On the stability of a pilot-aircraft system with input delay using controllers obtained by Artstein transform

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Abstract: In this paper the stability of a pilot-aircraft system with input delay and robust controllers obtained by Artstein transform is studied. The aircraft is modelled by the ADMIRE short-period dynamics together with the unsaturated first-order actuator model. The pilot model is expressed by a fixed gain together with a simple delay and is supposed to act on piecewise constant control signals. Beside the case when the states of the system are available, the case when the states are not observable is taken into account, using state observers. Also, the optimal stabilization with prescribed degree of stability is used in the synthesis of another robust controller for the simplified ADMIRE model. The robustness of the proposed control laws is investigated by numerical simulations.

Keywords: stability, time delay systems, piecewise controllers, robustness, short-period dynamics, Artstein transform, algebraic Riccati equation

1. INTRODUCTION

In this paper the stability of a particular case of the system

$$\dot{x}(t) = Ax(t) + B_0u(t) + B_1u(t - \tau) \tag{1}$$

is studied, where $\tau > 0$ is a delay which will be specified afterwards.

Remark 1. In the above equation, $A \in R^{n\times n}$,$\{B_0, B_1\} \in R^{n\times m}$, $u(t) \in R^{m\times 1}$, $\tau > 0$, the solution being defined for $t > 0$ if the initial conditions are given $(x_0, u(\cdot))$ together with the control input $u(t)$ for $t > 0$ where $u_0(\theta)$ represents a certain initial function defined for $\theta \in [-\tau, 0)$.

Remark 2. In the application used in this paper, the matrix $B_0 = 0_{n\times m}$.

Artstein transform introduces a linear transformation that applied to a delayed system leads to "an equivalent control system without delays" [Artstein (1982)]. For the obtained system, a discrete LQ controller is derived. Because the control is piecewise constant, the control problem is a suboptimal control problem.

Among the works which covers the same topic of deriving (robust) controllers for systems with delay, a classic reference is [Anderson and Moore (1969)]. In the paper [Olbrot (1978)], the stabilization of linear systems with general time delays is evidenciated as an algebraic rank condition. [Pandolfi (1991)] covers the area of dynamic stabilization by deriving a compensator through the use of a state-space technique. [Răsvan and Popescu (2001)] consider an elementary approach, based on variants of the Smith predictor, resulting a theoretical analysis of the compensator with suggestions for a digital control implementation. The paper [Popescu (2002)] deals with suboptimal control of systems with input delay using a quadratic performance index in order to show the robustness of the compensator, deduced using the discrete analogue of Artstein transform, relative to the delay. In [Popescu (2003)] a finite spectrum assignment technique, based on Artstein transform, is used for the distributed control delay law. In paper [Han (2005)] absolute stability of time-delay systems with sector-bounded nonlinearity is dealt with a stability criteria expressed in the form of linear matrix inequalities. H_∞ technique is applied by [Wang et al. (2008)], for a class of uncertain non-linear time-delay stochastic systems with the dynamics of the filtering error allowing robust asymptotic stability in the mean square. For a comprehensive overview, regarding results and open problems in the field of delayed systems, a good reference is [Richard (2003)].

This paper is organized as follows. In Section 2 theoretical considerations regarding the Artstein transform and its discrete analogue representation are presented. Also, the suitability to stabilizing controller design of the discrete analogue Artstein transform is mentioned. In Section 3, the low-order Aero Data Model in a Research Environment (ADMIRE) dynamics is treated. Starting from the so-called "low-order GAM-ADMIRE" theoretical aircraft and the simplified ADMIRE model equations, a new model is synthesized (the "simplified GAM-ADMIRE" system). The determination of the equilibrium points is made, together with the presentation of the associated linearized system and its derived short period approximation, in the context of the first order actuator dynamics. In Section 4 a piecewise constant control signal is derived in order to implement, in the next section, the robust controller. In this manner, a new pilot model is obtained with contributions from the classical characteristics of a human pilot model ($\kappa_P e^{-\tau s}$; τ, $\kappa_P > 0$) and, also, with features that seem to an automatic pilot (the stabilizing matrix which is obtained by the allocation of the system poles in the left complex plane). In Section 5 are presented numerical simulations, showing the robust stabilization of the simplified GAM-ADMIRE model through the utilization of the controller defined in the previous section. The case when the state of the system is not available is treated, in Section 6, using feedback based on state observers. The analysis is made both theoretical and numerical. The optimal stabilization with prescribed degree of stability is studied in Section 7 with the help of an optimal robust controller. Regarding the numerical considerations, these are made in Section 8 where is shown that the low order GAM-ADMIRE model is not optimally stabilizable. Conversely, the simplified ADMIRE theoretical aircraft is optimally stabilizable. The last section is dedicated to conclusions.

2. THEORETICAL CONSIDERATIONS

In what follows, the Artstein transform will be considered [Artstein (1982)]

$$z(t) = x(t) + \int_{-\tau}^{0} e^{-A(\theta+\tau)} B_1 u(t+\tau) d\tau \tag{2}$$

which leads to the following equivalence [Răsvan and Popescu (2004)]:

Proposition 3. Let $(x(t), u(t);\, t > 0)$ be a solution (admissible pair) for (1), defined by initial conditions $(x_0, u_0(\cdot))$. Then $(z(t), u(t); t > 0)$ with $z(t)$ defined by (2) is a solution (admissible pair) for the system

$$\dot{z}(t) = Az(t) + (B_0 + e^{-A\tau} B_1)u(t) \tag{3}$$

with the initial condition $z_0 = z(0)$.

Conversely, let $(z(t), u(t);\, t > 0)$ be a solution of (3) defined by initial condition z_0. Then, given $u_0(\cdot)$ defined on $(-\tau, 0)$ and taking

$$x_0 = z_0 - \int_{-\tau}^{0} e^{-A(\theta+\tau)} B_1 u_0(\theta) d\theta \tag{4}$$

the solution of (1) defined by these initial conditions and by $u(t)$, $t > 0$ is given by

$$x(t) = z(t) - \int_{-\tau}^{0} e^{-A(\theta+\tau)} B_1 u(t+\theta) d\theta \tag{5}$$

Using the steps from the previously cited book, the piecewise constant control signals are defined as follows

$$u(t) = u_k,\ k\delta \le t < (k+1)\delta,\ k \in N \tag{6}$$

where $\delta = \frac{h}{p}$, $p \in N^*$, $h > 0$ (h being the nominal delay).

Remark 4. δ is used in the discrete time implementation of the control law and is determined by the nominal delay h and a finite and fixed $p \in N^*$.

