (Alcaraz-González et al. 2003; Rapaport and Dochain 2005; Smith 1995). Such systems have the property to keep the partial order between two trajectories depending on the bounds of the uncertainties in the model: if the (unknown) initial condition of the real system can be bounded between two known values, the trajectories of the same system starting from these bounds will enclose the real trajectory (Rapaport and Dochain 2005; Mazenc and Bernard 2011; Moisan and Bernard 2005;).

In (Alcaraz-González et al. 2003), it is shown that starting from an asymptotic observer it is possible to construct an interval observer which is robust against the uncertainness of the inputs and nonlinearities of the system that is stable in the presence of time varying parameters in the dynamical matrices. Thus, an asymptotic observer can be achieved as follows (Alcaraz-González et al. 2003). The model (8) can be rewritten in the form:

$$\dot{\xi}(t) = K\varphi(\xi(t), t)) + A(t)\xi(t) + b(t), \qquad (24)$$

with $\xi \in \Re^n$ - state vector, $\varphi \in \Re^m$ - reaction rate vector, $K \in \Re^{n\times m}$ - yield matrix, $A \in \Re^{n\times n}$ expresses the linear dependence between the state variables and $b \in \Re^n$ contains all the model terms that are not a function of state. If we assume that $q \le n$ states are on-line measured, then the model (24) can be rewritten as (Alcaraz-González et al. 2003):

$$\dot{\zeta}_1(t) = K_1\varphi(\xi,t) + A_{11}\,\zeta_1 + A_{12}\,\zeta_2 + b_1(t)$$
$$\dot{\zeta}_2(t) = K_2\varphi(\xi,t) + A_{21}\,\zeta_1 + A_{22}\,\zeta_2 + b_2(t) \quad (25)$$

with ζ_1 ($\dim\zeta_1 = q$) - measured variables and ζ_2 ($\dim\zeta_2$) $= n - q = s$ unmeasured variables. Matrices K_1, K_2, A_{11}, A_{12}, A_{21}, A_{22}, b_1 and b_2 are the corresponding partitions of K, A and b respectively.

The following hypotheses are introduced (Alcaraz-González et al. 2000, 2003): (H1) K, A and b are known, $\forall t \geq 0$; (H2) $\varphi(\xi,t)$ is fully unknown, $\forall t \geq 0$; (H3) $rank\ K_1 = rank\ K = p$, with $p \leq m < n$. (H4) $A(t)$ is bounded, $A^- \leq A(t) \leq A^+$, $\forall t \geq 0$, where A^- and A^+ are two constant matrices. Hypothesis (H2) allows the observer's design so that it enables the reconstruction of the unmeasured states, whatever the uncertain or unknown kinetics is (Alcaraz-González et al. 2000, 2003). This can be achieved by finding a suitable linear combination of the states given by $w(t) = N\,\xi(t)$, where $N \in \Re^{s \times n}$ and w is an auxiliary variable ($\dim w = s$), such that (Alcaraz-González et al. 2003):

$$N K = N_1 K_1 + N_2 K_2 = 0\,. \quad (26)$$

Under hypotheses (H1)-(H4) the following system (Alcaraz-González et al., 2000, 2003):

$$\dot{\hat{w}}(t) = W(t)\,\hat{w}(t) + Z(t)\,\zeta_1(t) + N\,b(t), \quad \hat{w}(0) = N\,\hat{\xi}(0),$$
$$\hat{\zeta}_2(t) = N_2^{-1}(\hat{w}(t) - N_1\zeta_1(t)), \quad (27)$$

with

$$W(t) = (N_1 A_{12}(t) + N_2 A_{22}(t))N_2^{-1},$$
$$Z(t) = N_1 A_{11}(t) + N_2 A_{21}(t) - W(t)N_1, \quad (28)$$

where $N = [N_1 \vdots N_2]$ with N_2 chosen as $N_2 = kI_s$, $k > 0$, is a real arbitrary parameter, and $N_1 = -N_2 K_2 K_1^+$, is *a reaction rates independent asymptotic observer for the model* (24). The kinetic independence property of the observer (27)-(28) is guaranteed by (26). Note that the hypothesis (H4) is used to prove the stability of the asymptotic observer.

