INTERPOLATIVE-TYPE CONTROL SOLUTIONS FOR A "BALL AND BEAM" SYSTEM

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Abstract: *Controllers with interpolative blocks can replace fuzzy controllers in control structures. This is possible because fuzzy controllers belong also to the interpolative-type controller category, meaning controllers which implements interpolative-type reasoning. That kind of replacement is not only a formal operation, it is also associated with further corrections that confer to the structures with interpolative controllers enough flexibility to obtain better performances. The possibility of performances improvement is the main argument of the present paper. Another argument is the reduced calculus time, suited for the real-time implementation - it's about "look-up table" type solutions. In order to illustrate the above affirmations, a case study is developed in the paper. The controlled plant is an electromechanical one: ball and beam system, driven by the torque applied in the rotational joint. The model used for the system is a 4th order one. Given the system complexity, two structural solutions are proposed (TPS-4 –a structure with feedback from all four states- and TPS-2 –a structure with feedback from only two states-), both of them being tested - from the robustness point of view - at changes in system parameters. The case study is conceived also in order to underline the differences between them.*

Keywords: *interpolative controller, signed distance, synthetic input, robustness properties.*

1. INTRODUCTION

Mainly, interpolation represents a procedure by which a finite number of discrete information referring to cause-effect correlations (associated to some points from a limited set) are used to generate similar correlations regarding all set points.

In the field of automatic control especially the correlations referring to controller's input-output dependencies, named command characteristics, control laws, control algorithms, etc. are for great interest. A large scale of controllers is known as interpolative-type controllers: fuzzy, RIP (Rule based Interpolation), neural controllers and also the controllers contain interpolation blocks used for implementing some operations that appear in different kinds of control laws.

The possibility to treat the fuzzy and neural controllers as interpolative-type controllers was observed by Zadeh [11], who placed the fuzzy logic and the neural network's mechanisms in the interpolative-type reasoning, then by Dubois and Prade [13], Koczy and Hirota [12], who made for different situations consistent analytical studies, and finally by Drechsel [4] who associates interpolation tables to rule bases.

The advantages using interpolative blocks in control structures are, mainly, based on simplifying the solutions (easier implementation and reduced calculus time) and the possibility to ensure for the control systems, in a relative easy way, some robustness properties [10]. In the same time, as in [2], [3], the controllers based on interpolation table, on the same form as in RIP controllers, can be developed in much more situations than those based on linguistic rules. They can be conceived starting from solutions deducted in different ways, solutions that, finally, can be improved.

The goal of the paper is to underline the possibilities offered by the interpolative-type control with an example: the ball and beam system. The study suggests different issues from which the problem can be viewed, from the control and also robustness properties points of view.

Next, section 2 underlines some theoretical aspects concerning the methodology used to obtain interpolative controllers based on a synthetic input signal. The ball and beam system, the design of its control system and illustrative experimental results obtained through simulation are presented in section 3. Finally a few remarks conclude the presentation.

2. DESIGN STEPS TOWARD INTERPOLATIVE CONTROLLERS BASED ON SYNTHETIC INPUT

In many cases, the control of second order non-linear plants, can be successfully implemented using conventional fuzzy controllers based on error (e) and its derivative ($\dot{e}$), ($RG\_e\dot{e}$). For higher order plants, basically, it is necessary to use as inputs for the controller all the state variables to get acceptable results. In this case, the number of the rules, as well as the computational complexity, is prohibitive.

Due to relative simplicity in implementation and design, the two-input fuzzy controllers are used even for complex, higher order plants, as a sub optimal solution.

In [1] and [8] single-input fuzzy controllers were developed, based on some observations related to the similarities between fuzzy and sliding-mode control. In the present paper another solution is proposed, i.e. a single-input interpolative controller, similar to the fuzzy one having in addition some more advantages.

For conventional fuzzy controllers using the error and the change-of-error as input variables, the established rule table may be represented in a 2-dimensional space of the phase plane. Some authors, as Choi, Kwak and Kim in [1] and Palm in [8], observed that in such rule table the rules are skew-symmetric and the absolute magnitude of the control input is proportional to the distance of the characteristic point from its main diagonal line, which acts like a swhiching line in the normalized input space. The property also holds in n-dimensional case or for the PID-type fuzzy controllers that use the error, the sum-of-error, and the change-of-error as fuzzy input variables. Based on the mentioned observation, in [1] the authors suggest the using of the variable called *signed distance* (*sd*). Its value is the distance of an actual state from the input space to the main diagonal line (or hyper-plane) and it is positive or negative according to the position of the actual state related to the swhiching line/hyper plane. The derived signed distance is then used as single fuzzy input variable of a simple fuzzy controller.

The advantages of the single-input fuzzy controller developed in [1] (denoted as *RG_1F*) are as follows: ▪ it requires only one input linguistic variable, signed distance, regardless of the complexity of the controlled plants; so the control rule table is build in a 1-dimensional space; ▪ the number of rules and in fact the tuning parameters is greatly decreased and also the computational complexity; ▪ the single input variable, signed distance, implies knowledge of all state variables, hence, in the case of more than third-order controlled plants, the control performance can be superior to conventional

RG-$e\dot{e}$.

An alternative to the *RG_1F* an interpolative controller is offered in the present paper. To obtain a simpler control structure with better performances the *RG_1F* is replaced with a single input interpolative-type controller (denoted as *RG-1I*). Generally, the interpolative-type controllers implement the control algorithms through support points implanted in interpolative blocks, which are used either alone (Drechsel in [4] or [5]), or integrated in dynamic structures (as in [2], [3], [6] and [7]).

In essence, the interpolative blocks that are being used in [2], [3], [6] and [7], and also in the present paper, as controllers are interpolation tables, which contain a finite number of support values, collected from support points. The difference between these researches is the manner in which the support values are collected. More details are given in [10].