The controller is designed for the nominal delay h (a priori fixed) and will have to robustly stabilize the continous system (1) with the unknown delay τ. For the system (1), the following discrete time system is associated

$$x_{k+1} = A(\delta)x_k + B_0(\delta)u_k + B_1(\delta)u_{k-p} \tag{7}$$

where

$$A(\delta) = e^{A\delta},\ B_i(\delta) = \left(\int_0^{\delta} e^{A\theta} d\theta \right) B_i,\ i \in \{0, 1\} \tag{8}$$

Remark 5. The above equations can be derived in the following way

$$\begin{aligned} e^{A\delta} &\overset{t=\delta}{=} e^{At} = \mathcal{L}^{-1}\{(sI - A)^{-1}\} \\ \int_0^{\delta} e^{A\theta} d\theta &= A^{-1} e^{A\theta}|_0^{\delta} = A^{-1}(e^{A\delta} - I) \end{aligned} \tag{9}$$

where $\mathcal{L}^{-1}\{\cdot\}$ is the inverse Laplace transform.

Let $(x_0, u_0(\cdot))$ be the initial condition associated to (1). Because the discretized system is satisfied by

$$x_k = x(k\delta)$$

with $x(\cdot)$ - the solution of (1) with piecewise constant control signals, is natural to choose the initial condition $(x_0;\ u^0_{-i} = u_0(-i\delta),\ i \in \{0, \dots, p\})$, so that

$$z_k = x_k + \sum_{j=-p}^{-1} A(\delta)^{-(p+j+1)} B_1(\delta) u_{k+j} \tag{10}$$

The above equation represents the discrete analogue of the Artstein transform, and the discrete system associated to (3) can be determined as

$$z_{k+1} = A(\delta)z_k + (B_0(\delta) + A(\delta)^{-p} B_1(\delta))u_k \tag{11}$$

Let the system

$$\begin{aligned} x_{k+1} &= A(\delta)x_k + B_0(\delta)u_k + B_1(\delta)u_{k-p} \\ u_k &= Fx_k + \sum_{j=-p}^{-1} FA(\delta)^{-(p+j+1)} B_1(\delta) u_{k+j} \end{aligned} \tag{12}$$

where F is a stabilizing feedback for the transformed system (3) [Răsvan and Popescu (2004)]

$$u(t) = Fz(k\delta) = Fz_k,\ k\delta \le t < (k+1)\delta \tag{13}$$

The following proposition holds [Popescu (2003)]:

Proposition 6. Let the system (3) under the assumption that $(A, B_0 + e^{-A\tau} B_1)$ is a stabilizable pair. Then, $(A(\delta),\ B_0(\delta) + A(\delta)^{-p} B_1(\delta))$ is stabilizable and a stabilizing feedback for this last pair is, also, stabilizing for (3) under condition that $\delta > 0$ is sufficiently small. Here, $A(\delta)$, $B_0(\delta)$, $B_1(\delta)$ are defined by (8) and $\delta = \frac{h}{p}$. Moreover, a stabilizing feedback for (3) is stabilizing also if the implementation is made using samples, i. e. state-values measured at $k\delta$, $k \in \{0, 1, 2, \dots\}$.

3. LOW-ORDER ADMIRE DYNAMICS: THE SIMPLIFIED GAM-ADMIRE

In this section the simplified ADMIRE aircraft model [Balint and Balint (2011)] together with the "low-order GAM-ADMIRE" theoretical aircraft [Ioniţă et al. (2008)] - the quotes are necessary because this is a nomenclature which has not been used in the original cited paper - are linked together in order to derive a new system that describe the dynamics of both cited theoretical aircrafts (by appropiate constant parameters).

The ADMIRE simplified model is

$$\begin{aligned} \dot{\alpha} &= z_{\alpha}\alpha + q + \frac{g}{V_0}\cos\theta + z_{\delta_e}\delta_e \\ \dot{q} &= m_{\alpha}\alpha + m_q q + \frac{g}{V_0}(\bar{m}_{\dot{\alpha}}\cos\theta - \frac{c_2}{a} a_2 \sin\theta) + m_{\delta_e}\delta_e \\ \dot{\theta} &= q \end{aligned} \tag{14}$$

with the parameters values given in Table 1.

The low-order GAM-ADMIRE is described by

$z_\alpha = -1.6$	$g = 9.81\ \frac{m}{s^2}$	$V_0 = 84.569\ \frac{m}{s}$
$z_{\delta_e} = -0.5209$	$m_\alpha = 1.7251$	$m_{\delta_e} = -9.9729$
$m_q = -22.61$	$\bar{m}_{\dot{\alpha}} = -5.2642$	$c_2 = -0.029$
$a = -0.485$	$a_2 = 11.964$	$\frac{c_2}{a} a_2 = 0.7154$

Table 1. Coefficients for simplified ADMIRE model [Balint and Balint (2011)]

$$
\begin{aligned}
&\dot{\alpha} = z_\alpha \alpha + q + \frac{g}{V_0}\cos\theta + z_{\delta_e}\delta_e \\
&\dot{q} = \bar{m}_\alpha \alpha + \bar{m}_q q - \frac{1}{a}\alpha q + \frac{g}{V_0}(m_{\dot{\alpha}}\cos\theta - \bar{a}\sin\theta) + \bar{m}_{\delta_e}\delta_e \\
&\dot{\theta} = q
\end{aligned} \tag{15}
$$

with the coefficients given in Table 2.

$z_\alpha = -.7986$	$g = 9.81\ \frac{m}{s^2}$	$V_0 = 84.5\ \frac{m}{s}$
$z_{\delta_e} = -.2603$	$\bar{m}_\alpha = -6.5315$	$\bar{m}_{\delta_e} = -8.2668$
$\bar{m}_q = -.6957$	$m_{\dot{\alpha}} = -.162$	
$a = -.2424$	$\bar{a} = 1.424$	

Table 2. Coefficients for low-order GAM-ADMIRE [Ioniţă et al. (2008)]

Remark 7. Comparing the models (15) and (14) the fact that they look very similar is obvious (although the notations are different - m_{δ_e} vs. $\bar{m}_{\delta_e}$, m_α vs. $\bar{m}_\alpha$ or m_q vs. $\bar{m}_q$ - and the term $\frac{1}{a}\alpha q$ is present in just one of the models). Even if the coefficients have different values, the resemblance between the two models permit a unitary approach for writing a synthesized system.

Taking into account the previous remark, the following nonlinear model is derived, which will be called "the simplified GAM-ADMIRE model":

$$
\begin{aligned}
&\dot{\alpha} = \zeta_\alpha \alpha + q + \frac{g}{V_0}\cos\theta + \zeta_{\delta_e}\delta_e \\
&\dot{q} = \mu_\alpha \alpha + \mu_q q + \mu_f \alpha q + \frac{g}{V_0}(\mu_c\cos\theta + \mu_s\sin\theta) + \mu_{\delta_e}\delta_e \\
&\dot{\theta} = q
\end{aligned} \tag{16}
$$

In this paper, α is angle of attack, θ represents the pitch angle, $q = \frac{d\theta}{dt}$ pitch rate and, for simplified GAM-ADMIRE, δ_e is the elevon deflection. In the synthetized model (16), the coefficients will take either values from Table 1, either from Table 2, and - in this manner - Table 3 is considered.

