Application of CDM-PI-P Controller for SO₂ Emission Control Process

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Abstract: This paper focuses on the sulphur dioxide (SO₂) emission control using PI-P controller by considering Coefficient Diagram Method (CDM) as a candidate. To analyse the removal efficiency of SO₂, liquid-gas absorption column with mixing unit is modelled using SIMULINK in MATLAB platform. Sulphuric acid containing hydrogen peroxide is used as a scrubbing liquid. To compute the parameters of the CDM-PI-P controller, process is approximated as First Order Plus Time Delay (FOPTD) transfer function model. Performance of the CDM-PI-P controller is analysed and compared with conventional control techniques such as Ziegler-Nichols PI controller (ZN-PI) and Internal Model Control based PI controller (IMC-PI) in terms of time domain performance measures such as settling time, rise time, overshoot (tₛ, tᵣ, %Mp) and error indices (ISE, IAE, ITAE). The simulation results prove that the CDM-PI-P controller provides most consistent performance as compared to the conventional controllers.

Keywords: CDM-PI-P controller, SO₂ emission control, Liquid-Gas absorption column.

1. INTRODUCTION

One of the most taxing contaminant that must be removed from the industrial flue gas is sulphur dioxide (SO₂). The main sources of SO₂ emissions are coal fired boilers, sulphuric acid plants, chemical and metallurgical furnaces. Many techniques have been proposed for SO₂ removal process such as conversion of SO₂ into sulphur (Zhihui Ban, 2004), wet flue gas desulphurization, dry flue gas desulphurization, semi dry flue gas desulphurization (Yuegui Zhou et al., 2009), activated carbon (Azargohar et al., 2009) etc. Among all the techniques, wet type flue gas desulphurization produces 60-95% removal efficiency (Cofalaa et al., 2004).

Several researchers have tried many absorbents for wet technique to obtain better removal efficiency. Among these, lime stone based flue gas desulphurization technique is most widely used since it gives more than 90% SO₂ removal efficiency. Even though this technique provides better removal efficiency, it produces CO₂ as a secondary emission (Edward et al., 2004; Sheng-yu Liu et al., 2008). Thus, for the desulphurization process, sulphuric acid with added hydrogen peroxide is used as a reactant and it produces sulphuric acid as a by-product without generating any other pollutants (US patent no: 5,595,713). This technique is proved with experimental investigations and hence it is considered for this work.

Based on the results given in (Sandrine Colle et al., 2004), the flow rate and concentration of hydrogen peroxide has the direct influence on outlet SO₂ concentration. For regulating the flow rate of hydrogen peroxide, an appropriate controller is required to improve the removal efficiency by reducing outlet SO₂ concentration. From the literature review, Alpbaz (2006) has developed a self tuning PID controller for lime stone desulphurization process, but so far no controller is developed for H₂O₂ based desulphurization process. Hence a simple and robust controller is needed for this process.

Coefficient Diagram Method (CDM) is a systematic method introduced by Prof. Shunji Manabe in 1991. It is an algebraic approach that combines classical and modern control theories. In CDM, the polynomial matrices are used for plant and controller representation. CDM controller is designed in such a way that it produces a convenient bandwidth and response without an overshoot. CDM is a more complex controller as compared to conventional PID controller, but it provides convenient parameter selection rules to design a controller under the conditions of stability, robustness and time domain performances (Manabe, 1998). Hence a simple and robust controller, CDM based PI-P, is proposed for SO₂ emission control process.

The structure of the paper is as follows: A brief description about SO₂ emission control process and its model development is presented in section 2. The design of controllers such as ZN-PI, IMC-PI and CDM-PI-P controller are covered in section 3. Section 4 emphasizes the simulation results, comparative studies and performance measures. The final section summarizes the major conclusions of the proposed work. Various sources pertaining to this work are listed in the references.
2. PROCESS DYNAMICS AND MODELING

2.1 Process Description

Absorption is the process of transforming a gaseous pollutant from gas phase to a liquid phase. It involves the removal of gaseous pollutants by dissolving them in a liquid (Sinnott, 1991). For this process, packed column is designed based on the principle of gas-liquid interface to obtain proper liquid to gas mixing. It leads to the efficient removal of soluble SO₂ from gas stream. The liquid-gas absorption process using Liquid-Gas absorption column is illustrated in Figure 1.