The interpolative controller operates using the principle of interpolation between the values in the table. This offers the possibility to start from an already existing initial solution, and to correct the initial dependency in a quite simple manner. The changes operated in the tables, within the design operation of the interpolative controller, are made to improve performances for the control system they are started from. The improvement techniques used are different, according to each and every application. Some results obtained with an empiric improvement technique based on the state-plane behaviour studies are given in [2] and [3], and the usage of a genetic algorithm based improvement method is developed in [6] and [7]. In all cases, the improvement criterion assures, step by step, a structure with better performances than the initial one.

Three steps are performed in order to obtain interpolative controllers without loss in performances:

- *Step 1*: A two-input fuzzy controller RG_ $e\dot{e}$ (Figure 1) with skew-symmetric rule for the non-linear plant is designed.

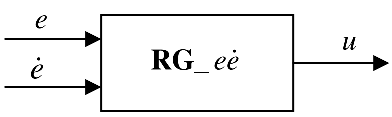


Figure 1. The initial *RG_* $e\dot{e}$ fuzzy controller

- *Step 2*: Starting from the fuzzy controller *RG-* $e\dot{e}$ designed in step 1, a simpler single-input fuzzy controller *RG_1F* based on a simplified rule base defined in terms of signed-distance is developed.

For the two-dimensional case the signed-distance is defined by

$$sd = \frac{\dot{e} + \lambda \cdot e}{\sqrt{1+\lambda^2}} \quad (1)$$

and for the n-dimensional case by

$$sd = \frac{e^{(n-1)} + \lambda_{n-1} e^{(n-2)} + \ldots + \lambda_2 \dot{e} + \lambda_1 e}{\sqrt{1 + \lambda_{n-1}^2 + \ldots + \lambda_2^2 + \lambda_1^2}}, \quad (2)$$

where λ / $\{\lambda_i\}$ correspond for the slope of the swhiching line / hyper plane, and $e^{(k)}$ is the k-th order derivative of the error. The scalar variable *sd* calculated with relation (1) or (2) is further used, like in Figure 2, as input variable of the fuzzy controller *RG_1F*.

By a correct design of the simplified rule base the control performances obtained with this structure remain almost the same as with the initial fuzzy controller, whereas the number of fuzzy rules is greatly reduced.

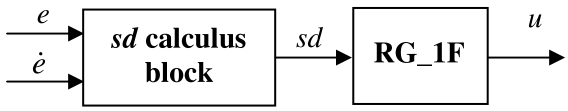


Figure 2. Signed distance *sd* calculus and the associated *RG_1F* controller.

- *Step 3*: Based on the RG_1F and in order to replace it, a simple single-input interpolative controller *RG_1I* is developed (Figure 3). The purpose of the replacement is to achieve a simpler single-input controller than the one developed in step two, and to improve the behaviour of the control system with *RG_1F*. Such an interpolative controller can be implemented as an interpolation table.

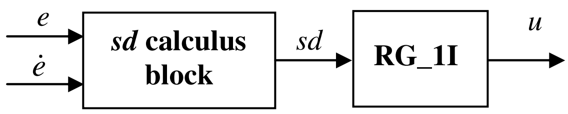


Figure 3. Signed distance *sd* calculus and the associated *RG_1I* controller.

Based on the single-input fuzzy controller, in present paper the support points are choose as middles of disjoint intervals associated to the linguistic terms of the linguistic variables of RG_$e\dot{e}$ fuzzy controller. Hence, real intervals are used instead the membership functions as Drechsel proceeds in [4] and [5] for RIP method. In this approach the linguistic terms are described by real intervals or singletons, in relation to the support set of the membership functions used in fuzzy RG_1F controller, and their distribution.

The interpolative controller has two degrees of freedom: ▪ the values of the control signal u from the interpolation table, and ▪ the interpolation method. Changes in each of these freedom degrees will change the performances of the system with interpolative controller. Finally, a simple single-input interpolative controller RG_1I is obtained, with performances at least equal (but improvable) than the initial structure, less calculus time and simpler implementation possibilities than the fuzzy two-dimensional or one-dimensional controllers.

In the next section, only modifications of the control signal values in the table will be used.

3. STUDY CASE: BALL AND BEAM SYSTEM

As controlled plant the non-linear system called ball and beam is considered. Three control methods regarding the controllers described in section 2 (Figures 1, 2 and 3) were studied comparatively, and next some simulated experimental results will be present.

The ball and beam system is a 4th order system for which it can be adopted as states variables a translation position, an angular position and their first derivatives (speeds), respectively. The goal of the control system is to stabilize the translation position (TPS).

Two solutions may be adopted: a solution based only on feedback for translation movement – TPS-2 (position r and velocity $\dot{r}$) and a solution that uses also feedback for rotational motion also (r, $\dot{r}$, θ, $\dot{\theta}$)– TPS-4. Both act like a lead-lag control (PD-type controllers) in a conventional configuration, respectively in a cascade configuration.

In section 3.2 and 3.3 there are developed and compared different variants for these two solutions. The TPS-2 solution has the advantage of simplicity but also the drawback of a weaker robustness range in respect to the parameters of the controlled plant.

3.1. Ball and beam model

Ball and beam system consists on a rigid T beam (an arm with the length L and a bearer symmetrically fixed on the arm edge) capable to rotate around a fixed point A, and a ball with the mass M_2, as is showed in Figure 4. An actuator that develops the rotational torque m controls the rotation.

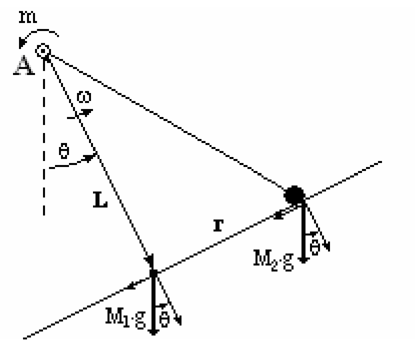


Fig.4. Ball and beam system

For the synthesis of the ball and beam model the following assumptions are made: ▪ The beam's arm is made of light material so that the beam's mass M_1 is concentrated in the middle of bearer. ▪ The moment of inertia of the beam in respect with the point A is J. ▪ The coefficient of kinetic friction between the ball of mass M_2 and the bearer is μ_f. ▪ For the motion around the point A the coefficient of viscous friction c_f is considered. ▪ The ball and beam system is viewed as a controlled plant with $\{m\} \to \{r\}$ orientation and state variables: $x_1 = r$, $x_2 = \dot{r}$, $x_3 = \theta$ and $x_4 = \dot{\theta}$. ▪ Concrete values of the parameters used are:

$M_1 = 1\,kg,\ M_2 = 0.5\,kg,\ L = 0.5\,m,\ \mu_f = 0.35,$

$c_f = 0.1 N \cdot s/m,\ J = 0.02\,kg \cdot m^2,\ g = 9.8\,m/s^2$

The model in (3) was established in [9].

