CONSTRAINED RECEDING HORIZON PREDICTIVE CONTROL FOR POSITION TRACKING OF AN INDUCTION MOTOR

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Abstract: *The receding horizon strategy can successfully manage controlling an industrial positioning system demanding high-performances. The most important difficulties rise when handling constraints imposed by the physical limitations or stability requirements. In the present paper, the feasibility analysis for the optimal control sequence will be used as a tuning procedure for the predictive strategy. Further the time-consuming on-line constrained optimization is avoided by means of an off-line computed linear law, whose limitations are adapted on-line. The linear dependence of the limitations on the context parameters is detailed and the linear expression of the adaptation mechanism is found.*

Keywords: *Predictive control, feasibility, induction motor.*

1. INTRODUCTION

The positioning systems control problems represent a wide domain of applications. The development of fast processors opens up the possibilities to further apply advanced techniques for the control of the electrical drives. Usually the PID controllers are used with success but their tuning turns to be error-prone when constraints have to be considered. The control laws based on the receding horizon principle can improve and optimize the performances of the control scheme due to their prediction capabilities and most important, can handle constraints due to their time-domain formulation.

Generalized predictive control (GPC) is such a technique with success in the industrial applications. Besides its qualities, the GPC provided in the unconstrained case linear laws easy to implement in their polynomial formulation (Clarke, et al., 1987) and can further be reinforced by adding equality constraints at the end of the prediction horizon (Clarke, and Scatollini, 1991) for stability reasons. The application to fast processes with constraints was delayed as the optimal control action was provided by an on-line optimization procedure, generally a time consuming process. Different techniques where proposed, from the active-set methods to LMI (Maciejowski, 2002). Lately, the idea of moving a part of the computational effort off-line emerged and alternative techniques (Bemporad, et al.,2002; Seron, et al., 2002) exist based on look-up tables of linear affine controllers for regions of the state-space.

Our approach is situated somewhere between the two mentioned possibilities by exploiting the special geometrical design characteristics of the constrained GPC (CGPC) law. There is no look-up table just because the search space is designed to be one-dimensional but there are multiple linear RST laws defined in order to adapt on-line the saturation limits of the control signal. This is possible due to the opportunity to define the constrained control interval by means of parameterized polyhedra. Thus, the real-time computational needs are not significantly augmented and the constraints are handled in an intelligent manner, not by a blind saturation.

2. THE INDUCTION MOTOR BENCHMARK

The considered plant is an experimental setup of a squirrel cage induction motor, a benchmark system for the positioning control laws of asynchronous machines (Mendes, and Barbot, 2002). Details regarding the general theory of electric machines and induction motors could be found in the literature (Leonhard, 2001). The machine considered is a three phase (squirrel) cage asynchronous motor with two pairs of poles in star connection, of power 1.1 KW, nominal torque 7Nm, inertia of 0.038 Kgm^2, viscous friction coefficient of $0.01\,\mathrm{Nm\,rad^{-1}s}$ and 300V alimentation, delivering 10A maximum per phase allowing a maximum machine torque of 21 Nm. The position sensor allows 14400 points per rotation. The motor load is a powder brake.

The internal torque/speed control follows the standard approach of an induction motor, a Direct Field Oriented Control (Leonhard, 2001) at a sample rate of 76.6µs (13.05 kHz). In the external loop the field weakening assures that the flux reference decreases when the nominal speed of the motor has been exceeded. The position controller has to provide the position tracking performance as in Fig. 1.

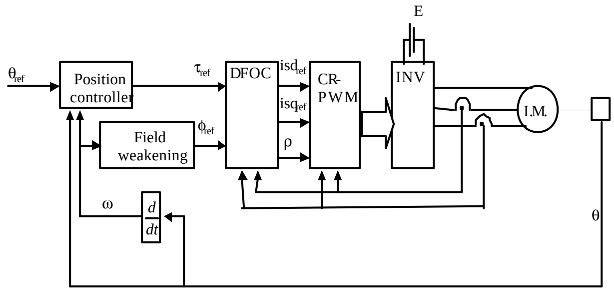

Fig. 1. Structure of the direct field-oriented control (DFOC) with a position loop controller

The pulse width modulation (PWM) control of the inverter is implemented in a PC with RT-Linux operating system and the experimental results presented in this paper have been obtained with such a control system build in a C environment. A nonlinear simulator is available also and its files could be further used with the real-time system.