Fig. 1. Schematic of SO₂ absorption process.

The SO₂ gas is entered at the inlet stream of the packed column. At the outlet, the mixing tank contains the sulphuric acid with little amount of untreated hydrogen peroxide which is recirculated through mixing the tank until the hydrogen peroxide solution completely reacts with the SO₂ gas. To improve the removal efficiency, the hydrogen peroxide is added in the mixing tank through final control element for the entire process. After treatment, SO₂ gas is exhausted through the outlet stream of the liquid-gas absorption column.

The SO₂ gas is entered at the inlet stream of the packed column. At the outlet, the mixing tank contains the sulphuric acid with little amount of untreated hydrogen peroxide which is recirculated through mixing the tank until the hydrogen peroxide solution completely reacts with the SO₂ gas. To improve the removal efficiency, the hydrogen peroxide is added in the mixing tank through final control element for the entire process. After treatment, SO₂ gas is exhausted through the outlet stream of the liquid-gas absorption column.

2.2. Process Dynamics

The following assumptions are made for model development,

1. The sulphuric acid with hydrogen peroxide solvent is recycled continuously.
2. The absorption process in the packed column is counter current.
3. The column operates with a constant temperature.
4. The H₂O₂ concentration is depleted on reacting with SO₂ at a certain time, t.
5. Chemical reactions take place only with hydrogen peroxide and SO₂.

The absorption process is described by the following dynamic equation based on the two-film double resistance theory proposed by (Whitman, 1923).

SO₂ lost by gas stream = SO₂ taken up by the liquid stream

\[
\frac{dC_{L1}(t)}{dt} = \frac{L_1}{G_1} \left[ C_{G1}(t) - C_{G2}(t) \right] - \frac{L_2}{G_2} \left[ C_{L1}(t) - C_{L2}(t) \right] \]

(2)

The concentration of SO₂ in the exhaust gas stream is obtained from the above equation as follows

\[
C_{G2}(t) = C_{G1}(t) - \frac{L_1}{G_1} \left[ C_{L1}(t) - C_{L2}(t) \right] \]

(3)

The above equation is used to represent the dynamics of liquid-gas absorption column. Material balance around the mixing tank is written by

\[
V \frac{dL_2(t)}{dt} = C_{L1} \frac{L_1(t) - L_2(t)}{V} + L_1 \left[ C_{L1}(t) - C_{L2}(t) \right] \]

(4)

The mass balance (4) is obtained by applying the conservation principle on the mixing tank with inlet concentrations (C₁ (t), C₃) and outlet concentration C₂ (t).

The rate equation is expressed for the mass transfer and chemical reaction between gas and liquid. It is represented as a first order differential equation since the complete conversion is not obtained without recirculation (Octave levenspiel, 2004). By considering the reaction rate of SO₂ on H₂O₂, the general rate equation (5) is formulated as,

\[
-r_A = \frac{dC_A}{dt} = K_2 C_A C_B \]

(5)

Where, \( r_A \) – reaction rate, \( C_A \) – concentration of component A, \( C_B \) – concentration of component B, \( K_2 \) - reaction rate constant. The above equation is rewritten for the liquid-gas absorption column is given by:

\[
\frac{dC_{L1}(t)}{dt} = K_2 C_{G1} C_{L2}(t) \]

(6)
The rate of change of outlet liquid concentration \( C_{L1}(t) \) depends both on the inlet SO\(_2\) concentration \( (C_{G1}) \) as well as the inlet liquid concentration \( (C_{L2}(t)) \).

2.3. Model development

Based on the dynamics of mixing tank (4) and liquid-gas absorption column (2) and (6) as shown in Figure 2, the SIMULINK model of SO\(_2\) emission control system in MATLAB platform is developed.