$\zeta_\alpha \in \left\{ \begin{matrix} -.7986, \\ -1.6 \end{matrix} \right\}$	$g = 9.81\ \frac{m}{s^2}$	$V_0 \in \{84.5, 84.569\}\frac{m}{s}$
$\zeta_{\delta_e} \in \left\{ \begin{matrix} -.2603, \\ -0.5209 \end{matrix} \right\}$	$\mu_\alpha \in \{\bar{m}_\alpha, m_\alpha\}$	$\mu_{\delta_e} \in \{\bar{m}_{\delta_e}, m_{\delta_e}\}$
$\mu_q \in \{\bar{m}_q, m_q\}$	$\mu_c \in \{m_{\dot{\alpha}}, \bar{m}_{\dot{\alpha}}\}$	$a \in \{-.2424, -.485\}$
$\mu_f \in \left\{-\frac{1}{a}, 0\right\}$	$\mu_s \in \{-\bar{a}, -\frac{c_2}{a}a_2\}$	

Table 3. Coefficients for the simplified GAM-ADMIRE model

3.1 *Analytical determination of the equilibrium points*

The equilibrium points of the system (16) were found using simple algebraic manipulations, the universal trigonometric substitution and the next procedure.
With an arbitrary, but fixed $\bar{\delta}_e$, bounded by

$$|\bar{\delta}_e| \le \frac{\sqrt{\varrho_a^2 + \varrho_b^2}}{|\varrho_c|} \tag{17}$$

- where the next notations (18) were employed -

$$
\begin{cases}
\varrho_a = \dfrac{g}{V_0}\left(\mu_c - \dfrac{\mu_\alpha}{\zeta_\alpha}\right) \\
\varrho_b = \mu_s \dfrac{g}{V_0} \\
\varrho_c = \mu_{\delta_e} - \dfrac{\zeta_{\delta_e}}{\zeta_\alpha}\mu_\alpha
\end{cases} \tag{18}
$$

the following results were obtained

$$\bar{\theta}_{\{1,\,2\}} = 2\tan^{-1}(\gamma_{a_{\{1,2\}}}) \tag{19}$$

where

$$\bar{\alpha} = -\frac{1}{\zeta_\alpha}\left(\frac{g}{V_0}\cos\bar{\theta} + \zeta_{\delta_e}\bar{\delta}_e\right) \tag{20}$$

$$\gamma_{a_{\{1,\,2\}}} = \frac{-\varrho_b \pm \sqrt{\varrho_a^2 + \varrho_b^2 - \varrho_c^2\bar{\delta}_e^2}}{\bar{\delta}_e\varrho_c - \varrho_a} \tag{21}$$

Remark 8. From (21) the following condition results

$$\bar{\delta}_e \neq \frac{\varrho_a}{\varrho_c} \tag{22}$$

and, in the case when this condition does not hold, γ_a is determined by the next relation

$$\gamma_a = -\frac{\gamma_{a_1}}{\gamma_{a_2}} \tag{23}$$

3.2 *The linearized system of the simplified GAM-ADMIRE model*

The system (16) is linearized around an equilibrium point $\bar{x}_2 = (\bar{\alpha}, 0, \bar{\theta})^T$ and the linearized system has the form

$$
\begin{cases}
\Delta\dot{x}_2 = A_\Lambda \Delta x_2 + b_\Lambda \Delta\delta_e \\
\Delta y = c_\Lambda^T \Delta x_2
\end{cases} \tag{24}
$$

where:

$$
\begin{aligned}
&A_\Lambda = \begin{pmatrix} \zeta_\alpha & 1 & -\dfrac{g}{V_0}\sin\bar{\theta} \\ \mu_\alpha & (\mu_q + \mu_f\bar{\alpha}) & \dfrac{g}{V_0}(\mu_s\cos\bar{\theta} - \mu_c\sin\bar{\theta}) \\ 0 & 1 & 0 \end{pmatrix} \\
&\Delta x_2 = \begin{pmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \end{pmatrix};\ b_\Lambda = \begin{pmatrix} \zeta_{\delta_e} \\ \mu_{\delta_e} \\ 0 \end{pmatrix};\ c_\Lambda^T = (0\ 0\ 1)
\end{aligned} \tag{25}
$$

Remark 9. In (24) the following notation has been used:

$$\Delta x_2 = x_2 - \bar{x}_2,\ x_2 = (\alpha, q, \theta)^T \tag{26}$$

3.3 *Short-period approximation with first-order unsaturated actuator model*

For the system (24) the short-period approximation is applied [Ioniţă (2009)]

$$
\begin{aligned}
\Delta\dot{\alpha} &= \zeta_\alpha\Delta\alpha + \Delta q + \zeta_{\delta_e}\Delta\delta_e \\
\Delta\dot{q} &= \mu_\alpha\Delta\alpha + \mu_{qf}\Delta q + \mu_{\delta_e}\Delta\delta_e
\end{aligned} \tag{27}
$$

where $\mu_{qf} \stackrel{def}{=} \mu_q + \mu_f\bar{\alpha}$.

Remark 10. A natural question is if always someone can use the short-period dynamics of an airplane in order to take conclusions about the stability of pilot-aircraft system. The answer is not positive because the modes of the longitudinal motion (the short-period and phugoid) can not be always decoupled. A precise answer is given by the theory of singular perturbations [Vidyasagar (1978)] or

by some necessary and sufficient conditions for decoupling [Falb and Wolovich (1967)].
The following assertion is considered: when $\theta = \alpha$, i.e. when the airplane is over the runway in the process of landing or when just flies in a straight line, the longitudinal dynamics can be decoupled.

Considering the first order actuator model with SAS (stability augmentation system)

$$\dot{\delta}_e = -\omega_0(\kappa_\alpha \alpha + \kappa_q q + \delta_e) \tag{28}$$

(where κ_α, $\kappa_q > 0$ are the SAS gains and $\omega_0 > 0$ is the actuator constant) a new model is derived from (27)+(28)

$$\begin{aligned} &\Delta\dot{\alpha}(t) = \zeta_\alpha \Delta\alpha(t) + \Delta q(t) + \zeta_{\delta_e}\Delta\delta_e(t) \\ &\Delta\dot{q}(t) = \mu_\alpha \Delta\alpha(t) + \mu_{qf}\Delta q(t) + \mu_{\delta_e}\Delta\delta_e(t) \\ &\Delta\dot{\delta}_e(t) = -\omega_0(\kappa_\alpha \Delta\alpha(t) + \kappa_q \Delta q(t) + \Delta\delta_e(t)) + \omega_0 u(t-\tau) \end{aligned} \tag{29}$$

with $u(t)$ defined in the next section ($\tau > 0$ being the delay of the human operator and $\Delta\delta_e(t) = \delta(t) - \bar{\delta}_e$).