The observer (27), (28) works only if the process input (vector b) is known (see (H1)). If some inputs are unmeasured, that is in (27) the vector b is unknown, then the asymptotic observer (27), (28) cannot be used. Therefore, based on the above described asymptotic observer, we present an interval observer for estimation of the unknown states able to handle the uncertainties in the input variables, model parameters, etc. For this purpose, the following supplementary hypotheses are introduced (Alcaraz-González et al. 2003; Rapaport and Dochain 2005): (H5) The input vector b is unknown but guaranteed bounds, possibly time varying, are given as $b^- \leq b(t) \leq b^+$; (H6) The initial conditions of the state vector are unknown but guaranteed bounds are given as $\xi^-(0) \leq \xi(0) \leq \xi^+(0)$. Also, the hypothesis (H1) is modified as: (H1$'$) The matrices K and A are known, $\forall t \geq 0$.

Remark 2. The operator $\leq$ applied between vectors or matrices must to be understood as a collection of inequalities between components (Alcaraz-González et al. 2003; Rapaport and Dochain 2005).

Under conditions (H5)-(H6), the idea is to design a *set-valued observer* in order to build guaranteed intervals for the unmeasured variables instead of estimating them precisely (Alcaraz-González et al. 2003; Rapaport and Dochain 2005). Interval observers work as a bundle of two observers: an upper observer, which produces an upper bound of the state vector, and a lower observer producing a lower bound, providing by this way a bounded interval in which the state vector is guaranteed to evolve (Alcaraz-González et al. 2003; Moisan and Bernard 2005); Rapaport and Dochain 2005).

Under hypotheses (H1$'$)-(H6), a robust interval observer for the system (24) presented in (Alcaraz-González et al. 2003; Langowski and Brdys 2007) can be reformulated as:

$$(S^+) = \begin{cases} \dot{w}^+(t) = W(t)w^+(t) + Z(t)\zeta_1(t) + M\,v^+(t) \\ w(0)^+ = N\xi(0)^+, \\ \zeta_2^+(t) = N_2^{-1}(w^+(t) - N_1\zeta_1(t)), \end{cases}$$
$$(S^-) = \begin{cases} \dot{w}^-(t) = W(t)w^-(t) + Z(t)\zeta_1(t) + M\,v^-(t) \\ w(0)^- = N\xi(0)^-, \\ \zeta_2^-(t) = N_2^{-1}(w^-(t) - N_1\zeta_1(t)), \end{cases} \quad (29)$$

where $W(t)$ and $Z(t)$ are given by (28), $\zeta_2^+(t)$ and $\zeta_2^-(t)$ are upper and lower bounds of the estimated state $\zeta_2(t)$, $M = [N_1 \vdots \widetilde{N}_1 \vdots N_2]$, $\widetilde{N}_1 = [|\,N_{1,ij}\,|]$, and

$$v^+(t) = [(b_1^+ + b_1^-)/2 \;\; (b_1^+ - b_1^-)/2 \;\; b_2^+]^T,$$
$$v^-(t) = [(b_1^+ + b_1^-)/2 \;\; -(b_1^+ - b_1^-)/2 \;\; b_2^-]^T,$$

with b_1^+, b_2^+ and b_1^-, b_2^- the corresponding partitions of the known upper and lower bounds of the input vector b.

If the matrix $W_\zeta = N_2^{-1}WN_2 = A_{22} - K_2K_1^+A_{12}$ is cooperative then, under hypotheses (H1)-(H6), the pair (S^+, S^-) from (29) constitutes a stable robust interval observer generating trajectories ζ_2^+ and ζ_2^- and it guarantees that $\zeta_2^- \leq \zeta_2 \leq \zeta_2^+$, $\forall t \geq 0$ as soon as $\xi^-(0) \leq \xi(0) \leq \xi^+(0)$ (Alcaraz-González et al. 2003; Langowski and Brdys 2007).