![Fig. 2. MATLAB-SIMULINK model of SO\(_2\) emission control process.](image)

The model parameters are tabulated in Table 1. The flow rate of hydrogen peroxide is varied from \((0 - 1)\) lph with a sampling interval of 0.01 lph and the corresponding steady state outlet SO\(_2\) concentrations are measured.

The typical pattern of a steady state process reaction curve as shown in Figure 3 is obtained by plotting the measured steady state outlet SO\(_2\) concentrations (mol/L) (controlled variable) against flow rate of hydrogen peroxide (lph) (manipulated variable). This reaction curve is normally used to describe the characteristics of SO\(_2\) emission control process. From Figure 3, it is clear that the behaviour of the process is non-linear and stable. Hence, First Order Plus Time Delay (FOPTD) transfer function \( G(s) = \frac{K_p}{\tau_p s + 1} e^{-\theta s} \) model is used to represent the SO\(_2\) emission control process.

**Table 1. Parameters for SO\(_2\) emission control process.**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Liquid flow rate for the column (( L_1 ))</td>
<td>150 lph</td>
</tr>
<tr>
<td>2.</td>
<td>Gas flow rate for the column (( G_1 ))</td>
<td>40 m(^3)/h</td>
</tr>
<tr>
<td>3.</td>
<td>Concentration of H(_2)O(<em>2) entering in to the mixing tank (( C</em>{L1} ))</td>
<td>0.1 mol/L</td>
</tr>
<tr>
<td>4.</td>
<td>Liquid flow rate in to the mixing tank (( L_2(t) ))</td>
<td>0-1 lph</td>
</tr>
<tr>
<td>5.</td>
<td>Initial concentration of Concentration of H(_2)O(_2)+H(_2)SO(<em>4) exit from the bottom of column ( C</em>{L1}(t) )</td>
<td>0.1 mol/L</td>
</tr>
<tr>
<td>6.</td>
<td>Concentration of H(_2)O(_2)+H(_2)SO(<em>4) entering into the top of column ( C</em>{L3}(t) )</td>
<td>0.1 mol/L</td>
</tr>
<tr>
<td>7.</td>
<td>Concentration of SO(<em>2) in inlet gas stream (( C</em>{G1} ))</td>
<td>0.12 mol/L</td>
</tr>
</tbody>
</table>

**Table 2. Identified model parameters at different operating points of outlet SO\(_2\) concentrations.**

<table>
<thead>
<tr>
<th>Operating point</th>
<th>Outlet SO(_2) Concentration (mol/L)</th>
<th>SO(_2) removal efficiency (%)</th>
<th>Step Magnitude</th>
<th>( K_p ) (%/%)</th>
<th>( \tau_p ) (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.048</td>
<td>60</td>
<td>60</td>
<td>10%</td>
<td>14.56</td>
<td>251</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-10%</td>
<td>27</td>
<td>249</td>
</tr>
<tr>
<td>0.016</td>
<td>86.67</td>
<td>86.67</td>
<td>10%</td>
<td>26.96</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-10%</td>
<td>2.56</td>
<td>245</td>
</tr>
<tr>
<td>0.0048</td>
<td>96.67</td>
<td>96.67</td>
<td>10%</td>
<td>0.44</td>
<td>244</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-10%</td>
<td>2.407</td>
<td>254</td>
</tr>
</tbody>
</table>

From Table 2, it is clear that the large gain variations are obtained at different magnitudes and it shows that the system is non-linear. The worst case model with the largest process
gain ($K_c=27$) and smallest time constant ($\tau_p=244$) is selected to represent the SO$_2$ emission control process. The identified FOPTD model of the process is represented as,

$$G(s) = \frac{K_p}{\tau_p s + 1} e^{-\theta s} = \frac{27}{244s + 1} e^{-s}$$  \hspace{1cm} (7)

Where, the process delay ($\theta$) is considered as 1 sec which is the sampling time used for simulation runs.