The predictive control law will be designed starting from a system similar with the one depicted in Fig. 2. It represents the mechanical and electrical part together with the zero order holder and the sampler. The electrical part is represented by a first order transfer from the torque setpoint to the effective torque. The t_e constant representing the current loop, the DFOC and the inverter dynamics will be neglected as its influence is insignificant, compared with the mechanical time constant.

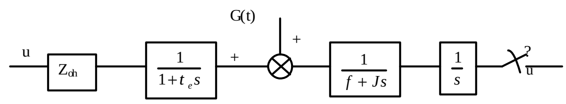

Fig. 2. Model for the MPC design

The mechanical part linked to the motor is characterized by the motor inertia J, the friction coefficient f and Γ the load torque. The discrete time transfer function between the electro-mechanical torque and the angular displacement for a sampling time of $T_e = 14 \times 76.6 = 1{,}0724\,\mathrm{ms}$ is given by:

$$\frac{q(q^{-1})}{u(q^{-1})} = \frac{10^{-4}(0.821q^{-1} + 0.8206\,q^{-2})}{(1-q^{-1})(1-0.998q^{-1})}. \tag{1}$$

3. GENERALIZED PREDICTIVE CONTROL

Generalized predictive control (GPC) is part of the model based predictive control (MPC) family. All these strategies are based on the fact that the system evolution can be predicted over a horizon taking into account the former control inputs, past plant outputs and future control sequence. The unconstrained GPC states the optimal control sequence in a polynomial form derived through analytical minimization of a cost function. The result is a control law under the RST form, an important implementation advantage.

The GPC algorithm major characteristics, delimita-ting it in the predictive control family are:

- The use of a CARIMA plant model:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + x(t)/D(q^{-1}) \tag{2}$$

where u, y are the system input and output respectively, $x(t)$ is a centred Gaussian white noise, $\Delta(q^{-1}) = 1 - q^{-1}$ is the difference operator, A and B are polynomials in the backward shift operator q^{-1} of respective degree n_a, n_b.

- The cost function is quadratic in the tracking error and control effort over the receding horizon:
-

$$J(t) = \sum_{j=N_1}^{N_2} [w(t+j) - \hat{y}(t+j)]^2 + l \sum_{j=1}^{N_u} [\Delta u(t+j-1)]^2 \tag{3}$$

with $\hat{y}(t+j)$ the output prediction, N_u the control horizon, N_1, N_2 the minimum and maximum costing horizons, l the weighting factor and w the setpoint.

Based on the model previously mentioned (2), an optimal j-step ahead predictor can be constructed:

$$\hat{y}(t+j) = \underbrace{F_j(q^{-1})y(t) + H_j(q^{-1})Du(t-1)}_{\text{freeresponse}} + \underbrace{+ G_j(q^{-1})Du(t+j-1)}_{\text{forced response}} \tag{4}$$

where the F_j, G_j, H_j polynomials are solutions of the Diophantine equations:

$$\begin{aligned} D(q^{-1})A(q^{-1})J_j(q^{-1}) + q^{-j}F_j(q^{-1}) &= 1 \\ G_j(q^{-1}) + q^{-j}H_j(q^{-1}) &= B(q^{-1})J_j(q^{-1}) \end{aligned} \tag{5}$$

Replacing $\hat{y}(t+j)$ in (3) and solving:

$$\min_{Du(t),\ldots,Du(t+N_u-1)} J(N_1, N_2, N_u, t) \tag{6}$$

leads to the optimal control sequence. Only the first control action is effectively applied to the plant input. The procedure is restarted at the next sampling period leading to a linear RST law (Fig. 3a) with improved performances and stable behaviour.