4. ON THE STABILITY OF THE PILOT-AIRCRAFT SYSTEM WITH INPUT DELAY

Consider the delayed-system

$$\dot{x}(t) = Ax(t) + B_0 u(t) + B_1 u(t-\tau)$$

where, employing (29), the following notation has been used: $x \overset{def}{=} \Delta x$, $x \in \{\alpha, q, \delta_e\}$

$$A = \begin{pmatrix} \zeta_\alpha & 1 & \zeta_{\delta_e} \\ \mu_\alpha & \mu_{qf} & \mu_{\delta_e} \\ -\omega_0\kappa_\alpha & -\omega_0\kappa_q & -\omega_0 \end{pmatrix};\ x = \begin{pmatrix} \alpha \\ q \\ \delta_e \end{pmatrix} \tag{30}$$

$$B_0 = (\, 0_{3\times 1}\,);\ B_1 = (\, 0_{1\times 2}\ \omega_0\,)^T;\ c^T = (1\ 0_{1\times 2})$$

Using (3) the next system is obtained

$$\dot{z}(t) = Az(t) + Bu(t) \tag{31}$$

where

$$B \overset{def}{=} B_0 + e^{-A\tau}B_1 = e^{-A\tau}B_1 \tag{32}$$

Let the $det(sI - A)$ expression

$$det(sI - A) = s^3 + \zeta_{SA_2}s^2 + \zeta_{SA_1}s + \zeta_{SA_0} \tag{33}$$

where ζ_{SA_i}, $i = 0, 1, 2$ are given by

$$\begin{aligned} &\zeta_{SA_2} = \omega_0 - \mu_{qf} - \zeta_\alpha \\ &\zeta_{SA_1} = \omega_0\kappa_\alpha\zeta_{\delta_e} - \mu_\alpha - (\omega_0 - \mu_{qf})\zeta_\alpha + \omega_0(\mu_{\delta_e}\kappa_q - \mu_{qf}) \\ &\zeta_{SA_0} = \omega_0[\zeta_{\delta_e}(\mu_\alpha\kappa_q - \kappa_\alpha\mu_{qf}) + \kappa_\alpha\mu_{\delta_e} - \mu_\alpha + \\ &\qquad + \mu_{qf}\zeta_\alpha - \zeta_\alpha\mu_{\delta_e}\kappa_q] \end{aligned}$$

with $\zeta_{SA_i} > 0$ and $\omega_0 = 20\ \frac{rad}{s}$, $\bar{\alpha} > 0$, $\kappa_q = .0728$, $\kappa_\alpha = 0.401$ [Balint and Balint (2011)]

then, considering the nominal delay h

$$B \overset{t=-h}{=} \mathcal{L}^{-1}\{(sI - A)^{-1}\}B_1 \tag{34}$$

$$\begin{aligned} e^{A\delta} &\overset{t=\delta}{=} \mathcal{L}^{-1}\{(sI - A)^{-1}\} = \\ &= \mathcal{L}^{-1}\left\{ \begin{pmatrix} a_{\delta 11}(s) & a_{\delta 12}(s) & a_{\delta 13}(s) \\ a_{\delta 21}(s) & a_{\delta 22}(s) & a_{\delta 23}(s) \\ a_{\delta 31}(s) & a_{\delta 32}(s) & a_{\delta 33}(s) \end{pmatrix} \frac{1}{det(sI - A)} \right\} = \\ &= \begin{pmatrix} l_{\delta 11}(t) & l_{\delta 12}(t) & l_{\delta 13}(t) \\ l_{\delta 21}(t) & l_{\delta 22}(t) & l_{\delta 23}(t) \\ l_{\delta 31}(t) & l_{\delta 32}(t) & l_{\delta 33}(t) \end{pmatrix} \end{aligned} \tag{35}$$

where

$$\begin{aligned} &a_{\delta 11}(s) = s^2 - (\mu_{qf} - \omega_0)s + (\mu_{\delta_e}\kappa_q - \mu_{qf})\omega_0 \\ &a_{\delta 12}(s) = s + (1 - \kappa_q\zeta_{\delta_e})\omega_0 \\ &a_{\delta 13}(s) = \zeta_{\delta_e}s + (\mu_{\delta_e} - \zeta_{\delta_e}\mu_{qf}) \\ &a_{\delta 21}(s) = \mu_\alpha s + (\mu_\alpha - \mu_{\delta_e}\kappa_\alpha)\omega_0 \\ &a_{\delta 22}(s) = s^2 - (\zeta_\alpha - \omega_0)s - (\zeta_\alpha - \zeta_{\delta_e}\kappa_\alpha)\omega_0 \\ &a_{\delta 23}(s) = \mu_{\delta_e}s + (\zeta_{\delta_e}\mu_\alpha - \zeta_\alpha\mu_{\delta_e}) \\ &a_{\delta 31}(s) = -\omega_0\kappa_\alpha s + (\mu_{qf}\kappa_\alpha - \mu_\alpha\kappa_q)\omega_0 \\ &a_{\delta 32}(s) = -\omega_0\kappa_q s + (\zeta_\alpha\kappa_q - \kappa_\alpha)\omega_0 \\ &a_{\delta 33}(s) = s^2 - (\mu_{qf} + \zeta_\alpha)s + (\zeta_\alpha\mu_{qf} - \mu_\alpha)^2 \end{aligned} \tag{36}$$

and $l_{\delta ..}(t)$ are given by the inverse Laplace transform, in functions of the roots of (33).

Remark 11. From the previous considerations

$$B \overset{\tau=h}{=} \omega_0 \begin{pmatrix} l_{\delta 13}(-h) \\ l_{\delta 23}(-h) \\ l_{\delta 33}(-h) \end{pmatrix} \tag{37}$$

The signal defined in Răsvan and Popescu (2004) is balanced by the pilot gain κ_P resulting

$$u(t) = \kappa_P F\left[x_k + \sum_{j=-p}^{-1} A(\delta)^{-(p+j+1)}B_1(\delta)u_{k+j}\right], \quad k\delta \le t < (k+1)\delta,\ k \in \mathbf{N}, \mathbf{p} \in \mathbf{N}^* \tag{38}$$

where F is a line-matrix which may be obtained by imposing allocation of the system poles (31) in left complex plane, x_k is the sampled signal at the k time and $\delta = \frac{h}{p}$.

Remark 12. In order to apply an algorithm derived from equation (38) the matrices $A(\delta)$ and $B_1(\delta)$ must be computed - they are defined at (8) and (9).