3. CONTROLLER DESIGN

3.1. Conventional PI controllers

Many previous researchers have used the performance of a conventional PI controller as the benchmark to analyse the performance of a proposed controller. In this way, the conventional PI controllers such as Ziegler-Nichols PI controller (Ziegler and Nichols, 1942) and Internal Model Control based PI controller (Sigurd Skogestad, 2001) are considered for comparative purpose. For convenience, these control techniques are abbreviated as ZN-PI and IMC-PI respectively. The settings of the above said controllers are computed based on the FOPTD model given in (7) and are provided in Table 3.

Table 3. Conventional PI Controller Parameters.

<table>
<thead>
<tr>
<th>Controllers</th>
<th>Tuning Rules</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZN-PI</td>
<td>$K_c = \frac{0.9 \tau_p}{K_p}$ and $T_i = 3.330$</td>
<td>$K_c = 8.133$, $T_i = 3.33$, $K_i = 2.442$</td>
</tr>
<tr>
<td>IMC-PI</td>
<td>$K_c = \frac{\tau_p + 0.5 \theta}{K_p (\lambda)}$ and $T_i = \tau_p + 0.50$</td>
<td>$K_c = 5.326$, $T_i = 244.5$, $K_i = 0.0217$</td>
</tr>
</tbody>
</table>

where, $K_c$ - Controller gain; $K_p$ - Process gain; $K_i$ - Integral gain; $\tau_p$ - Process time constant; $T_i$ - Integral time constant; $\theta$ - Process delay; $\lambda$ - Tuning factor (Recommended value 1.70)

3.2 CDM-PI-P controller

The CDM-PI-P controller parameters are computed based on design procedure given in (Meenakshipriya et al., 2011). The general form of two-degrees of freedom (2DOF) PI control structure is shown in Figure 4.

$$C(s) = \frac{B(s)}{A(s)} = \frac{k_i s + k_0}{s}$$  \hspace{1cm} (8)

$$C_f(s) = \frac{F(s)}{B(s)} = \frac{k_0}{k_i s + k_0}$$  \hspace{1cm} (9)

Where, $A(s)$ - forward denominator polynomial; $B(s)$ - feedback numerator polynomial; $F(s)$ - reference numerator polynomial; $k_0$ and $k_i$ - Coefficients of CDM controller polynomials. By using the CDM controller polynomials of $A(s)$ and $B(s)$ from (8), the characteristic polynomial $P(s)$ is obtained as follows.

$$P(s) = A(s)D(s) + B(s)N(s) = \sum_{i=0}^{n} a_i s^i, a_i > 0$$  \hspace{1cm} (10)

Where, $N(s)$ and $D(s)$ are the numerator and denominator polynomials of the plant transfer function.

The target characteristics polynomial ($P_{target}(s)$) is determined as

$$P_{target}(s) = a_0 \left[ \frac{\lambda}{\sum_{i=2}^{n} \left( \frac{i-1}{\prod_{j=1}^{i-1} (i-j)} \lambda^i \right) s^i} + s + 1 \right]$$  \hspace{1cm} (11)

Where $\tau$ - Equivalent time constant ($\tau = \tau_p / 2.5 = \tau$) and $\gamma_i$ - Stability indices ([2.5 2 2 ...]). By equating the polynomials (10) and (11), the values of the CDM controller polynomials ($k_1$ and $k_0$) and equivalent time constant ($\tau$) are computed as follows.

The process considered for this study is approximated as FOPTD transfer function model. Hence the general mathematical model is given below.

$$G_p(s) = \frac{N(s)}{D(s)} = \frac{K_p}{\tau_p s + 1} e^{-\theta s}$$  \hspace{1cm} (12)

Using a simple first order Pade approximation for dead time in the Laplace domain, $e^{-\theta s} = \frac{2 - \theta s}{2 + \theta s}$, (12) becomes

$$G_p(s) = \frac{N(s)}{D(s)} = \frac{2K_p - K_p \theta s}{\tau_p \theta s^2 + (2\tau_p + \theta) js + 2}$$  \hspace{1cm} (13)