In the case of the alcoholic fermentation bioprocess to estimate a lower and an upper bound of unmeasurable state X we use an asymptotic observer whose structure is achieved by particularization of the equations (27) and (28). So, we consider the state partition $\zeta_1 = [S\ P]^T$, $\zeta_2 = X$, that induces on the matrices K, A and b from (24) the following partitions:

$$K = [K_1^T \vdots K_2^T] = \begin{bmatrix} -1/Y_{X/S} & 0 \vdots 1 \\ -1/Y_{P/S} & 1 \vdots 0 \end{bmatrix}^T, \text{ with } rank\ K = 2,$$

$$\varphi(\xi,t) = [\varphi_1 \;\; \varphi_2]^T = [\mu(\cdot)X \;\; \nu_p(\cdot)X]^T,$$

$$A(t)=\begin{bmatrix} A_{11} & \vdots & A_{12} \\ \cdots & \vdots & \cdots \\ A_{21} & \vdots & A_{22} \end{bmatrix}=\begin{bmatrix} -D & 0 & \vdots & 0 \\ 0 & -D & \vdots & 0 \\ \cdots & \cdots & \vdots & \cdots \\ 0 & 0 & \vdots & -D \end{bmatrix}, \text{ with } D=\frac{F_{in}}{V},$$

$$b=[b_1^T \vdots b_2^T]^T=\left[F_{in}S_{in}/V \ \ 0 \vdots 0\right]^T.$$

If the matrix N_2 is chosen as $N_2 = \theta > 0$, then the matrix N takes the form: $N=[N_1 \vdots N_2]=\theta\cdot\left[Y_{X/S} \ \ Y_{X/S}/Y_{P/S} \ \vdots 1\right]$.

The unmeasured state X can be obtained by using the asymptotic observer (27), (28) where W and Z are described by the following matrices: $W(t)=-D$, $Z=0$. It is obvious that if $0 \le D^- \le D(t) \le D^+$, where D^- and D^+ represent a lower and respectively an upper bound of D, then $W^- \le W(t) \le W^+$ with $W^{\mp}(t)=W(t)$ when $D(t)=D^{\pm}(t)$.

It is straightforward to very that if $\theta = 1/Y_{X/S}$, then $N=\left[1 \ \ 1/Y_{P/S} \ \vdots 1/Y_{X/S}\right]$ and, by using the first equation from (27), we find that the dynamic of auxiliary variable $w(t)=N\,\xi$ takes the form: $\dot{\hat{w}}(t)=-(F_{in}/V)\hat{w}+(F_{in}/V)S_{in}$. Then the unmeasurable variable $\hat{\zeta}_2$ from (27) is given by $\hat{\zeta}_2=\hat{X}=Y_{X/S}\left(\hat{w}-(1/Y_{P/S})P-S\right)$. As a result, the achieved asymptotic observer is identically with the observer given by (13)-(14).

Then a *robust interval observer* for the system (8), which estimates a lower and an upper bound of the unmeasurable state X, is defined by the following equations:

$$\begin{aligned} &\dot{\hat{w}}^+(t)=-(F_{in}/V)\hat{w}^+ +(F_{in}/V)S_{in}^+, \quad w^+(0)=N\xi^+(0) \\ &\hat{X}^+=Y_{X/S}\left(\hat{w}^+-(1/Y_{P/S})P-S\right) \\ &\dot{\hat{w}}^-(t)=-(F_{in}/V)\hat{w}^- +(F_{in}/V)S_{in}^-, \quad w^-(0)=N\xi^-(0) \\ &\hat{X}^-=Y_{X/S}\left(\hat{w}^--(1/Y_{P/S})P-S\right) \end{aligned} \tag{30}$$

To develop a *robust-adaptive controller*, let's assume that the dynamics of S in (2) can be rewritten as follows (Alcaraz-Gonzalez et al. 2000, 2005):

$$\dot{S}(t)=f(\zeta_1(t),\zeta_2(t),p(t))+g(\zeta_1(t),\zeta_2(t),p(t))\,u(t) \tag{33}$$

where f and g are two scalar functions (usually nonlinear and possible time-varying) and the vector p contains all uncertain parameters (both the uncertain process inputs as well as the uncertain kinetic parameters), some of them possible time-varying. Assume also that p is bounded as $p^- \le p(t) \le p^+$. Then, if p is bounded and both the maximum bound and the minimum bound of ζ_2 are known (by using the interval observer) it is possible to establish the maximum and the minimum values of f and g at each time (Alcaraz-Gonzalez et al. 2000, 2005). Under these assumptions, the following parameters can be defined (Alcaraz-Gonzalez et al. 2000):