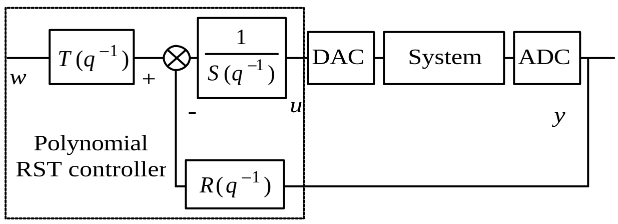

Fig. 3a. Control law under RST form

GPC can be implemented by a low-complexity controller due to the possibility of an analytical expression of the minimum in (3).
In order to validate different controllers and their capabilities, a position benchmark cycle is used [10], where the speed reference increases to the nominal speed and descends to zero with different profiles. Fig. 3b presents the position and motor load trajectories, while the rotor flux is constant. This cycle enables to test the position loop behavior at very slow and zeros speeds (Fig. 4) with load variations, at nominal speed and during parabolic position tracking. This last part is useful as the PID control laws are characterized generally by a non-zero steady state error.

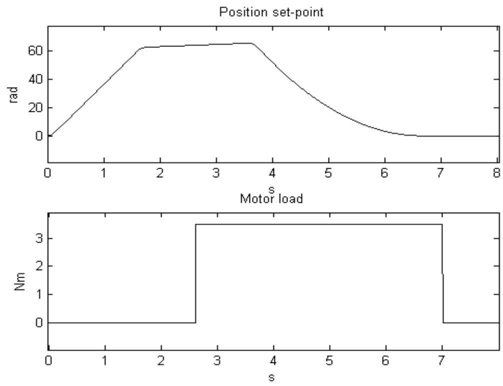

Fig. 3b. Position trajectory and motor load

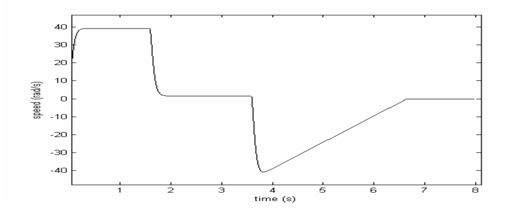

Fig. 4. Benchmark related speed trajectory

3.1 Inequality Constraints

Generally, by considering inequality constraints, the search domain can be restricted to an intersection of half planes (3). The resulting domain is a convex polyhedron and by the fact that the cost function is quadratic in the predicted error and the control effort, we are still dealing with a convex optimization problem. For the unconstrained case the search domain is represented by the universe polyhedron.

i) Constraints on the input update can be given by the actuator's limitation. The mathematic form is:

$$\underline{Du} \le u(t+k) - u(t+k-1) \le \overline{Du}, 1 \le k \le N_u \tag{7}$$

and the matrix representation:

$$\mathbf{1}\underline{Du} \le Ik_u \le \mathbf{1}\overline{Du} \tag{8}$$

with $k_u = [\Delta u_1, \cdots, \Delta u_{N_u}]^T$ a vector of future control updates, I a N_u-dimensional eye matrix, $\underline{\Delta u}, \overline{\Delta u}$ the upper and lower bounds, and **1** an $N_2 - N_1$ vector whose entries are one.

ii) Other performance constraints. Such constraints are related to the application area, for example in robotics: the position trajectory can be prohibited in some zones. These limitations are generally translated by challenging performances related to the overshoot of the response and could be prescribed as:

$$y(t+k) \le p \cdot w(t+k), N_1 \le k \le N_2 \tag{9}$$

where p is the percentage corresponding to the desired overshoot (e.g. $p = 1.01 \text{ for } 1\%$) or simpler:

$$\hat{y}(t+k) \le \overline{y}, \qquad N_1 \le k \le N \tag{10}$$

iii) The predictor. All the constraints on variables linked to the system dynamics use as key element the output predictor with its two parts: one related to the future control actions and the other dependent on the context of the system. In a matrix form:

$$\hat{\mathbf{y}} = \mathbf{G}\,\mathbf{k}_u + \underbrace{\mathbf{if}\;\mathbf{y}_{past}(t) + \mathbf{ih}D\,\mathbf{u}_{past}(t)}_{l} \tag{11}$$