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1 \cdot x_4^2 - g \cdot \sin x_3 + M_2 \cdot \mu_f \cdot \operatorname{sgn} x_2 \cdot \cos x_3 - L \cdot \dfrac{1}{\left(J + M_1 \cdot L^2 + M_2 \cdot x_1^2\right)} \cdot [m - M_1 \cdot L \cdot g \cdot \sin x_3 - \\ \quad - M_2 \cdot x_1 \cdot g \cdot \cos x_3 - M_2 \cdot L \cdot x_1 \cdot x_4^2 - 2 \cdot M_2 \cdot x_1 \cdot x_4 \cdot x_2 - c_f \cdot x_4 - L \cdot \mu_f \cdot M_2 \cdot g \cdot \operatorname{sgn} x_2 \cdot \cos x_3] \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{\left(J + M_1 \cdot L^2 + M_2 \cdot x_1^2\right)} \cdot [m - M_1 \cdot L \cdot g \cdot \sin x_3 - M_2 \cdot x_1 \cdot g \cdot \cos x_3 - M_2 \cdot L \cdot x_1 \cdot x_4^2 - 2 \cdot M_2 \cdot x_1 \cdot x_4 \cdot x_2 - \\ \quad - c_f \cdot x_4 - L \cdot \mu_f \cdot M_2 \cdot g \cdot \operatorname{sgn} x_2 \cdot \cos x_3] \\ y = x_1 \end{cases} \tag{3}$$

3.2. Control structure with complete state feedback (TPS-4)

Step 1: The control system is as in Figure 5, where $e_r = r - w$, $e_\theta = \theta_{ref} - \theta$. The reference w is considered constant over time subintervals (ladder signal). Because of the structure of the controller, denoted $RG4\_e\dot{e}$, the system can be considered as a control system with inner feedback loop even if the internal controlled plant structure can not be represented by a minimal type serial connection having θ as intermediate measure. $RG4\_e\dot{e}$ consists on two controller blocks denoted by $RG4\_e\dot{e}\_r$ and $RG4\_e\dot{e}\_\theta$. The translation fuzzy controller $RG4\_e\dot{e}\_r$ uses two inputs: position error (e_r) and velocity ($v_r = \dot{r} = -\dot{e}_r$). Its output is an angular position (θ_{ref}) representing the reference for the internal loop. The rotational fuzzy controller $RG4\_e\dot{e}\_\theta$ uses also two inputs: angular position error (e_θ) and velocity ($v_\theta = \dot{\theta} = -\dot{e}_\theta$). Its output is the motor torque (m) that drives the frame in order to reach and maintain the ball in the desired position w on the bearer.

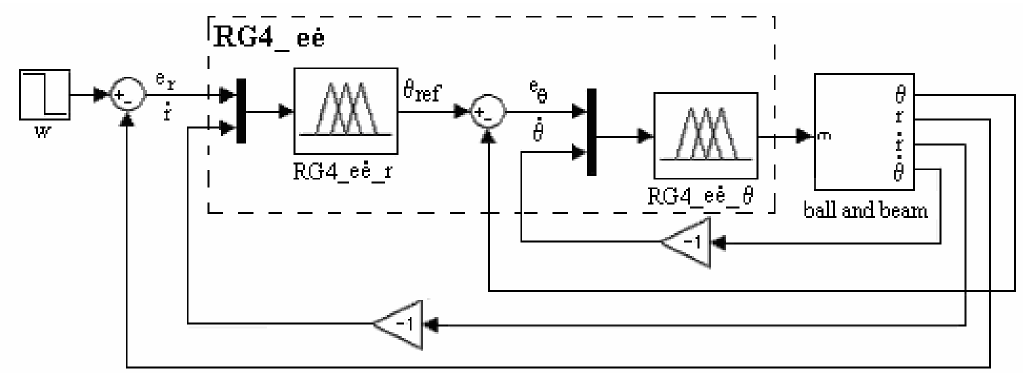


Fig.5. Fuzzy control scheme with $RG4\text{-}\,e\dot{e}$ for the ball and beam system

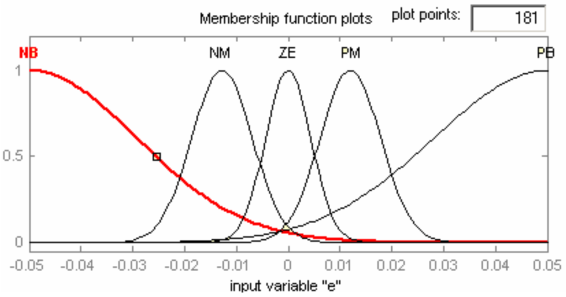
The domains of values (D) and their ranges (L) for the input and output variables of $RG4\_e\dot{e}\_r$ are: D_{e_r} = [-0.05, 0.05] m and L_{e_r} = 0.1 m for e_r, $D_{\dot{r}}$ = [-0.2, 0.2] m/s and L_{vr} = 0.4 m/s for $\dot{r}$, respectively $D_{\theta_{ref}}$ = [-2, 2] rad and $L_{\theta_{ref}}$ = 4 rad for the reference angular position θ_{ref}.

For $RG4\_e\dot{e}\_\theta$ that are considered: D_{e_θ} = [-0.4, 0.4] rad and L_{e_θ} = 0.8 rad for e_θ, $D_{\dot{\theta}}$ = [-1, 1] rad/s and $L_{\dot{\theta}}$ = 2 rad/s for $\dot{\theta}$, respectively D_m = [-2, 2] N·m and L_m = 4 N/m for the motor torque m.