5. NUMERICAL CONSIDERATIONS

In the robust controller synthesis (38), the value of the nominal delay is considered $h = 0.3$ s, for the low-order GAM-ADMIRE, respectively $h = 0.35$ s for the simplified ADMIRE model.

In all numerical simulations the value of the pilot gain is $\kappa_P = 0.1$.

In the case of simplified ADMIRE model the A matrix has the following numerical evaluation:

$$\begin{aligned} A &= \begin{pmatrix} \zeta_\alpha & 1 & \zeta_{\delta_e} \\ \mu_\alpha & \mu_{qf} & \mu_{\delta_e} \\ -\omega_0\kappa_\alpha & -\omega_0\kappa_q & -\omega_0 \end{pmatrix} = \\ &= \begin{pmatrix} -1.6 & 1 & -.5209 \\ 1.7251 & -22.61 & -9.9729 \\ -8.02 & -1.4568 & -20 \end{pmatrix} \end{aligned} \tag{39}$$

The values for ζ_{SA_i}, $i = 0, 1, 2$ are

$$\zeta_{SA_2} = 44.21,\ \zeta_{SA_1} = 499.9448,\ \zeta_{SA_0} = 490.0247 \tag{40}$$

and the roots for (33) are: -1.08, -18.1433, -24.9857
Applying the inverse Laplace transform, the following time-domain functions are derived

$$\begin{aligned}
l_{\delta 11}(t) &= -0.02e^{-24.99t} + 0.05e^{-18.14t} + 0.96e^{-1.08t}\\
l_{\delta 12}(t) &= -0.03e^{-24.99t} - 0.02e^{-18.14t} + 0.05e^{-1.08t}\\
l_{\delta 13}(t) &= -0.05e^{-24.99t} + 0.11e^{-18.14t} - 0.05e^{-1.08t}\\
l_{\delta 21}(t) &= 0.44e^{-24.99t} - 0.71e^{-18.14t} + 0.28e^{-1.08t}\\
l_{\delta 22}(t) &= 0.69e^{-24.99t} + 0.3e^{-18.14t} + 0.01e^{-1.08t}\\
l_{\delta 23}(t) &= 1.42e^{-24.99t} - 1.41e^{-18.14t} - 0.01e^{-1.08t}\\
l_{\delta 31}(t) &= 0.1e^{-24.99t} + 0.33e^{-18.14t} - 0.43e^{-1.08t}\\
l_{\delta 32}(t) &= 0.16e^{-24.99t} - 0.14e^{-18.14t} - 0.02e^{-1.08t}\\
l_{\delta 33}(t) &= 0.33e^{-24.99t} + 0.65e^{-18.14t} + 0.02e^{-1.08t}
\end{aligned} \tag{41}$$

and then, taking $t = \delta$, $A(\delta)$ is evaluated to

$$A(\delta) = \begin{pmatrix} 0.9487 & 0.0238 & -0.0165 \\ 0.0703 & 0.4584 & -0.1667 \\ -0.1974 & -0.0273 & 0.5028 \end{pmatrix} \tag{42}$$

According to (37)

$$B = B_0 + e^{-Ah}B_1 = e^{-Ah}B_1 = \begin{pmatrix} -5502.01 \\ 162281.15 \\ 48750.23 \end{pmatrix} \tag{43}$$

with $B_1(\delta)$ determined from the second equation of (8), respectively (9): $B_1(\delta) = (-0.01 \ -0.08 \ 0.51)^T$

In the case of low-order GAM-ADMIRE theoretical aircraft the A matrix is given by

$$\begin{aligned}
A &= \begin{pmatrix} \zeta_\alpha & 1 & \zeta_{\delta_e} \\ \mu_\alpha & \mu_{qf} & \mu_{\delta_e} \\ -\omega_0\kappa_\alpha & -\omega_0\kappa_q & -\omega_0 \end{pmatrix} = \\
&= \begin{pmatrix} -0.7986 & 1 & -0.2603 \\ -6.5315 & \simeq -0.0632 & -8.2668 \\ -8.02 & -1.4568 & -20 \end{pmatrix}
\end{aligned} \tag{44}$$

The values for ζ_{SA_i}, $i = 0, 1, 2$ are

$$\zeta_{SA_2} = 20.8618,\ \zeta_{SA_1} = 9.6864,\ \zeta_{SA_0} = 58.0663 \tag{45}$$

and the roots (33): $-20.5277, \ -0.167 \pm \imath 1.67355$

Applying the inverse Laplace transform the following are obtained

$$\begin{aligned}
l_{\delta 11}(t) &= -.42e^{-20.5t} + \\
&\quad + 7.65e^{-.17t}[.19\cos(1.67t) + .98\sin(1.67t)]\\
l_{\delta 12}(t) &= .69e^{-20.5t} + 21.08e^{-.17t}[.03\cos(1.67t) + \sin(1.67t)]\\
l_{\delta 13}(t) &= -.46e^{-20.5t} + 8.48e^{-.17t}[.05\cos(1.67t) + \sin(1.67t)]\\
l_{\delta 21}(t) &= -2.56e^{-20.5t} + \\
&\quad + 66.92e^{-.17t}[.04\cos(1.67t) + \sin(1.67t)]\\
l_{\delta 22}(t) &= .58e^{-20.5t} + 17.84e^{-.17t}[.02\cos(1.67t) - \sin(1.67t)]\\
l_{\delta 23}(t) &= .02e^{-20.5t} - 6.38e^{-.17t}\sin(1.67t)\\
l_{\delta 31}(t) &= .59e^{-20.5t} + 7.91e^{-.17t}[.07\cos(1.67t) + \sin(1.67t)]\\
l_{\delta 32}(t) &= -.45e^{-20.5t} + 9.6e^{-.17t}[.05\cos(1.67t) + \sin(1.67t)]\\
l_{\delta 33}(t) &= 1.11e^{-20.5t} + 6.93e^{-.17t}[.02\cos(1.67t) + \sin(1.67t)]
\end{aligned} \tag{46}$$

and, then, taking $t = \delta$ the following evaluation for $A(\delta)$ is determined

$$A(\delta) = \begin{pmatrix} 1.5607 & 2.1085 & 0.6327 \\ 4.4982 & -0.1615 & -0.295 \\ 1.2986 & 0.6834 & 1.0621 \end{pmatrix} \tag{47}$$

From (37)

$$B = B_0 + e^{-Ah}B_1 = e^{-Ah}B_1 = \begin{pmatrix} -4455.28 \\ 211.66 \\ 10467.02 \end{pmatrix} \tag{48}$$

with $B_1(\delta)$ is determined from the second equation of (8), respectively (9): $B_1(\delta) = (4.6 \ 15.54 \ -3.04)^T$

The following simulations were obtained: Figures 1 and 2 for the simplified ADMIRE model and, for the GAM-ADMIRE theoretical aircraft, Figures 3 and 4.