$$f^*(t)=\begin{cases} \min\limits_{\substack{\zeta_{2i}\in[\zeta_{2i}^-,\zeta_{2i}^+],\forall i \\ p_j\in[p_j^-,p_j^+],\forall j}} f(\zeta_1(t),\zeta_{2i}(t),p_j(t)), & \text{if } S(t)<S^* \\ \max\limits_{\substack{\zeta_{2i}\in[\zeta_{2i}^-,\zeta_{2i}^+],\forall i \\ p_j\in[p_j^-,p_j^+],\forall j}} f(\zeta_1(t),\zeta_{2i}(t),p_j(t)), & \text{if } S(t)>S^* \end{cases} \tag{34}$$

$$g^*(t)=\begin{cases} \min\limits_{\substack{\zeta_{2,i}\in[\zeta_{2i}^-,\zeta_{2i}^+],\forall i \\ p_j\in[p_j^-,p_j^+],\forall j}} g(\zeta_1(t),\zeta_{2i}(t),p_j(t)), & \text{if } S(t)<S^* \\ \max\limits_{\substack{\zeta_{2,i}\in[\zeta_{2i}^-,\zeta_{2i}^+],\forall i \\ p_j\in[p_j^-,p_j^+],\forall j}} g(\zeta_1(t),\zeta_{2i}(t),p_j(t)), & \text{if } S(t)>S^* \end{cases} \tag{35}$$

Using the parameters defined in (34) and (35) we can formulate the following result.

Theorem 1. If the functions f and g in (33) are so that the condition $f(\zeta_1,\zeta_2,p)/g(\zeta_1,\zeta_2,p)<0$ is fulfilled, then a control law of the form

$$F_{in}^*=\left(1/g^*(t)\right)\cdot\left(\dot{S}^*+\lambda_1(S^*-S)-f^*(t)\right) \tag{36}$$

with $g^* \ne 0, \forall t \ge 0$, asymptotically stabilizes S towards S^*.

Proof. Let's define the tracking system error as $e_t(t)=S^*(t)-S(t)$ and consider the following candidate Lyapunov function: $V(t)=(S^*(t)-S(t))^2$ (see also (Alcaraz-Gonzalez et al. 2000)). Using (36), its time derivative along the trajectory (33) takes the form:

$$\dot{V}(t)=(S^*-S)\left(\dot{S}^*-f(t)-\frac{g(t)}{g^*(t)}(\dot{S}^*+\lambda_1(S^*-S)-f^*(t)\right).$$

Using the definitions of f^* and g^* and the condition $f/g<0$, one can obtain that $\dot{V}(t)\le -\lambda_1(S^*-S)^2 \le 0$, $\forall\lambda_1>0$. So S asymptotically converges towards S^*.

It must be noted that in the case of analysed alcoholic fermentation bioprocess, the functions f and g from (33) are given by $f(\cdot)=-(1/Y_{X/S})X\mu-(1/Y_{P/S})X\nu_p$ and $g(\cdot)=(S_{in}-S)/V$. Note also that in a normal operation of the bioreactor, the function $g(\cdot)>0, \forall t \ge 0$.

4. SIMULATION RESULTS AND COMMENTS

The performance of the designed adaptive and robust-adaptive controllers by comparison to exactly linearizing controller (10) has been tested by performing extensive simulation experiments. For a proper comparison, the simulations were carried out by using the process model (1)-(4) under identical conditions. The values of the yield and of the kinetic coefficients are (Queinnec 1991; Petre 2005):

$\mu_{\max}^0=0.54\,h^{-1}$, $K_S=5\,g/l$, $K_I=201\,g/l$, $\nu_{\max}^0=2.1\,h^{-1}$, $K_S^{'}=9\,g/l$, $K_I^{'}=297\,g/l$, $P_m=70\,g/l$, $Y_{X/S}=1.5$, $Y_{P/S}=0.43$, $S_{in}^0=160\,g/l$, $F_{in}\in[0,2]$ l/h, $V\in[V_0,V_{\max}]=[4,16]\,l$.