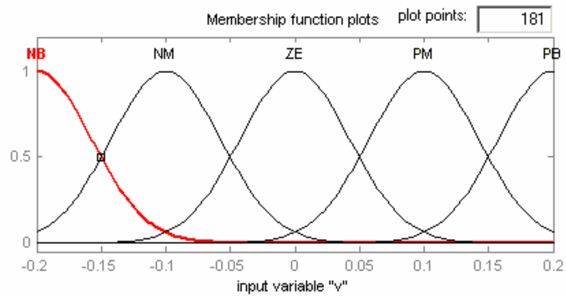
All variables are defined by 5 linguistic terms: NB, NM, ZE, PM, PB. The rule bases that were used, both skew-symmetric, are given in table 1 and 2 for the $RG4\_e\dot{e}\_r$ and $RG4\_e\dot{e}\_\theta$, respectively.

The shape of the membership functions for error (e), speed (v) and control signals for both controllers are detailed in Fig.6a), Fig.6b) and Fig.6c), respectively.

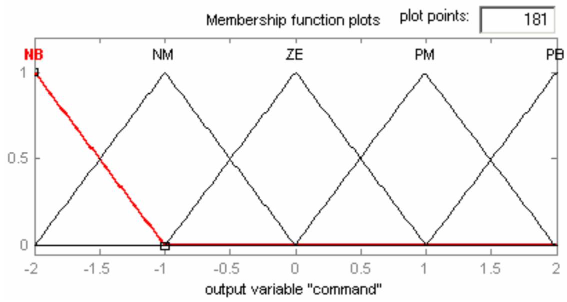
The slopes of the corresponding swhiching lines are: $\lambda_r = \dfrac{L_{\dot{r}}}{L_{e_r}} = \dfrac{0.4}{0.1}$ and $\lambda_\theta = \dfrac{L_{\dot{\theta}}}{L_{e_\theta}} = \dfrac{2}{0.8}$.



6a)



6b)



6c)

Figure 6. Membership functions for *RG4-* $e\dot{e}$.

Table 1 – Rule base for the *RG4-* $e\dot{e}$*_r*

$v_r \backslash e_r$	NB	NM	ZE	PM	PB
PB	ZE	NM	NM	NB	NB
PM	PM	ZE	NM	NM	NB
ZE	PM	PM	ZE	NM	NM
NM	PB	PM	PM	ZE	NM
NB	PB	PB	PM	PM	ZE

Table 2 – Rule base for the *RG4-* $e\dot{e}$*_θ*

$v_\theta \backslash e_\theta$	NB	NM	ZE	PM	PB
PB	ZE	PM	PM	PB	PB
PM	NM	ZE	PM	PM	PB
ZE	NM	NM	ZE	PM	PM
NM	NB	NM	NM	ZE	PM
NB	NB	NB	NM	NM	ZE

In order to show that the system is able to stabilize in a certain state, different from the origin, is considered a scenario based on the step reference signal

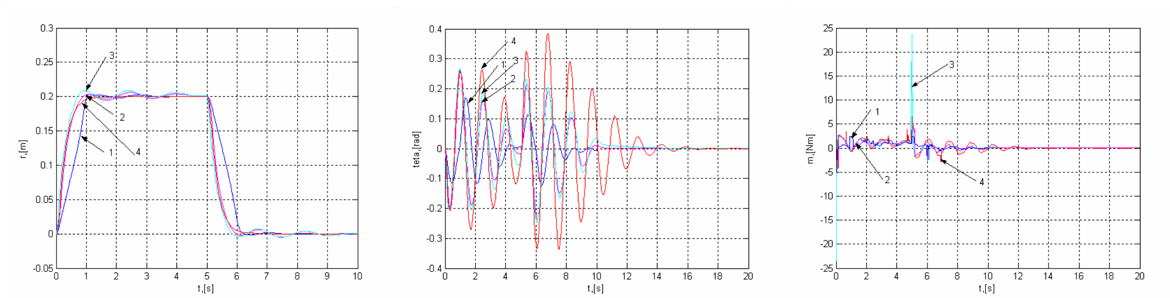
$$w(t) = \begin{cases} -0.2 \cdot \sigma(t) & \text{if } 0 \le t \le 5 \\ 0.2 \cdot \sigma(t) & \text{if } t > 5 \end{cases} \qquad (4)$$

applied in zero initial conditions

$$(r, \dot{r}, \theta, \dot{\theta}) = (0,0,0,0) .$$

The reference was chosen such that to study the manner in which the ball can be driven on the bearer, from the central position to a quite “far” position and than back to central position. The signals denoted with 1 in Figure 7 ($r(t)$ in 7a), $\theta(t)$ in 7b) and $m(t)$ in 7c)) shows the obtained results.

Translation position stabilization is just a bit oscillatory and is obtained in very well damped oscillations conditions for the rotation movement, using a torque value less than 2 Nm. The return to the central position keeps the same features; the behaviour is very resembling with a linear system’s one. Hence, the conventional fuzzy control structure seams to present acceptable stabilization and transitory features, but the calculus time is much too large. In order to reduce the computational complexity, step 2 will be followed.



a) b) c)

Figure 7. Comparative control system responses (1 – system with *RG4_* $e\dot{e}$, 2 – system with *RG4_1F*, 3 – system with *RG4_1I* and 4 – system with *improved RG4_1I*) in translation position r (a), angular position θ (b) and motor torque m (c) at reference signal (4) and initial condition $(r, \dot{r}, \theta, \dot{\theta}) = (0,0,0,0)$.

Step 2:

In the structure of *RG4_* $e\dot{e}$ (Figure 5) each fuzzy controller is replaced like in Figure 2 by a signed distance generator for sd_r and sd_θ respectively. For generators holds relation (1) with $e = e_r$, $\dot{e} = \dot{e}_r = -\dot{r}$ for sd_r and $e = e_\theta$

$\dot{e} = \dot{e}_\theta = -\dot{\theta}$ for sd_θ, to obtain the single-input fuzzy controllers *RG4_1F_r* for the translation controller and *RG4_1F_θ* for the rotational controller. There is obtained the control system in Figure 8 with inner feedback loop and with the fuzzy controller *RG4-1F*.