By substituting (8) and (13) in (10), the characteristic polynomial of the control system is obtained as

$$P(s) = s(\tau_p \theta s^2 + (2\tau_p + \theta) js + 2) + (k_i s + k_0)(2K_p - K_p \theta s)$$

$$= \tau_p \theta s^3 + (2\tau_p + \theta - k_j K_p \theta) s^2 + (2 + 2k_j K_p - K_p k_j \theta) s + 2K_p k_0$$  \hspace{1cm} (14)

By substituting the value of equivalent time constant ($\tau$) and the stability indices $\gamma_i = \{\gamma_1, \gamma_2\}$ in (11), the target characteristic polynomial is formulated as

$$P_{target}(s) = a_0 \left[ \frac{\lambda}{\gamma_1 \gamma_2 s^2 + \gamma_1^2 s + \gamma_2^2 s + s + 1} \right]$$  \hspace{1cm} (15)
The stability indices ($\gamma_1$ and $\gamma_2$) are selected based on Manabe’s recommended value or the designer can change the value of stability indices, if required. However, the equivalent time constant ($\tau$) is not specified and it is considered as another variable to be solved.

By equating the coefficients of the terms of equal power of (14) and (15), equivalent time constant ($\tau$) and CDM controller parameters ($k_1$ and $k_0$) are computed as follows.

\[
\tau = \left(\frac{P_0 \gamma_1 \gamma_2}{a_0}\right) \sqrt{a} \quad k_1 = \left(\frac{a_0 \tau + K_p k_0}{a_0}\right) \quad k_0 = \left(\frac{a_0}{2K_p}\right)
\]  

By using the above equation, CDM-PI-P controller parameters ($K_c$, $T_i$ and $K_f$) in terms of CDM controller polynomials ($k_1$ and $k_0$) are obtained as follows.

The block diagram (Figure 5) shows the general PI-P control structure where $G(s)$, $G_{PI}(s)$ and $G_P(s)$ represent the transfer function models of the plant, PI controller ($G_{PI}(s) = K_c (I + \frac{1}{T_i s})$) and P controller ($G_P(s) = K_f$) respectively. The PI-P control system given in Figure 5 is reduced to its equivalent system as shown in Figure 6.

\[
G_f(s) = \frac{G_{PI}(s)}{G_{PI}(s) + G_P(s)} = \frac{i(K_c(1 + T_i s) + K_c}{(K_c + K_f)T_i s + K_c}
\]  

By using the relation $K_c = \lambda K_f$ (Astrom and Hagglund, 1984) in (17) and (18), CDM-PI-P controller parameters and pre-filter elements are found to be

\[
K_c = \frac{ik_f}{(1 + \lambda)} \quad T_i = \frac{ik_f}{(1 + \lambda)T_0}
\]

3.2.1. Selection of tuning factor ($\lambda$) and stability indices ($\gamma_1, \gamma_2$)

The selection of tuning factor ($\lambda$) and stability indices ($\gamma_1, \gamma_2$) are playing a predominant role to obtain the optimal controller settings for CDM-PI-P controller. These parameters have the direct influence on transient control of the closed loop response. The effect of stability indices on closed loop response is analysed at the operating point of 0.06 mol/L SO2 outlet concentration for minimum $\lambda$ value of 0.1. Initially, for this analysis, the stability indices are chosen based on the Manabe’s recommended value as $\gamma_1=2.5$ and $\gamma_2=2$. The closed loop responses are recorded as shown in Figure 7 by increasing each stability indices value at the sampling interval of 0.5.

From Figure 6, it is observed that $G_{PI}(s)$ and $G_f(s)$ becomes,

\[
G_{PI}(s) = G_{PI}(s) + \frac{(K_c + K_f)T_i s + K_c}{T_i s}
\]  

Fig. 7. Effect of stability indices $\gamma_1, \gamma_2$ on the closed loop response for $\lambda=0.1$.