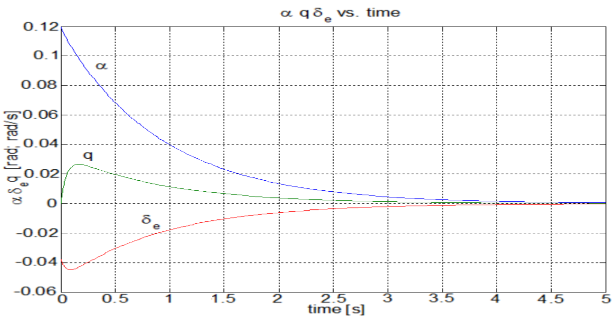

Fig. 1. α q δ_e for the simplified ADMIRE model $\tau = 0.2$ s

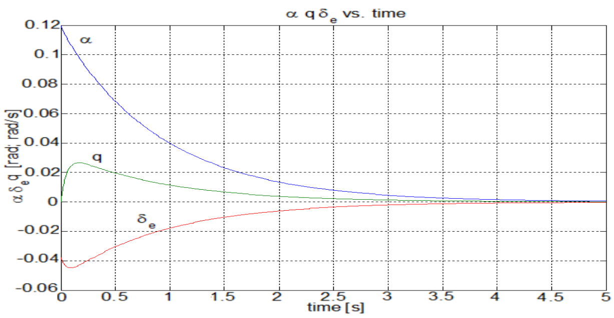

Fig. 2. α q δ_e for the simplified ADMIRE model $\tau = 0.4$ s

Remark 13. In the case of the simplified ADMIRE model the initial conditions were $x_0 = (.1196, 0, -.0378)$.

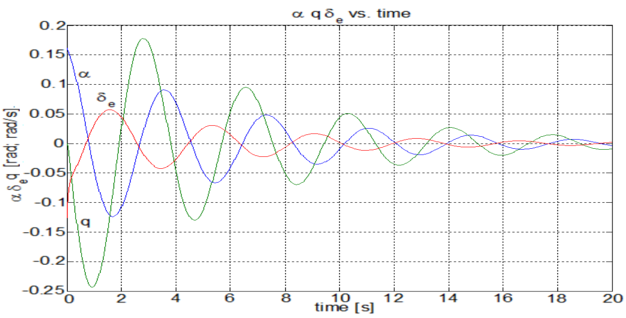

Fig. 3. α q δ_e for the low-order GAM-ADMIRE theoretical aircraft $\tau = 0.2$ s

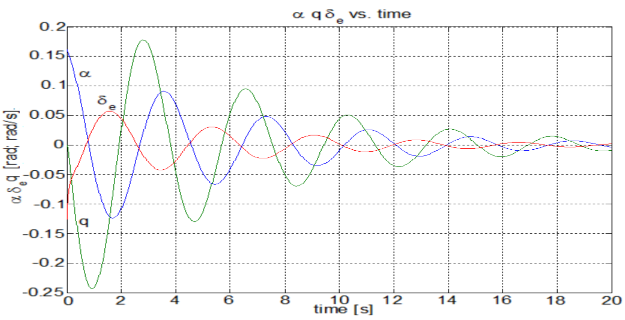

Fig. 4. α q δ_e for the low-order GAM-ADMIRE theoretical aircraft $\tau = 0.4$ s

Remark 14. For the low-order GAM-ADMIRE theoretical aircraft the initial conditions $x_0 = (.1612, 0, -.1255)$.

6. FEEDBACK BASED ON STATE OBSERVERS

The feedback based on state observers is considered when the state of the system is not accessible, the only information available being the measured output $y(k\delta) = y_k$

$$y_k = C^T x_k \tag{49}$$

(supposing that the pair (C^T, A) is observable).
In this case, a state observer will be used. For the discrete system, the observer used is [Răsvan and Popescu (2004)]

$$\begin{aligned}\tilde{x}_{k+1} = & \left[A(\delta) - K_0(\delta)C^T\right]\tilde{x}_k + B_0(\delta)u_k \\ & + B_1(\delta)u_{k-p} + K_0(\delta)y_k\end{aligned} \tag{50}$$

where the $K_0(\delta)$ vector is choosed in such a manner that the matrix $A(\delta) - K_0(\delta)C^T$ is stable.

Remark 15. The existence of $K_0(\delta)$ results from the stabilizability of the controllable pair $(A^T(\delta), C)$ and the controllability of this pair is obtained from the observability of the pair (C^T, A). $K_0(\delta)$ is obtained through the use of a closed-loop pole assignment method.

Remark 16. For simplified GAM-ADMIRE $C^T = (1\ 1\ 1)$.

6.1 Simplified ADMIRE model with feedback based on state-observers

In the case of simplified ADMIRE model the following evaluation is obtained

$$K_0(\delta) = (\,9\ \ -11.4614\ 4.97\,)^T$$

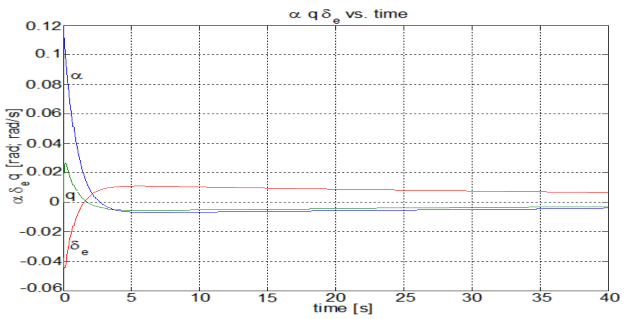

Fig. 5. $\alpha\ q\ \delta_e$ for the simplified ADMIRE model $\tau = 0.2$ s, with state-observer

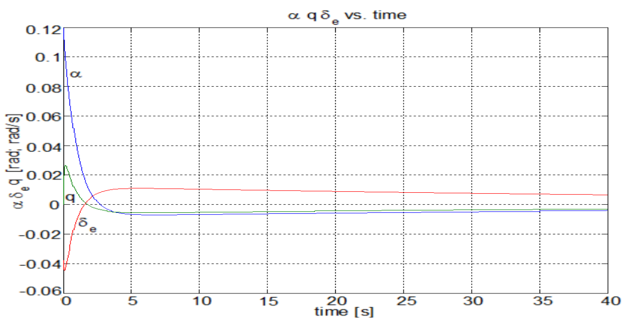

Fig. 6. $\alpha\ q\ \delta_e$ for the simplified ADMIRE model $\tau = 0.4$ s, with state-observer