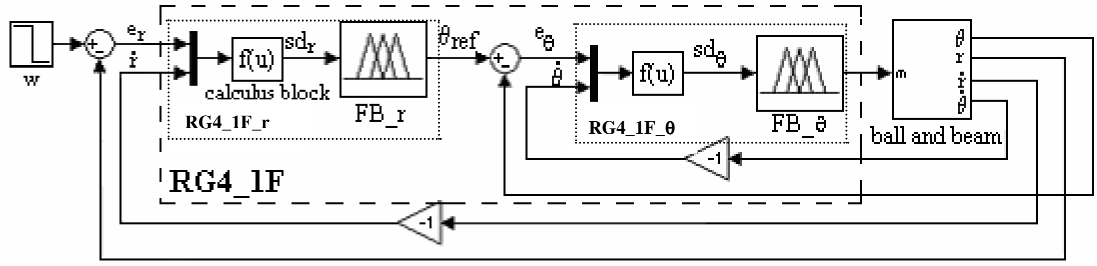


Figure 8. Simplified fuzzy control scheme with *RG4-1F* for the ball and beam system.

The values domain limits of the fuzzified variables sd_r and sd_θ, determined by geometric means [10] are: D_{sdr} = [-0.09, 0.09] m and $D_{sd\theta}$ = [-0.74, 0.74] rad.

The associated fuzzy variables for sd_r, sd_θ, θ_{ref} and m are described by 5 linguistic terms also, with membership functions as in Fig.6a) for sd_r, and sd_θ and like in Fig.6c) for θ_{ref} and m. The simplified rule bases obtained according to the mentions in section 2 are presented in table 3 and 4.

Table 3 – Rule base for the *RG4-1F-r*

sd_r	NB	NM	ZE	PM	PB
θ_{ref}	PB	PM	ZE	NM	NB

Table 4 – Rule base for the *RG4-1F-θ*

sd_θ	NB	NM	ZE	PM	PB
m	NB	NM	ZE	PM	PB

Time response of the system with *RG4_1F* for the same scenario as in step 1 is depicted in Figure 7 also (the signals denoted with 2: $r(t)$ in 7a), $\theta(t)$ in 7b) and $m(t)$ in 7c)). Indeed, the performances of the system with *RG4_1F* remain the same, but the complexity of the controllers was reduced and also the time needed for calculations.

Step3: In order to obtain an even simpler controller, easier to implement and with a reduced calculus time, the *RG4_1F* is replaced with an interpolative controller *RG4_1I* as in Figure 9. The main difference between this structure and those from Figure 8 consist in the replacing of the controllers *RG4_1F_r* and *RG4_1F_θ* with the interpolative blocks (or controllers) *RG4_1I_r* and *RG4_1I_θ.* The interpolative controllers use also the synthetic inputs sd_r and sd_θ, respectively, calculated with the same formulae and for the same variation domain as in the step 2. For the interpolative controllers, implemented as two "look-up table", the linguistic terms are described as in table 5 and 6, by disjoint intervals or single values, and the rule bases are the same in table 3 and 4.

Table 5 – Linguistic terms of *RG4_1I_r*

l.t.	**NB**	**NM**	**ZE**	**PM**	**PB**
sd_r	-0.09	[-0.02,0]	0	[0,0.02]	0.09
θ_{ref}	-2	[-0.8,0]	0	[0,0.08]	2

Table 6 – Linguistic terms of *RG4_1I_θ*

l.t.	**NB**	**NM**	**ZE**	**PM**	**PB**
sd_θ	-0.74	[-0.16,0]	0	[0,0.16]	0.74
m	-5	[-2,0]	0	[0,2]	5

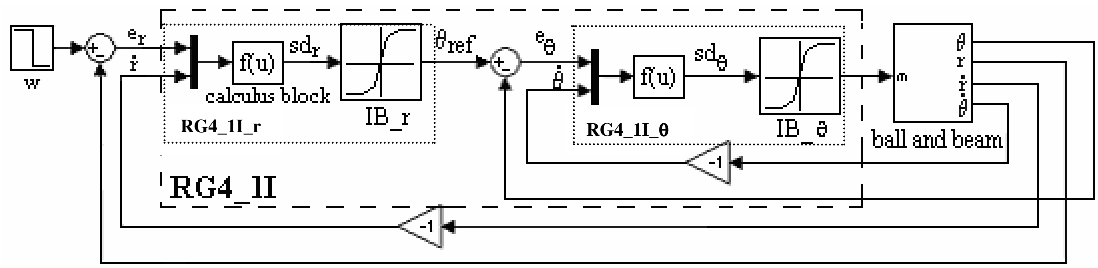


Figure 9. Interpolative based control scheme with *RG4_1I* for the ball and beam system.

The time responses of the control system with *RG4_1I* for the same scenario as in step 1 and 2 were depicted in Figure 7 (curves denoted by 3). The results are slightly better regarding the translation and angular positions (for example the rise time on *r(t)*) than in the previous steps. However, the driving effort for t = 0 sec. and t = 5 sec., also in the commutation moments of reference signal, is considerably more intensive like in step 2. Therefore, the control must be further improved by modifying the values of the control signals (outputs). In order to reduce the torque peaks without affecting the response speed of the system the I/O characteristic slope was increased, in the symmetry point neighbourhood for both *RG4_1I_r* and *RG4_1I_θ* controllers. After some trials the tables 5 and 6 was replaced by the tables 7 and 8.

Table 7 – Linguistic terms of *improved RG4_1I_r*

l.t.	NB	NM	ZE	PM	PB
sd_r	-0.09	[-0.04,0]	0	[0,0.04]	0.09
θ_{ref}	-2	[-3,0]	0	[0,3]	2

Table 8 – Linguistic terms of *improved RG4_1I_θ*

l.t.	NB	NM	ZE	PM	PB
sd_θ	-0.74	[-0.16,0]	0	[0,0.16]	0.74
m	-3	[-3,0]	0	[0,3]	3

Excepting the equilibrium points corresponding to the linguistic values ZE, in every case was necessary to modify all support points in the interpolation tables.