From the recorded response, the performance measures are computed and tabulated in Table 4. From the table, it is clear that the performance for $\gamma_1=3.5$ and $\gamma_2=3$ is provided fair transient response with minimum settling time and no peak overshoot. Hence $\gamma_1=3.5$ and $\gamma_2=3$ are selected as the optimal stability indices values. Similarly, the effect of $\lambda$ on the closed loop response is studied at the same operating point of 0.06 mol/L SO2 outlet concentration, by considering the optimized stability indices value of $\gamma_1=3.5$ and $\gamma_2=3$. 
Table 4. Performance measures for the effect of stability indices \( \gamma_1, \gamma_2 \) on the closed loop response for \( \lambda = 0.1 \).

<table>
<thead>
<tr>
<th>Performance measures</th>
<th>( \gamma_1 = 2.5, \gamma_2 = 2 )</th>
<th>( \gamma_1 = 3, \gamma_2 = 2.5 )</th>
<th>( \gamma_1 = 3.5, \gamma_2 = 3 )</th>
<th>( \gamma_1 = 4, \gamma_2 = 3.5 )</th>
<th>( \gamma_1 = 4, \gamma_2 = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISE</td>
<td>0.015</td>
<td>0.014</td>
<td>0.015</td>
<td>0.015</td>
<td>0.017</td>
</tr>
<tr>
<td>IAE</td>
<td>0.40</td>
<td>0.33</td>
<td>0.39</td>
<td>0.39</td>
<td>0.49</td>
</tr>
<tr>
<td>ITAE</td>
<td>2.74</td>
<td>1.39</td>
<td>1.46</td>
<td>2.27</td>
<td>3.92</td>
</tr>
<tr>
<td>( t_\text{r}(\text{Sec}) )</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( t_\text{s}(\text{Sec}) )</td>
<td>128</td>
<td>51</td>
<td>47</td>
<td>102</td>
<td>126</td>
</tr>
<tr>
<td>%Mp</td>
<td>21</td>
<td>6.33</td>
<td>NIL</td>
<td>NIL</td>
<td>NIL</td>
</tr>
</tbody>
</table>

The closed loop responses are shown in Figure 8 and the performance measures are computed and tabulated in Table 5.

Table 5. Performance measures for the effect of tuning factor \( \lambda \) on the closed loop response for \( \gamma_1 = 3.5 \) and \( \gamma_2 = 3 \).

<table>
<thead>
<tr>
<th>Performance Measures</th>
<th>( \lambda = 0.1 )</th>
<th>( \lambda = 0.3 )</th>
<th>( \lambda = 0.5 )</th>
<th>( \lambda = 0.7 )</th>
<th>( \lambda = 0.9 )</th>
<th>( \lambda = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISE</td>
<td>0.014</td>
<td>0.010</td>
<td>0.009</td>
<td>0.007</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>IAE</td>
<td>0.34</td>
<td>0.28</td>
<td>0.23</td>
<td>0.19</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td>ITAE</td>
<td>1.128</td>
<td>0.812</td>
<td>0.571</td>
<td>0.402</td>
<td>0.491</td>
<td>0.684</td>
</tr>
<tr>
<td>( t_\text{r}(\text{Sec}) )</td>
<td>NIL</td>
<td>NIL</td>
<td>NIL</td>
<td>NIL</td>
<td>NIL</td>
<td>4</td>
</tr>
<tr>
<td>( t_\text{s}(\text{Sec}) )</td>
<td>44</td>
<td>43</td>
<td>42</td>
<td>39</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>%Mp</td>
<td>NIL</td>
<td>NIL</td>
<td>NIL</td>
<td>NIL</td>
<td>NIL</td>
<td>33</td>
</tr>
</tbody>
</table>

From Figure 8 and Table 5, it is observed that, at \( \lambda = 0.7 \), the closed loop response with minimum error indices and less settling time is obtained when compared to other \( \lambda \) values. Hence \( \lambda = 0.7 \) is selected as the optimized value for further analysis.

3.2.2. Computation of CDM-PI-P controller parameters

The CDM-PI-P controller parameters are computed based on the procedure given in the section 3.2 as follows.