6.2 Low-order GAM-ADMIRE theoretical aircraft with feedback based on state-observers

For the low-order GAM-ADMIRE theoretical aircraft:

$$K_0(\delta) = (\,1.9886\ 0.3107\ 0.76195\,)^T$$

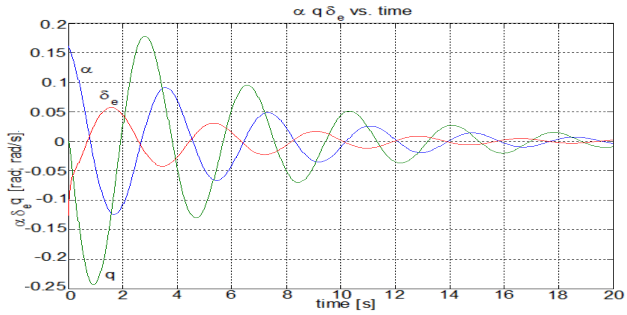

Fig. 7. $\alpha\ q\ \delta_e$ for low-order GAM-ADMIRE theoretical aircraft $\tau = 0.2$ s, with state-observer

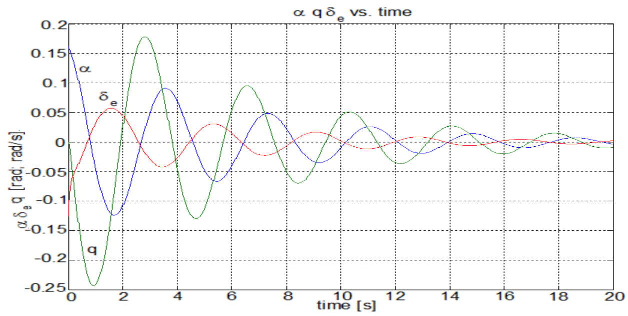

Fig. 8. $\alpha\ q\ \delta_e$ for low-order GAM-ADMIRE theoretical aircraft $\tau = 0.4$ s, with state-observer

Remark 17. In all graphic representations the unknown delay τ of pilot was considered to have the values of the frontier of the interval $[0.2\ 0.4]$ s. This interval represents the reaction time for a trained pilot [Brieger (2000)].

7. OPTIMAL STABILIZATION WITH PRESCRIBED DEGREE OF STABILITY

In this section optimal stabilization problem with prescribed degree of stability is discussed. For the system (1) the following quadratic performance criterion is associated [Anderson and Moore (1969)]

$$J(u) = \int_0^\infty \left(x^T(t)Qx(t) + u^T(t)Mu(t)\right)e^{2\varphi t}dt \tag{51}$$

where Q is a nonegative definite symmetric and constant matrix and M is a positive definite matrix (in this paper $M = I$, i.e. identity matrix) and with Remark (2) enforced.

Remark 18. For $\varphi = 0$ the optimal stabilization does not have a prescribed degree of stability but, in the case when $\varphi > 0$, the optimal stabilization has a prescribed degree of stability.

Because on the interval $(0, \tau)$ the control signal is determined exclusively by the initial condition $u_0(\cdot)$, $x(t)$ on $(0, \tau)$ can not be influenced through optimization.

From [Răsvan and Popescu (2004)], in the case of $\varphi = 0$, the following measure is considered

$$J(v) = \Phi(x_0, u_0(\cdot)) + \\ + \int_\tau^\infty \left(x^T(t)Qx(t) + v^T(t)Mv(t)\right) dt \quad (52)$$

where $v(t) = u(t-\tau)$, $t > \tau$ and $\Phi(x_0, u_0(\cdot))$ is a quadratic functional defined on the product space of the initial conditions: $R^n \times L^2((-\tau, 0); R^m)$

$$\begin{aligned}\Phi(x_0, u_0(\cdot)) &= x_0^T e^{A^T\tau} M(-\tau,\tau)e^{A\tau}x_0 + \\ &+ x_0^T e^{A^T\tau}\int_{-\tau}^0 M(-\tau,\xi)e^{-A\xi}B_1u_0(\xi)d\xi + \\ &+ \left(\int_{-\tau}^0 u_0^T(\xi)B_1^T e^{-A^T\xi}M(\xi,-\tau)d\xi\right)e^{A\tau}x_0 + \\ &+ \int_{-\tau}^0\int_{-\tau}^0 u_0^T(\xi)B_1^T e^{-A^T\xi}M(\xi,\lambda)e^{-A\lambda}B_1u_0(\lambda)d\lambda\end{aligned} \quad (53)$$

where

$$M(\xi,\lambda) = \begin{cases}\int_\xi^0 e^{A^*\varpi}Me^{A\varpi}d\varpi, & -\tau \le \lambda \le 0,\ \lambda \le \xi \le 0 \\ \int_\lambda^0 e^{A^*\varpi}Me^{A\varpi}d\varpi, & -\tau \le \lambda \le 0,\ -\tau \le \xi \le \lambda\end{cases} \quad (54)$$

with $M = I$.

In the next part of the section the optimal stabilization is considered with a prescribed degree of stability ($\varphi > 0$).

Remark 19. For $J(\cdot)$, (51), to be finite (with (A, B_1) assumed completly controllable [Anderson and Moore (1969)]) the poles of the closed-loop system (1) must have the real part less than $-\varphi$.

For obtaining the obtimal control the following notation is made

$$x_1(t) = x(t)e^{\varphi t} \quad (55)$$

so that

$$\dot{x}_1(t) = A_\varphi x_1(t) + B_\varphi u_1(t-\tau) \quad (56)$$

where

$$\begin{aligned}u_1(t) &= e^{\varphi t}u(t) \\ A_\varphi &= A + \varphi I \\ B_\varphi &= B_1 e^{\varphi\tau}\end{aligned} \quad (57)$$

The performance criterion (51) becomes

$$J(u_1) = \int_0^\infty \left[x_1^T(t)Qx_1(t) + u_1^T(t)u_1(t)\right] dt \quad (58)$$

Considering

$$u(t) = u_1(t)e^{-\varphi t} \quad (59)$$

results that is necessary for the pair (A_φ, B_φ) to be stabilizable. From [Răsvan and Popescu (2004)] the following lemma is applied.

Lemma 20. If the pair (A, B_1) is controllable, then the pair (A_φ, B_φ) is controllable.

Using the same method as the one considered in the case of (52) the performance criterion equivalent with (58), has the following form

$$J(v) = \Phi(x_{10}, u_{10}(\cdot)) + \\ + \int_\tau^\infty \left(x_1^T(t)Qx_1(t) + v^T(t)Mv(t)\right) dt \quad (60)$$

where $v(t) = u_1(t-\tau)$, $t > \tau$, and $\Phi(x_0, u_0(\cdot))$ is a quadratic functional defined on the product space of the initial conditions: $R^n \times L^2((-\tau, 0); R^m)$.