The time responses of the control system with this improved *RG4_1I* controller are presented in Figure 7 also. The main advantage are: the reduced complexity of the controllers, reduced calculus time and generally better performances comparatively to those obtained in the second design step (step 2).

Next, also in the third design step, sampled-time structures were developed based on the corresponding continuous time control with improved interpolative controllers *RG4_1I*, for different sample periods.

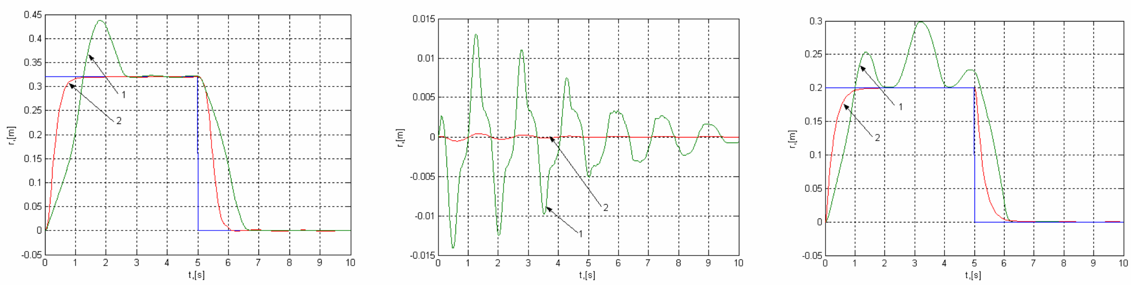
The sampled system preserves the performances of the continuous one until a $h_{\max} = 0.01\,\text{sec}$ value of the sample period. This value is small enough to preserve the control performances and, in the same time, large enough to implement a real-time control.

The controlled plant (ball and beam system) is strongly non-linear and all the designed controllers have a fixed structure.

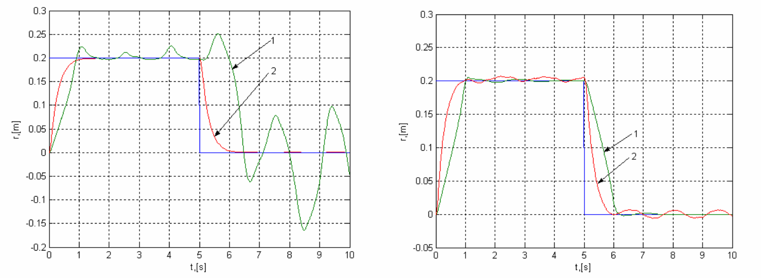
In such conditions it is obvious the necessity to investigate if the designed control systems preserve their properties at parameters value modifications of the plant (position reference *w*, initial conditions $(r, \dot{r}, \theta, \dot{\theta})$, ball mass value M_2, coefficient of kinetic friction μ_f and sample period *h*. The main property that must be preserved is the system stabilization at different reference values for the ball position *r* on the bearer. The secondary properties refer to the control systems dynamics.

Some results of the investigation on the robustness of these properties were summarised in Figure 10 (for the control systems with $RG4\_e\dot{e}$ and improved *RG4_1I*) and by table 9 for the systems with $RG4\_e\dot{e}$, *RG4_1F* and improved *RG4_1I*.

In all situations the interpolative control system behaves better from the robustness point of view and permits larger ranges for the parameter modifications. In the same conditions, the control system with conventional fuzzy controller looses completely the control of the ball in some of those cases.



a) b) c)



d) e)

Figure 10. Some time responses for control systems (1 - with $RG4\_e\dot{e}$ and 2 – with improved *RG4_1I* at parameters value modifications: reference value $w = 0.32$ (a), initial conditions $(r, \dot{r}, \theta, \dot{\theta}) = (0, 0, \frac{\pi}{10}, 0)$ (b), mass value $M_2 = 1$ kg (c), coefficient of kinetic friction $\mu_f = 0.0001$ (d), sample period value $h = 0.01$ sec (e)

Table 9 –Robustness analysis of the control structures with complete state feedback (TPS-4)

Control structure type	**Parameters value modifications**					
	translation position reference *w*	**initial conditions** $(r, \dot{r}, \theta, \dot{\theta})$	**ball mass value M_2**	**coefficient of kinetic friction μ_f**	**sample period value *h***	**More parameters (same time)**
$RG4\_e\dot{e}$	oscillations; overshoot	oscillations; exponential instability	exponential instability	oscillations	stable; perf. are preserved	exponential instability
RG4_1F	oscillations	oscillations	oscillations	oscillations	stable; perf. are preserved	exponential instability
improved RG4_1I	stable; performances are preserved	stable; perf. are preserved	stable; perf. are preserved	stable; perf. are preserved	stable; perf. are preserved	stable; perf. are preserved

3.3. Control structure with translation state feedback (TPS – 2)

It is well known that for systems with inner feedback loop the inner loop has the role to improve the output control of the internal loop which represents an internal measure of the controlled plant. If we renounce at the internal loop, we renounce also at the control of this measure. Both solutions achieve the control of the main measure.

These specifications are available in principle for the control structures in section 3.2 also. In the hypothesis of adopting such a solution one can renounce to the internal loop with feedback from the rotational motion of the frame and it's preserved only the loop with feedback from the translation motion of the ball on the bearer. The rule base of the fuzzy translation controller $RG4\_e\dot{e}\_r$ remain the same and as well the membership functions (Figure 6) but it is necessary to compensate the gain of the inner feedback loop of the initial control system.

In this context, it is natural to use a control scheme for the ball and beam system with a controller with only two inputs, with feedback from the translation position and speed only. Alternatively to the block schemes in Figure 5, 8 and 9 the schemes in Figure 11 were considered: in Figure 11a) the control system with fuzzy conventional controller with two inputs $RG2\_e\dot{e}$, in Figure 11b) the control system with simplified fuzzy controller with two inputs *RG2_1F* and in Figure 11c) the control system with interpolative two-input controller *RG2_1I.*

The structures in Figure 11 are not containing anymore the control loop of the angular position. The controllers responsible to the angular position control were discarded and the controllers that insure the translation position (and the corresponding feedback) was preserved. The $RG2\_e\dot{e}$, *RG2_1F* and *RG2_1I* controllers are the same with $RG4\_e\dot{e}\_r$, *RG4_1F_r* and *RG4_1I_r* respectively.