Case 1. The behaviour of closed-loop system using indirect adaptive controller (23), by comparison to exactly linearizing control law (10) is presented in Fig. 3 - the time evolution of the controlled variable S, and Fig. 4 - the control input F_{in}. In this case the influent concentration S_{in} is time varying as it is shown in Fig. 2, but it is assumed measurable, and the kinetic coefficients $\mu_{\max}$ and $\nu_{\max}$ are two time varying parameters upon some sinusoidal patterns as:

$$\mu_{\max}(t) = \mu^0_{\max}(1 + 0.2\sin(\pi t/2)) , \tag{37}$$

$$\nu_{\max}(t) = \nu^0_{\max}(1 - 0.2\cos(\pi t/3)) . \tag{38}$$

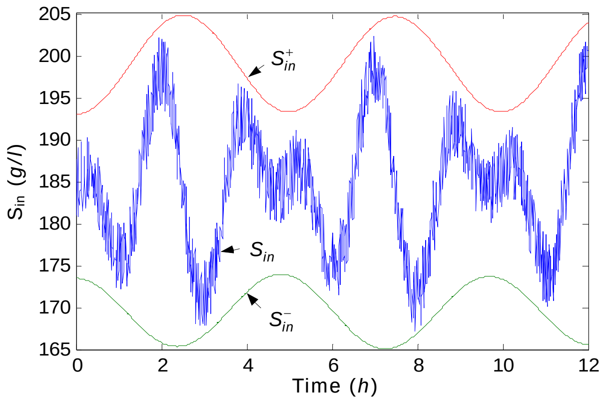

Fig. 2. Time evolution of S_{in} and of its bounds.

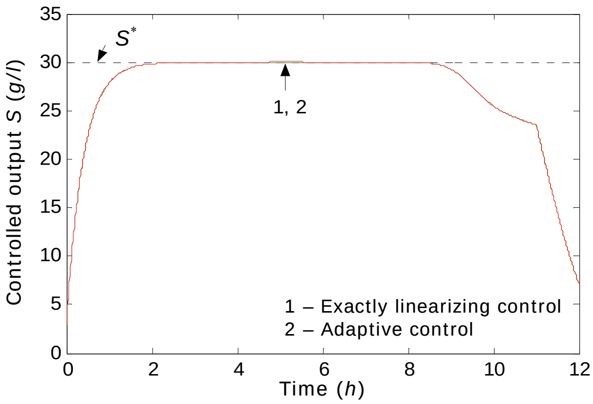

Fig. 3. Time evolution of output S.

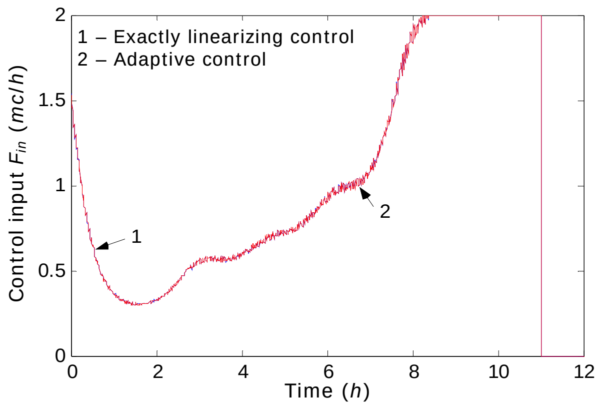

Fig. 4. Profile of control input F_{in}.

From Fig. 3 and Fig. 4 it can be observed that the substrate concentration S tracks the reference profile S^*, and the control inputs F_{in} is kept in the physical limits required by the process. The gain of control laws (10) and (23) is $\lambda_1 = 2.5$, and the tuning parameters of adaptive controller have been set to the values: $\omega = 100$, $\gamma_1 = \gamma_2 = 1.5$, $\lambda = 0.45$.

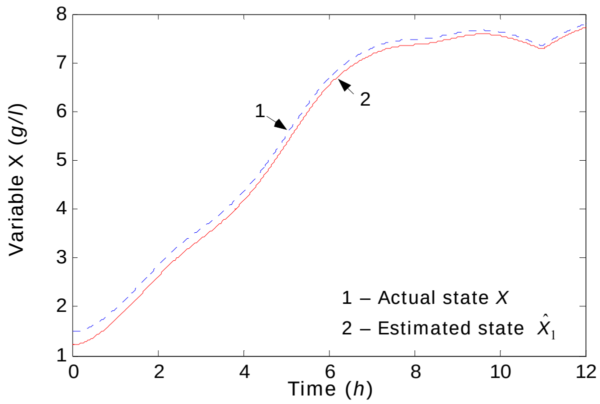

Fig. 5. Time evolution of estimate of unknown variable X.