STEP 1: FOPTD Model of SO\(_2\) emission control system represented in (7) is

\[ G(s) = \frac{K_p}{\tau p s + 1} e^{-t_0 s} = \frac{27}{244s + 1} e^{-s} \]

STEP 2: Equivalent transfer function of the above said FOPTD model using first order Pade’s approximation technique is

\[ G(s) = \frac{-27s + 5s}{244s^2 + 489s + 2} \]

STEP 3: First degree CDM controller polynomials (A(s) and B(s)) are chosen as \( A(s) = s \) and \( B(s) = k_1 s + k_0 \)

STEP 4: Selected stability indices values (\( \gamma_1, \gamma_2 \)) and tuning factor (\( \lambda \)) are

\( \gamma_1 = 3.5, \gamma_2 = 3 \) and \( \lambda = 0.7 \)

STEP 5: Calculated \( P(s) = s \left( 244s^2 + 489s + 2 \right) + (k_1 s + k_0) \left( -27s + 5 \right) \)

STEP 6: Computed coefficients of CDM controller polynomials are \( k_1 = 3.8778 \) and \( k_0 = 0.54 \)

STEP 7: Comparing the CDM controller polynomials with PI-P controller transfer function, the CDM-PI-P controller parameters (\( K_c, T_i \) and \( K_f \)) and pre-filter element (\( G_f(s) \)) are computed as follows:

4. SIMULATION RESULTS

The simulation runs are carried out to analyse the performance of the CDM-PI-P controller with the existing conventional PI control techniques such as ZN-PI and IMC-PI. For analysis, PI controller structure for SO\(_2\) emission control system is modelled using MATLAB-SIMULINK as shown in Figures 9 and 10. From the figures, it can be seen that it is the unity feedback controller which makes the system less sensitive to external disturbances.

Fig. 8. Effect of tuning factor \( \lambda \) on the closed loop response for \( \gamma_1 = 3.5 \) and \( \gamma_2 = 3 \).

Fig. 9. Model of SO\(_2\) emission control system with conventional PI controller.
4.1. Performance Analysis

The performance along with controller output of ZN-PI, IMC-PI and CDM-PI-P controllers are analyzed at the operating point of 0.06 mol/L outlet SO₂ concentration and it is recorded in Figures 11 and 12.

From the figures, it is clear that the CDM-PI-P controller is forced to follow the set point at short duration of time and maintain the steady state without overshoot as compared to ZN-PI and IMC-PI control techniques.

Similarly, the same analysis are carried out with different operating points such as 0.09 mol/L (25% removal efficiency), 0.06 mol/L (50% removal efficiency), 0.03 mol/L (75% removal efficiency), 0.02 mol/L (83.4% removal efficiency), 0.01 mol/L (92% removal efficiency) to ensure the robustness of the CDM-PI-P controller.

Closed loop simulated transient responses obtained at different operating points are shown in Figure 13. The figure reveals that the performance of CDM-PI-P provides better performance with the same settings for different operating points. Among the controller tuning rules, CDM-PI-P tolerates the perturbations in the model parameters and provides the most consistent and robust response when the operating point changes.
The performance measures in terms of error indices such as Integral Squared Error (ISE), Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE), and Quality indices such as (rise time \(t_r\), settling time \(t_s\), peak overshoot \(%M_p\)) derived from Figure 13 are tabulated in Table 6. The numerical values presented in Table 6 reveal that the minimum error indices and good quality indices are obtained by the CDM-PI-P controller.

From the simulation results, it becomes clear that the CDM-PI-P controller is effective for controlling the nonlinear process. Though there are variations in the process gain, better control action is achieved by bringing the system to the desired set value without any modifications in the controller design and structure. The developed \(\text{SO}_2\)-H\(_2\)O\(_2\) absorption model provides fair simulation results over different range of operating points.

4.2 Disturbance rejection test

The disturbance rejection performance is investigated at the operating point of 0.06 mol/L outlet \(\text{SO}_2\) concentration. A step disturbance is introduced into the process by way of increasing the outlet \(\text{SO}_2\) concentration to 0.09 mol/L after 30th second as shown in Figure 14.