It can be considered that the linear quadratic problem defined by (60) for the solutions of the system is

$$\dot{x}_1(t) = A_\varphi x_1(t) + B_\varphi v(t) \quad (61)$$

This problem has the solution [Răsvan and Popescu (2004)]

$$v(t+\tau) = u_1(t) = -B_\varphi^T P x_1(t+\tau) \quad (62)$$

where P represents the positive definite solution of the matrix Ricatti equation

$$A_\varphi^T P + PA_\varphi - PB_\varphi B_\varphi^T P + Q = 0 \quad (63)$$

Computing $x_1(t+\tau)$ from (61) with $v(t) = u_1(t-\tau)$ and replacing the value from (62) and taking the nominal delay h the following control law is obtained

$$u_1(t) = -B_\varphi^T P e^{A_\varphi h}\left[x_1(t) + \int_{-h}^0 e^{-A_\varphi}(\theta + h)B_\varphi u_1(t+\theta)d\theta\right] \quad (64)$$

Using the last two notations from (57), the Riccati equation (63) can be written as follows:

$$(A^T + \varphi I)P + P(A + \varphi I) - e^{2\varphi h}PB_1B_1^TP + Q = 0 \quad (65)$$

From (59), with u_1 given by (64) and using the last two notations from (57), after x_1 and u_1 are replaced by their expressions in terms of x and u, the optimal control is derived

$$u(t) = -e^{2\varphi h}B_1^T P e^{A_\varphi h}\left[x(t) + \int_{-h}^0 e^{-A(\theta+h)}B_1u(t+\theta)d\theta\right] \quad (66)$$

In terms of practical implementation of the controller associated to (66), proceeding similarly as in (38), the following piecewise constant control law is used

$$\begin{aligned}u(t) &= -\kappa_P e^{2\varphi h}B_1^T P e^{A_\varphi h}\cdot \\ &\cdot\left[x_k + \sum_{j=-p}^{-1} A(\delta)^{-(p+j+1)}B_1(\delta)u_{k+j}\right], \\ &k\delta \le t < (k+1)\delta,\ k \in \mathbf{N}, \mathbf{p} \in \mathbf{N}^*\end{aligned} \quad (67)$$

Remark 21. Altough the stabilization law (66) is optimal, the practical implementation (67) - through piecewise constant control signals - of the controller is suboptimal.

8. NUMERICAL CONSIDERATIONS

In the following two subsections numerical simulations are considered in the case of low-order GAM-ADMIRE model and simplified ADMIRE theoretical aircraft with the following values $\rho = \{1\}$, $\varphi = \{3\}$.

Remark 22. Considering, as in [Răsvan and Popescu (2004)]

$$Q = \begin{pmatrix}\rho & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \quad (68)$$

$S = B_1B_1^T$ and the Hamiltonian matrix - [Bellon (2008)], [Anderson and Moore (1989)] -

$$H = \begin{pmatrix}A & -S\\ -Q & -A^T\end{pmatrix} \quad (69)$$

then, for the system (1) to be optimally stabilizable, it is necessary that the eigenvalues of (69) to have the real part non-zero [Jora et al. (1996)].

8.1 Numerical considerations for simplified ADMIRE

In the case of simplified ADMIRE theoretical aircraft, the Hamiltonian matrix is

$$H = \begin{pmatrix} -1.6 & 1 & -0.5209 & 0 & 0 & 0 \\ 1.7251 & -22.61 & -9.9729 & 0 & 0 & 0 \\ -8.02 & -1.4568 & -20 & 0 & 0 & 400 \\ -1 & 0 & 0 & 1.6 & -1.7251 & 8.02 \\ 0 & 0 & 0 & -1 & 22.61 & 1.4568 \\ 0 & 0 & 0 & 0.5209 & 9.9729 & 20 \end{pmatrix} \tag{70}$$

whose eigenvalues are

$$\mp 24.9724; \ \mp 18.1870; \ \mp 0.4967 \tag{71}$$

and it is obvious that the system (1) can be optimally stabilizable using the Remark 22.

In figures 9 and 10 are given numerical simulations for $\varphi = 3$.

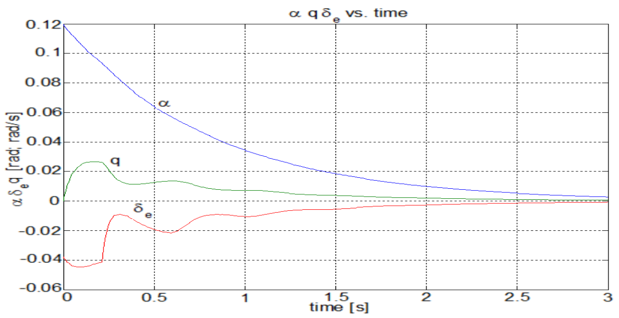

Fig. 9. φ q δ_e for the simplified ADMIRE model $\tau = 0.2$ s, $\varphi = 3$

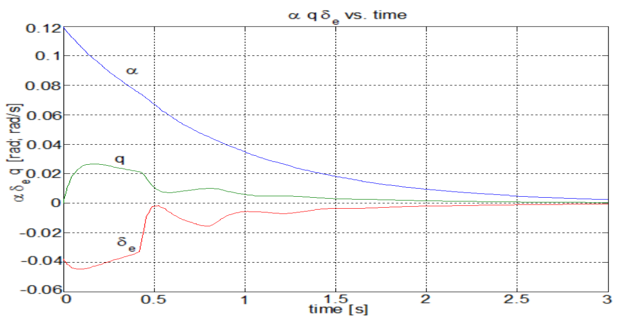

Fig. 10. φ q δ_e for the simplified ADMIRE model with $\tau = 0.4$ s, $\varphi = 3$

8.2 Numerical considerations for low-order GAM-ADMIRE model

For the low-order GAM-ADMIRE model the Hamiltonian matrix is

$$H = \begin{pmatrix} -0.7986 & 1 & -0.26 & 0 & 0 & 0 \\ -6.5315 & -0.063 & -8.267 & 0 & 0 & 0 \\ -8.02 & -1.457 & -20 & 0 & 0 & 400 \\ -1 & 0 & 0 & 0.7986 & 6.5315 & 8.02 \\ 0 & 0 & 0 & -1 & 0.063 & 1.457 \\ 0 & 0 & 0 & 0.26 & 8.267 & 20 \end{pmatrix} \tag{72}$$

whose eigenvalues are

$$\mp 20.5255; \ \mp 2.3039; \ 0 \pm \imath 3.28 \tag{73}$$

and it is clear that the system (1) is not optimally stabilizable using the Remark 22.

9. CONCLUSIONS

The stability of a pilot-aircraft system with input delay was emphasized using robust controllers obtained by Artstein transform. The delay was considered as unknown parameter and the input was derived using piecewise constant control signals. Using numerical simulations, the robustness of the proposed control laws (38), (67) was proved in the context of the admissible human pilot delays. The case when the states of the system are not observable, for controller (38), was investigated through the use of state observers. The optimal stabilization with prescribed degree of stability was used to synthesize the robust controller (67), in the case of the simplified ADMIRE model.

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