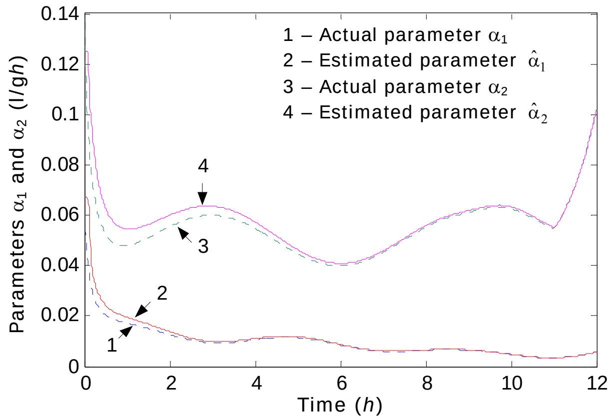

Fig. 6. Profile of estimates of unknown parameters α_1 and α_2.

The time evolution of the estimate of unmeasured variable X provided by the asymptotic observer (13)-(14) is presented in Fig. 5, and the time evolution of estimates of unknown specific reaction rates α_1 and α_2 provided by the recursive least-square parameter (20)-(22) is presented in Fig. 6. From this figures, it can be noticed a good behaviour of both state observer and of parameter estimator.

From graphics in Figs. 3-6 it can be seen that the behaviour of overall system with adaptive controller (23), as we expected, is good, despite the high variation of S_{in} and time variation of process parameters, being very close to the behaviour of closed loop system in the ideal case obtained using the linearizing controller (10) when the process model is completely known.

Case 2. Now we analyse the behaviour of closed-loop system using an adequately structure of robust-adaptive controller (34)-(36). The system's behaviour is analysed assuming that the influent concentration S_{in} is not measurable but some lower and upper bounds, denoted by S^-_{in} and S^+_{in}, respectively, as in Fig. 2, are given, and that the kinetic coefficients $\mu_{\max}$ and $\nu_{\max}$ are two uncertain and time-varying parameters, but some lower and upper bounds of them, possible time-varying, are known i.e. $\mu^-_{\max} \le \mu_{\max}(t) \le \mu^+_{\max}$ and $\nu^-_{\max} \le \nu_{\max}(t) \le \nu^+_{\max}$. In our analysis we assume that the time-variations of $\mu_{\max}$ and $\nu_{\max}$ are given by (37) and (38), respectively, that is

$$\mu_{\max} \in [\mu^{-}_{\max}, \mu^{+}_{\max}] = [0.8\mu^{0}_{\max}, 1.2\mu^{0}_{\max}], \text{ and}$$

$$\nu_{\max} \in [\nu^{-}_{\max}, \nu^{+}_{\max}] = [0.8\nu^{0}_{\max}, 1.2\nu^{0}_{\max}].$$

The *control objective* is the same as in the previous case, i.e. to maintain the output S at a desired value $S^{*} \in \Re^{+}$ despite the unknown and variation of S_{in} as well as the time variation of some process parameters.

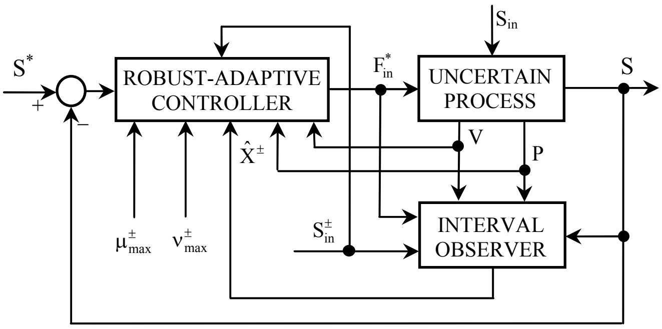

Fig. 7. The structure of the robust-adaptive system.