$$\begin{gathered}
\mathbf{k}_u = [\Delta u(t) \cdots \Delta u(t+N_u-1)]^{\mathrm{T}} \\
\mathbf{u}_{past}(t) = [u(t-1) \cdots u(t-n_b)]^{\mathrm{T}} \\
\mathbf{y}_{past}(t) = [y(t) \cdots y(t-n_a)]^{\mathrm{T}} \\
\hat{\mathbf{y}} = [\hat{y}(t+N_1) \cdots \hat{y}(t+N_2)]^{\mathrm{T}} \\
\mathbf{ih} = \begin{bmatrix} H_{N_1}(1) & \cdots & H_{N_1}(n_b-1) \\ \vdots & & \vdots \\ H_{N_2}(1) & \cdots & H_{N_2}(n_b-1) \end{bmatrix}; \mathbf{if} = \begin{bmatrix} F_{N_1}(1) & \cdots & F_{N_1}(n_a) \\ \vdots & & \vdots \\ F_{N_2}(1) & \cdots & F_{N_2}(n_a) \end{bmatrix}
\end{gathered} \tag{12}$$

The elements g_k representing the step response coefficients are in fact the G_j polynomials coeffi-cients at each prediction time and compose the matrix $\mathbf{G} \in \Re^{(N_2-N_1+1)\times N_u}$:

$$\mathbf{G} = \begin{bmatrix} g_{N_1} & g_{N_1-1} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ g_{N_2-1} & \cdots & \vdots & \vdots \\ g_{N_2} & \cdots & \cdots & g_{N_2-N_u+1} \end{bmatrix} \tag{13}$$

3.2 End-point Constraints

Another important class of limitations is imposed by stability purposes (Clarke, and Scatollini, 1991) in terms of constraints at the end of the prediction horizon. The idea is to force the evolution of the system to a close neighbourhood of the setpoint. For infinite prediction horizons this is expressed as:

$$\|\hat{y}(t+k) - w(t+N_2)\| \le b(t) \cdot e^{-dk}, \forall k > N_2 \tag{14}$$

for a given function $b(t)$ and a fix d .

In the case of GPC and a finite horizon, the frame-work changes and one can impose:

$$|\hat{y}(t+k) - w(t+N_2)| \le b(t) \cdot e^{-dk}, N_2 < k \le N_2 + m \tag{15}$$

where m is the number of end-point constraints. Despite the theoretical developments, the terminal constraints could be expressed by equalities at the end of the prediction horizon:

$$\hat{y}(t+k) = w(t+N_2),\ N_2 < k \le N_2 + m \tag{16}$$

For the induction motor benchmark, the use of this approach may not be the best choice for the position setpoint in Fig. 3. The equality constraints at the end of the prediction horizon (16) could raise problems due to their inability of handling higher degree set-points which is the case of our application. For a set-point of order k, zero steady state error is obtained if:

$$\left.\frac{d^j R(q^{-1})}{(dq^{-1})^j}\right|_{q^{-1}=1} = \left.\frac{d^j T(q)}{(dq^{-1})^j}\right|_{q^{-1}=1},\ j = 1:k \tag{17}$$

Obviously, in the general case, this relation is impossible to fulfill for $k > 1$ if the system doesn't include integrators. In the induction motor case, the controller can be designed to lead to convenient results in the unconstrained case. If terminal constraints as in (16) are considered, the steady state error is non-zero even if not very important.
In the following a simpler and direct approach which verifies the necessary conditions of zero steady error for the ramp part of the trajectory will be used:

$$\hat{y}(t+k) = w(t+k), N_2 < k \le N_2 + m \tag{18}$$

4. OPTIMAL CONTROL SEQUENCE

4.1 Unconstrained case

For a GPC law, the cost index (3) can be written as:

$$\begin{aligned} J &= (\mathbf{G}\,\mathbf{k}_u + \mathbf{l} - \mathbf{w})^{\mathrm{T}}(\mathbf{G}\,\mathbf{k}_u + \mathbf{l} - \mathbf{w}) + l\,\mathbf{k}_u^{\mathrm{T}}\mathbf{k}_u \\ &= 0.5\,\mathbf{k}_u^{\mathrm{T}}\,\mathbf{Q}\,\mathbf{k}_u + \mathbf{f}^{\mathrm{T}}\,\mathbf{k}_u + J_0 \end{aligned} \tag{29}$$

where $\mathbf{w} = [w(t+N_1)\ \cdots\ w(t+N_2)]^{\mathrm{T}}$. The optimum of J could be found analytically by making the gradient equal to zero leading to $\mathbf{k}_u = -\mathbf{Q}^{-1}\mathbf{f}$. Further these results allow the representation of the GPC law by a RST formulation.