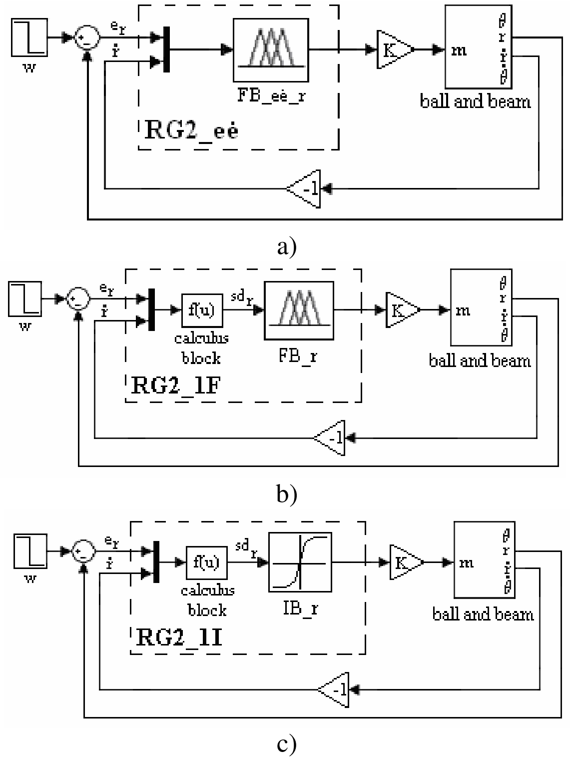


Figure 11. Fuzzy control system with $RG2\_e\dot{e}$ (a), *RG_-1F* (b) and *RG2_1I* (c) for the ball and beam system

The gain K of the command (motor torque) m has the $K = 10$ value for the interpolative system and $K = 20$ value for the fuzzy ones. These values were determined in a quasi-empirical manner.

On the one hand, the simulations made with structures in Figure 11 in the same conditions as in 3.2 shows that the new structures present, for the nominal regime, the same performances as the initial structures – from the controlled position r point of view. Some differences appear in the angle θ and torque m variations (see Figure 12a) for the angle and figure 12b) for the torque): worth damped oscillations and a too large settling time for the rotational movement (see the variation of angle θ), as well as the peak in the driving torque m.

By using the two-input controller with feedback only from the translation movement the motor torque values are increasing regarding the case in that the four-inputs controller is used. Therefore the advantage of the control structure simplicity is diminished by the necessity of a larger torque availability and larger energy consumption, respectively. The observation can be extrapolated: a more complicated control structure can present some advantages regarding a simpler control structure with the same function, advantages related to the variation domain of the execution measure and the installed power requisite.

On the other hand, the comparative study of these schemes - intended for the same function – realized only for nominal conditions, is not sufficient. As one can see in 3.2, the parameters of the system as ball mass, initial position etc. can take different values. This is the reason for a careful study - in this situation also – of the systems behavior in changing parameters condition. The study was made with the same parameters and in the same scenarios as in section 3.2. From the simulations resulted that the robustness of the new structures regarding the parameters modifications from the nominal values is different: the robustness domains of the new structures are reduced regarding those of the structures in 3.2.

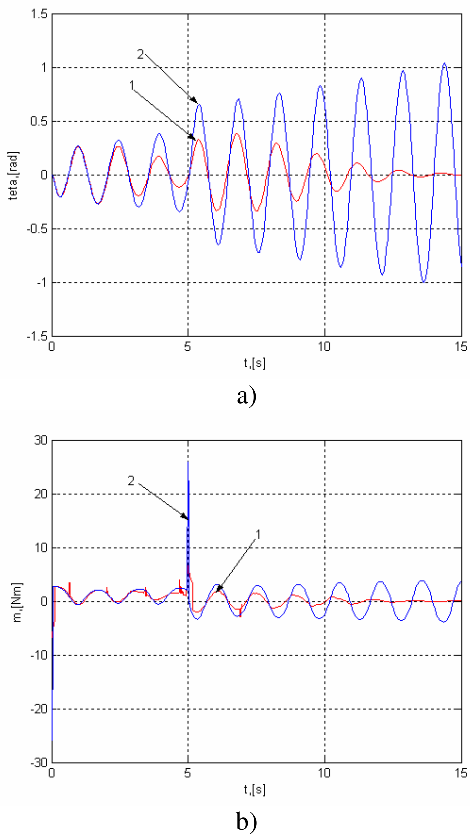
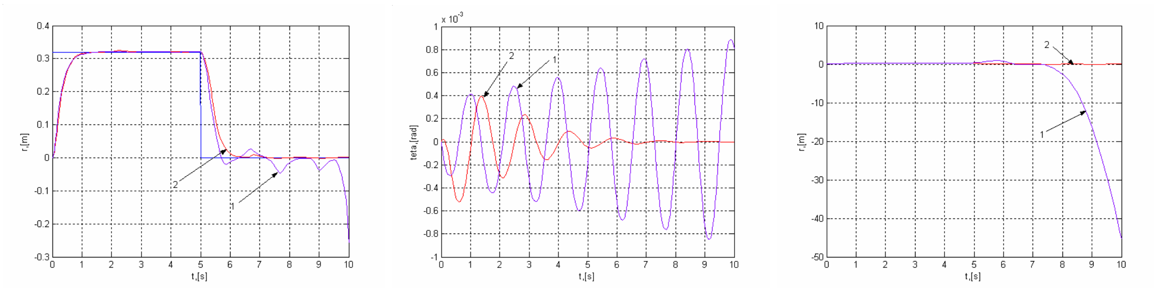


Figure 12. Variations of the angle θ (a) and torque m (b) for the control systems with 1 – *improved RG4_1I* and 2 – *RG2_1I*

In this context, we consider Figure 13 referring to the improved interpolative controller *RG2_1I* system and to the system with four-input controller from which it derived.