In the control law (36) the definitions of the functions f^{*} and g^{*} are particularized as follows:

$$f^{*}(t) = \begin{cases} -(1/Y_{X/S})X^{+}\mu^{+} - (1/Y_{P/S})X^{+}\nu^{+}_{P}, & \text{if } S(t) < S^{*} \\ -(1/Y_{X/S})X^{-}\mu^{-} - (1/Y_{P/S})X^{-}\nu^{-}_{P}, & \text{if } S(t) > S^{*} \end{cases} \tag{40}$$

$$g^{*}(t) = \begin{cases} (S^{-}_{in} - S)/V, & \text{if } S(t) < S^{*} \\ (S^{+}_{in} - S)/V, & \text{if } S(t) > S^{*} \end{cases} \tag{41}$$

with

$$\mu^{\mp} = \mu^{\mp}_{\max}\left(1 - P/P_{m}\right)\cdot S/(K_{S} + S + S^{2}/K_{I})) \text{ and}$$

$$\nu^{\mp}_{P} = \nu^{\pm}_{\max} S/(K'_{S} + S + S^{2}/K'_{I}), \tag{42}$$

and X^{-} and X^{+} corresponding to S^{-}_{in} and S^{+}_{in}, respectively.

A block diagram of the proposed robust-adaptive system is shown in Fig. 7.

The behaviour of closed-loop system using robust-adaptive controller (36), (40)-(42) by comparison to the exactly linearizing law (10) is presented in Fig. 8 and in Fig. 9.

The graphics shown in Fig. 8 correspond to the controlled output S, and graphics in Fig. 9 correspond to the control input F_{in}.

The time evolution of the estimates of the lower and upper bounds of the unmeasured variable X is presented in Fig. 10.

The estimated values $\hat{X}^{+}$ and $\hat{X}^{-}$ are obtained by using the interval observer (29), where the input vector v contains the known bounds S^{+}_{in} and S^{-}_{in}, respectively. The initial conditions of these states are unknown but some guaranteed lower and upper bounds are assumed known as $0.5 = X^{-}(0) \le X(0) \le X^{+}(0) = 2.5$ (g/l). The gain of control law (36) has been set to the same value as in the first two cases, i.e. $\lambda_{1} = 2.5$.

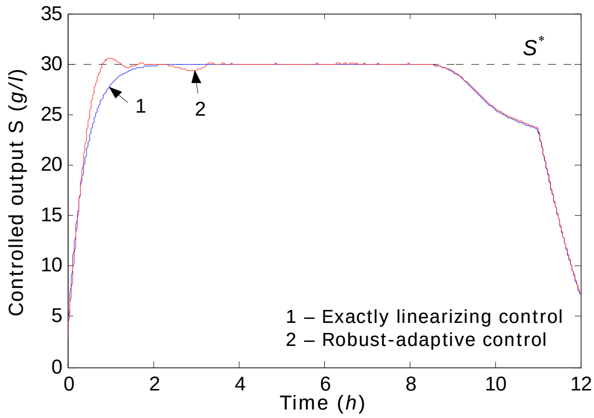

Fig. 8. Time evolution of controlled output S.

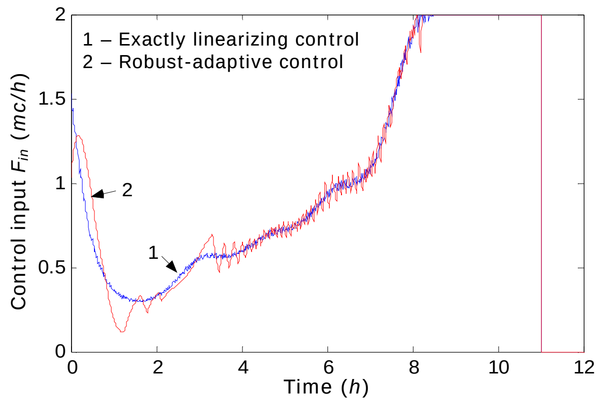

Fig. 9. Profile of control input F_{in}.

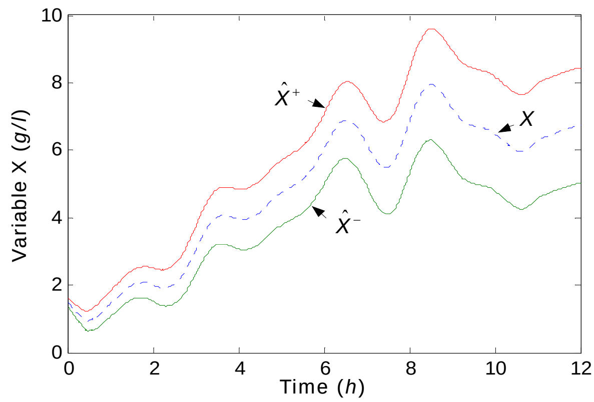

Fig. 10. Profile of estimates of upper and lower bounds of unknown variables X.