4.2 Terminal constraints

With the cost function (29) and adding the end-point constraints one has to find the optimal sequence for:

$$\begin{aligned} &\min_{\mathbf{k_u}}\ J = 0.5\,\mathbf{k}_u^{\mathrm{T}}\,\mathbf{Q}\,\mathbf{k}_u + \mathbf{f}^{\mathrm{T}}\,\mathbf{k}_u \\ &s.t.\ G_c\mathbf{k}_u = l_c - w_c \end{aligned} \tag{31}$$

Appling the Lagrange multipliers:

$$\begin{aligned} \mathbf{k}_u = &\left(Q^{-1}G_c{}^T(G_cQG_c{}^T)^{-1}G_cQ^{-1} - Q^{-1}\right)(l - w) + \\ &+ Q^{-1}G_c{}^T(G_cQG_c{}^T)^{-1}(l_c - w_c) \end{aligned} \tag{32}$$

which is linearly dependent on the past inputs, past outputs and the setpoint and can easily be translated in an RST formulation.

4.3 Non-parameterized inequality constraints

If inequality constraints are defined on the future control updates, the search domain is intersected by half-planes and the unconstrained optimal may be retrieved outside the allowed polyhedron Fig. 5.

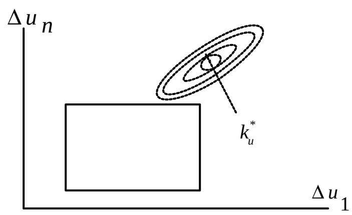

Fig. 5. Global optimum outside the feasible domain

One choice is to saturate the values lying outside the bounds. The performances are poor as generally the saturated values are far from the constrained optimal combination. In order to overcome this difficulty, a transformation of the search space is performed:

$$\mathbf{k_u}^* = \mathbf{Q}^{1/2}\mathbf{k_u} \tag{30}$$

The isocost ellipses are changed into circles, Fig. 6.

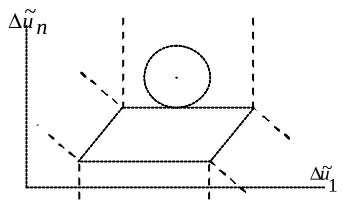

Fig. 6. Regions with the same RST linear law

Thus a delimitation of the search space is made and a different linear RST law can be defined for each part. The control laws are memorized in a look up table and at each sample time a search procedure retrieves the RST for the context of the system. There are two disadvantages, first related to the number of laws which is growing with the dimension and the complexity of the feasible domain. The second is that constraints may depend on the context parameters and thus the search space cutting might not be fixed. In this case the search in the look-up table becomes even more time-consuming. These aspects have to be considered when designing the constrained GPC law.

4.4 Mixed constraints - Parameterized polyhedra

The equality constraints offer the advantage of decreasing the degrees of freedom for the optimization problem. The unconstrained case supposes a N_u dimensional polyhedron. Using m equality constraints the dimension of the domain is $N_u - m$. Solving on-line an optimization with mix constraints employs active-set or interior point methods.
For the induction motor, a GPC law having $N_u = 2$ and $m = 1$, the inequality constraints with terminal constraints restrain the search to a segment Fig. 7a.
The support of the line given by the equality constraints has a variable offset dependent on the current setpoint and on the system history (Fig 7b). This fact may lead to infeasibility if the line does not intersect the domain of available input updates. Such cases are to be avoided and have to pass an infeasibility analysis.