From the figure, it is ensured that, only CDM-PI-P controller damp the disturbance in a shorter time with no undershoot as compared to the ZN-PI and IMC-PI controllers. The error and quality indices of output signal are used to evaluate the disturbance rejection performance of the controllers and are given in Table 7.
From the results, it is proved that the CDM-PI-P control strategy is also successful in disturbance rejection with minimum error indices and good quality indices.

5. CONCLUSIONS

In this work, SO₂ emission control was achieved through the CDM-PI-P controller. SO₂ emission control process was developed based on the Liquid-Gas absorption kinetics principle in SIMULINK at MATLAB platform. The developed SO₂-H₂O₂ absorption model provides fair simulation results over different range of operating points. From the process, it was observed that the outlet SO₂ concentration depends on the flow rate of hydrogen peroxide. Since hydrogen peroxide is a stringent component, it boosts up the oxidation process during the absorption of SO₂. Thus perfect control over the flow rate of hydrogen peroxide was made by the CDM-PI-P controller. From the simulation results, it was proved that the CDM-PI-P controller provides fair transient as well as steady state behaviour with minimum error indices and good quality indices than the ZN-PI and IMC-PI controllers. Among the controller tuning rules, CDM-PI-P controller tolerated the perturbations in the model parameters when the operating point changes. Also, it produces successful performance in disturbance rejection test. Finally it is concluded that CDM furnishes a convenient and flexible design under the conditions of stability, robustness, and time domain performance which provides superior performance in terms of set point tracking and disturbance rejection.

GREEK SYMBOLS

\( \tau \) : Equivalent time constant
\( \tau_p \) : Process time constant
\( \theta \) : Process delay
\( \gamma_i \) : Stability indices

ABBREVIATIONS

CDM : Coefficient Diagram Method
CDM-PI-P : Coefficient Diagram Method-Proportional Integral-Proportional
FOPTD : First Order Plus Time Delay
ISE : Integral Squared Error
IAE : Integral Absolute Error
IMC-PI : Internal Model Control-Proportional Integral
PI : Proportional Integral
ZN-PI : Ziegler Nichols-Proportional Integral

NOMENCLATURE

\( C_A \) : Concentration of component A(mol/L)
\( C_B \) : Concentration of component B(mol/L)
\( C_{L1}(t) \) : Concentration of \( \text{H}_2\text{O}_2+\text{H}_2\text{SO}_4 \) exit from the bottom of column (mol/L)
\( C_{L2}(t) \) : Concentration of \( \text{H}_2\text{O}_2+\text{H}_2\text{SO}_4 \) entering into the top of column (mol/L)
\( C_{L3} \) : Concentration of \( \text{H}_2\text{O}_2 \) entering in to the mixing tank (mol/L)
\( C_{G1} \) : Concentration of SO₂ inlet to the Gas-Liquid absorption column (mol/L)
\( C(s) \) : Main controller
\( C_f(s) \) : Feed forward controller
\( F(s) \) : Reference numerator polynomial of the controller transfer function
\( G_1 \) : Gas flow rate for the column (m³/hr)
\( \text{H}_2\text{O}_2 \) : Hydrogen peroxide
\( \text{H}_2\text{SO}_4 \) : Sulphuric acid
\( l_a, k_i \) and \( a_i \) : Coefficients of CDM controller polynomials
\( L_1 \) : Liquid inlet flow rate for the column (lph)
\( L_2(t) \) : Liquid Flow rate in to the mixing tank (lph)
\( \% \text{Mp} \) : Percentage peak overshoot
\( N(s) \) : Numerator polynomial of the plant transfer function
\( P(s) \) : Characteristic polynomial of the closed-loop system
\( P_{\text{target}}(s) \) : Target characteristic polynomial of the closed-loop system
\( r_A \) : Reaction rate
\( \text{SO}_2 \) : Sulphur dioxide
\( t_r \) : Rise time
\( t_s \) : Settling time
\( V \) : Volume of the tank (L)

REFERENCES


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