The time evolution of the uncertain but bounded time varying parameters μ and ν_{P} as well as of their bounds given in (42) is shown in Fig. 11.

From graphics in Fig. 8 it can be seen that the behaviour of overall system with robust-adaptive controller (36), (40)-(42), even if this controller uses much less a priori information, is good, being close to the behaviour of closed loop system with adaptive controller (32) as well as to the behaviour of closed loop system in the ideal case obtained using the linearizing controller (21) when the process model is completely known. The controller is able to maintain the controlled output S close to its desired value S^*, despite the unknown high variation of the unmeasurable concentration of S_{in} and time variation of the uncertain process parameters $\mu_{\max}$ and $\nu_{\max}$, respectively μ and ν_P. Also, as in the first case, the control input F_{in}^* is kept in the physical limits required by the process.

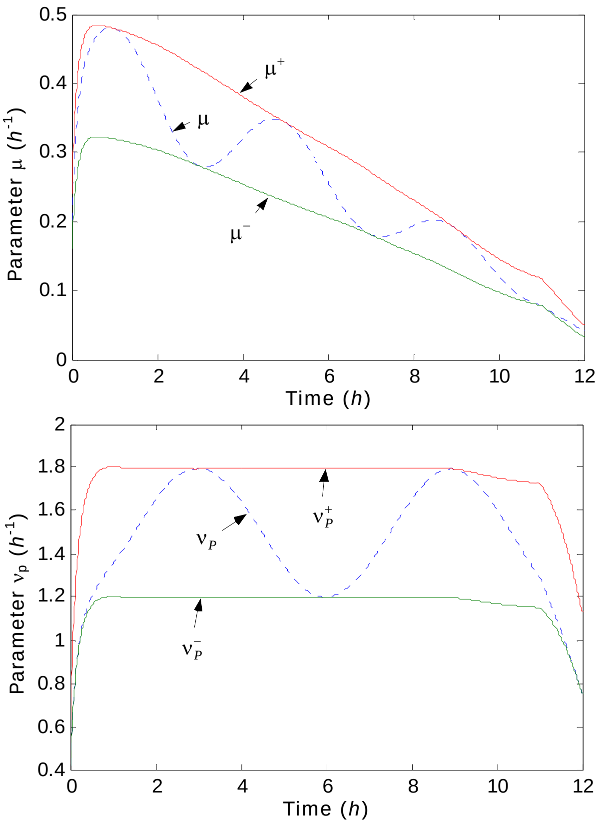

Fig. 11. The time evolution of parameters μ and ν_P.

7. CONCLUSION

In this paper some adaptive and robust-adaptive control strategies for a class of fed-batch fermentation processes were designed and analysed. The designed algorithms were applied to a particular alcoholic fermentation process and the effectiveness of the designed algorithms was validated by numerical simulations. Since the proposed control strategies involve an unmeasurable process state, this must has to be estimated based on the known measurements by using appropriately state estimators. Therefore, in the paper, an asymptotic observer and a robust interval observer for the analysed class of fed-batch bioprocesses were presented.

The adaptive control structure was achieved by combining a linearizing control law with an asymptotic observer and a parameter estimator used for on-line estimation of the bioprocess unknown kinetics. This control structure was developed under the assumption that the specific reaction rates were completely unknown, but the influent substrate concentration was measurable. As we expected the obtained results were very good.

The robust-adaptive control structure was developed under the assumption that the reaction rates are highly uncertain and time-varying and the influent substrate concentration is completely unknown, but some lower and upper bounds both of influent substrate concentration and of reaction rates uncertainties are known. This control structure was achieved by combining a linearizing control law with an interval observer able to estimates a lower and an upper bound of unmeasurable state. Also, in the proposed control strategy the uncertain process parameters are replaced by their lower and upper bounds assumed known. From the obtained results one concludes that this approach can handle time-varying uncertainties simultaneously on the kinetic and on the feed concentrations.

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