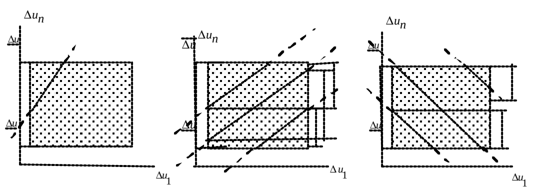

Fig. 7 Domains resulting from mixed constraints

If the infeasibility possibility is obviate, one can still distinguish specific cases for the first control action Fig 7b,c. This first increment is a very important variable as it is the one effectively applied at the plant input and in the same time is the only effective degree of freedom because *Nu-m=1.*

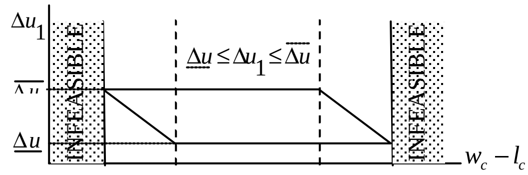

Fig. 8. ? u_1 variation – a parameterized polyhedron

The limits of the interval are dependent on the stated bounds (7), $\underline{\Delta u} \le \Delta u_1 \le \overline{\Delta u}$ but depend also on the intersection of the line with the limits on the control increment at the second sampling time. As a result, this interval can be represented as a parameterized polyhedra on the parameter $w_c - l_c$, Fig. 8.

4.5 Implementation

The linear control law in the RST form could be found when terminal constraints are considered. For the induction motor, the GPC law in (29) will provide at each sampling time a control sequence:

$$\mathbf{k}_u^* = \{\Delta u_1^*, \Delta u_2^*\} \tag{33}$$

The first input increment Δu_1^* is applied to the plant. This update must be done only after a check is made to verify if this value is included in the allowed interval of variation at the current sampling time (the limits of the interval must be available). These limits correspond to the equations:

$$\begin{aligned} A &: \underbrace{\begin{bmatrix} G_c \\ 0 \quad 1 \end{bmatrix}}_{A_c} \begin{bmatrix} \Delta u_{1a} \\ \Delta u_{2a} \end{bmatrix} = \begin{bmatrix} w_c - l_c \\ \underline{\Delta u} \end{bmatrix} \\ B &: \begin{bmatrix} G_c \\ 0 \quad 1 \end{bmatrix} \begin{bmatrix} \Delta u_{1b} \\ \Delta u_{2b} \end{bmatrix} = \begin{bmatrix} w_c - l_c \\ \overline{\Delta u} \end{bmatrix} \end{aligned} \tag{34}$$

If the matrix A_c is not full rank, we are dealing with degeneracy and the values Δu_{1a} and Δu_{1b} should be set to infinity as the line will not touch the limits on the second input update.

The procedure is the same for control laws with $m > 1$ and $N_u = m + 1$. The difference is that for each inequality constraint such a limit value will exist. However, the number of inequalities is definitely lower than the number of regions computed by other methods.

For the study case followed along this paper, we note

$$A_c^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \tag{35}$$

the equation A and B are solved and the solutions are:

$$\begin{aligned} Du_{1a}(t) &= a_{11}(w_c(t+N_2+1) - l_c(t+N_2+1)) + a_{12}\underline{Du} = \\ &= a_{11}w_c - a_{11}if_c y(t) - a_{11}ih_c Du(t-1)) + a_{12}\underline{Du} \\ Du_{1b}(t) &= a_{11}(w_c(t+N_2+1) - l_c(t+N_2+1)) + a_{12}\overline{Du} = \\ &= a_{11}w_c - a_{11}if_c y(t) - a_{11}ih_c Du(t-1)) + a_{12}\overline{Du} \end{aligned} \tag{36}$$

Remark : The linear dependence of Δu_{1a} and Δu_{1b} on the past input, past outputs and future setpoint is not exactly a RST form as it includes an affine part depending on $\underline{\Delta u}$ and $\overline{\Delta u}$. Its implementation is the same as for the classic RST one.

Finally in order to be validated, the optimal control increment Δu_1^* has to satisfy:

$$\max(\underline{\Delta u}, \min(\Delta u_{1a}, \Delta u_{1b})) \le \Delta u_1^* \le \min(\overline{\Delta u}, \max(\Delta u_{1a}, \Delta u_{1b})) \tag{37}$$

otherwise is replaced by the violated bound.