a) b) c)

Fig.13. Time responses for control systems (1 - with *RG2_1I* and 2 – with improved *RG4_1I*) at parameters value modifications: prescribed value $w = 0.32$ (a), initial conditions $(r, \dot{r}, \theta, \dot{\theta}) = (0, 0, \frac{\pi}{10}, 0)$ (b), mass value $M_2 = 1.9\,\text{kg}$ (c)

Table 10 - Robustness analysis of the control structures with translation state feedback (TPS-2)

Control structure type	**Parameters value modifications**					
	translation position reference *w*	**initial conditions ($r, \dot{r}, \theta, \dot{\theta}$)**	**ball mass value M_2**	**coefficient of kinetic friction μ_f**	**sample period value *h***	**More parameters (same time)**
improved RG4_1I	stable; perf. are preserved	oscillations	stable; perf. are preserved	stable; perf. are preserved	small oscillations	small oscillations
RG2_1I	oscillations; instable	stable; perf. are preserved	exponential instability	stable; perf. are preserved	small oscillations	exponential instability

Figure 13a) shows that domain of the reference must be reduced. The return of the ball from the *r* = 0.32 m position in central *r* = 0 position is made with some oscillations. As Figure 13b) shows, the initial equilibrium angular domain must be reduced also: the system with *RG2-1I* can't manage to re-ecquilibrate for $\theta > \frac{\pi}{12}$.

Figure 13c) illustrates that the structure with improved *RG4_1I* is stable even for very large values of the ball mass (M_2 = 1.9 kg). In the same time, the structure with *RG2_1I* becomes unstable. The conclusions of the comparative study regarding the *RG2_1I* and *RG4_1I* are synthesized in table 10. It can be observed that the simple structure with only two feedbacks is less performing than the four feedbacks one.

The comparative study made to the rest of the schemes leads to similar observations.

Finally we conclude that the simplification of the control scheme is advantageous only when the parameters of the plant and the reference signal are placed between reduced variation bounds and at the actuator level is available enough power. Hence, the initial scheme, more complicated, is recommended by its robustness and by a reduced installed power for actuators.

4. CONCLUSIONS

The paper is based on the fact that controllers with interpolative blocks can replace fuzzy controllers in control structures. This fact is possible because fuzzy controllers belong to the interpolative-type controller category, meaning controllers which implements interpolative-type reasoning. The mentioned replacement is not only a formal operation; it is also associated with further corrections that confer to the structures with interpolative controllers enough flexibility to obtain better performances. The possibility of performances improvement is the main argument that justifies the demarche made in the present paper. Another argument is the reduced calculus time, suited for the real-time implementation - it's about "look-up table" type solutions.

In order to illustrate the above affirmations, a case study is presented in the paper. The controlled plant is an electromechanical one:

ball and beam system, driven by the torque applied in the rotational joint. The model used for the system is a 4th order one. The case study is conceived also in order to underline the difference between capabilities of two types of control structures. Structures with feedback from all four states and structures with feedback from only two states are considered. The study showed that the structures in the first category are more performing from at least two points of view: ▪ the usage of an execution element with reduced available power, ▪ larger robustness domain (for the control systems performances) regarding to changes in plant parameters, from the nominal values. In the same time, the study underlines the usefulness of using control structures with synthetic intermediate measures, as signed distance is.

From the applicative point of view, the conclusion of the case study is the following: the best solution is an inner based control structure that uses an improved interpolative controller with feedback from all four plant states *RG4-1I*, real-time implemented. We consider that the methodological aspects that appear in the paper are even more important.

REFERENCES

[1] Choi, B.-J., Kwak, S.-W., Kim, B.K. – "Design and stability analysis of single-input fuzzy controller", IEEE Transactions on Systems, Man and Cybernetics, part B: Cybernetics, vol.30/2, pp. **303-309**, 2000.

[2] Dragomir, T.L., Dale, S. – "Cases of using interpolator type blocks in control structures", Proceedings of SACCS'01, Iaşi, CD, 2001.

[3] Dragomir, T.L., Dale, S., Bălaş, M.M. – "Some aspects regarding interpolative structures", Proceedings of IWICS'01, Bucureşti, pp. **6-19**, 2001.

[4] Drechsel, D. – „Regelbasierte Interpolation und fuzzy control", Vieweg, 1996.

[5] Drechsel, D. – „RIP control", CEAI 1/1999, Bucureşti, pp. **7-17**.

[6] Dragomir, T.L., Vladu, E.E., Dale, S. – "Interpolative-type controller synthesis using genetic algorithms", Proceedings of CSCS'14, Bucureşti, pp. **70-75**, 2003.

[7] Dale, S., Vladu, E.E. – „Using genetic algorithms to improve hybrid adaptive-interpolative system performances", Proceedings of RSEE'02, Băile-Felix, pp. **7-23**, 2002.

[8] Palm, R. - "On the compatibility of fuzzy control and conventional control techniques", from "Fuzzy logic – Implementation and Applications", edited by M.J. Patira and D.M. Mlynek, Wiley-Teubner, pp. **63-105**.

[9] Dale, S., Bara, A. – "Simple Intelligent Controllers based on Rules", Proceedings of SOFA'05 International Workshop, Szeged-Arad, pp. **68-72**, 2005.

[10] Dale, S., - "Contribuţii la studiul sistemelor de reglare cu regulatoare de tip interpolativ", Editura Politehnica, Timişoara, 2006.

[11] L.A. Zadeh – „Interpolative reasoning as a common basis for inference in fuzzy logic, neural network theory and the calculus of fuzzy If/Then rules", Opening Talk, 2nd International conference on fuzzy logic and neural networks, Iizuka, pp. **XIII-XIV**, 1992.

[12] L.T. Kóczy and K. Hirota – „Approximate reasoning by linear rule interpolation and general approximation", Internat. J. Approx. Reason. 1993/9, pp. **197-225**, 1993.

[13] D. Dubois, H. Prade and M. Grabisch – „Gradual rules and the aproximation of control laws", John Wiley & Sons, pp.**147-183**, 1995.