5. EXPERIMENTS

For the induction motor presented in section II, the selected constrained GPC law was implemented. During the experiments we proceeded from simulations in a Matlab environment allowing the feasibility analysis to real-time experiments. More than that, for the GPC law with terminal constraints, the direct implementation of the RST without the limitation mechanism performed well, without passing the upper and lower bound {2; -2}, Fig. 9.

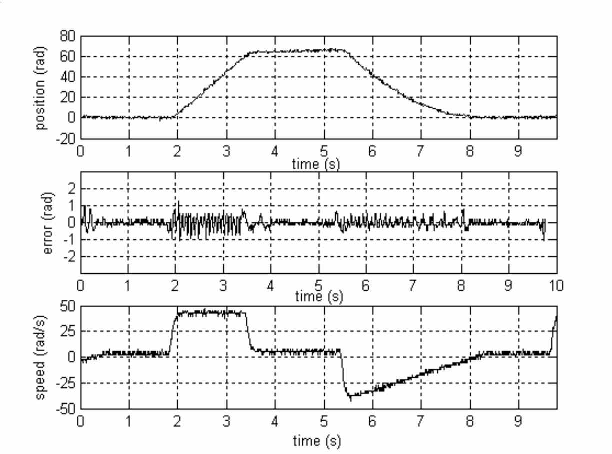

Fig. 9. The position response, the error and speed

The opportune choice of the end-point type of constraints proves to provide a zero steady error for the ramp part of the trajectory. The results verified the tuning statements. Some experiments were affected by the measurement noise and thus a robustification procedure was employed, (Rodriguez, and Dumur, 2002).

A closer look on the alchemy of the GPC law proves that the optimal sequences without the limitation mechanism are violating the constraints on the part of the commands which is not effectively applied to the plant input Fig 10.

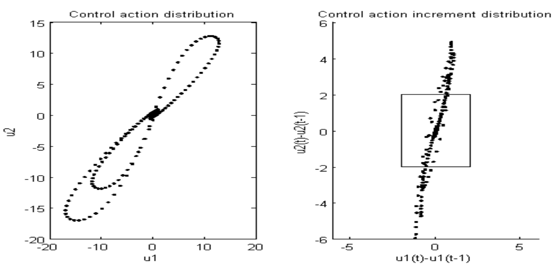

Fig. 10. The distribution of control actions

If the mechanism of limitation is taken into conside-ration the slight violations will be overcome, Fig 11.

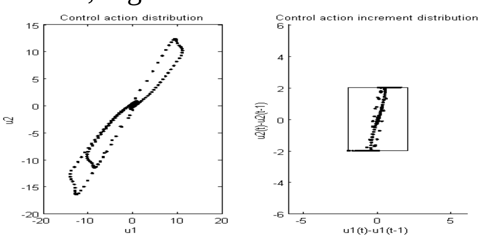

Fig. 11. The distribution of control actions

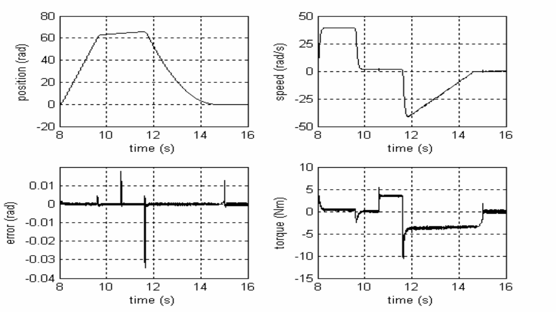

Fig. 12. The position response for the CGPC law

The real-time software used for the control procedure does not support affine expression of a linear control law. In order to prove the qualities of the control laws the nonlinear simulator of the induction motor was used, confirming the theoretical results, Fig. 12.

6. CONCLUSIONS

The paper presented an application of the constrained predictive control to an induction motor plant. The contributions where made towards the real-time implementation of the constrained optimal sequence by avoiding the on-line optimization. In counterpart, three linear laws where memorized, one linear RST for the unconstrained part of the evolution and two linear affine RST descriptions for the on-line adaptation of the saturation limits. The procedure proved to be appropriate for all the cases where the geometrical representation of the search space is represented by a parameterized interval. If this domain is of higher dimension, look-up tables with affine control laws for each critical region of the context parameters can be used.

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