Kalman Filtering for Model-Based Networked Control Systems

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Abstract: In this paper a control problem of a continuous linear plant which measured outputs are transmitted to a controller via a fading communication channel is considered. The attention is focused on the state reconstruction of the plant state between the sampling moments. Based on this estimation, a continuous-time state feedback control law is designed and implemented. Comparative numerical results are also presented emphasizing the benefits of the proposed method with respect to the classical one based on discrete-time filtering and zero order holders.

Keywords: Communication networks, Optimal estimation, Optimal control, Stochastic systems, Multiplicative noise, Riccati equations, Numerical algorithms.

1. INTRODUCTION

The control systems which components are interconnected using communication channels received much attention over the last two decades. Such systems often called *networked control systems* naturally arise in many engineering applications. For instance automated highway systems (Hedrick et al. 1994, Carbaugh et al. 1999, Lee et al. 2001), unmanned air vehicles (UAVs) and satellites formations (Seiler 2001, Seiler and Sengupta 2005, Giulietti and Mengali 2004, Smith and Hadaegh 2007), manufacturing factories (Marshall 2001, Shah and Raman 2003, Moyne and Tilbury 2007) are only some of these applications intensively analyzed in the recent control literature. The main difficulty generated by the use of communication networks is that the information is not instantaneously transmitted. Moreover the packets of data representing the digitized information can be perturbed or even lost at some moments of time. In order to analyse the effect of delays and packets loses or to attenuate their influence, deterministic and stochastic models of the fading communication channels have been proposed (see for instance, Chan and Özgüner 1995, Walsh et al. 2001, Elia 2005, Dong et al. 2010, Stefan et al. 2011 and their references). Another issue of distributed configurations is related to the network bandwidth. Indeed, the fast transmission of a large amount of information requires a large bandwidth. This represents a major constraint since the data networks typically have limited bandwidth. A solution of this problem is proposed in (Brockett 1997) where an optimal control problem is solved penalising the derivatives norms $\|\partial u / \partial t\|$ and $\|\partial u / \partial x\|$ in order to reduce the data transfer rate.

The purpose of the present paper is to analyse the interaction between the control and estimation performances of a networked system with fading communication channels in terms of the sampling period and to propose a method to reduce the performance lose when small sampling rate is used. The control law uses the process state estimations based on the measurements at the sampling moments. The key component of the configuration considered in this paper is a *Kalman type filter* which estimates the states. A common approach is to consider a discrete-time filter estimating the process states at the sampling moments and then based on these estimations, to implement control laws designed in order to accomplish some specific desired requirements. Usually the control signals are piecewise constant between the sampling instants. When the sampling period is small, which situation corresponds to a high data transfer rate, the approximation of the continuous-time state of the plant with a piecewise constant estimation is acceptable. The difference between the true state and its piecewise constant approximation increases for larger sampling period of the transmitted measurements. Enlarging the sampling period means to reduce the number of data packets transmitted through the network. The benefit of reducing the data transfer rate as much as possible is that more bandwidth becomes available to allocate more resources. These aspects have been noted and analyzed for instance in (Walsh and Ye 2001) and in (Montestruque and Antsaklis 2002) where it is concluded that *reducing the number of transmitted packets brings more benefits than data compression*. In order to preserve the control performance when reduced transfer rate is used, a reconstruction of the plant state between the sampling moments is required. In (Brockett 1997, Walsh et al. 1997) and in (Montestruque and Antsaklis 2002) a model of the plant dynamics is used to generate an open-loop state approximation between the sampling instants using the information about the initial state and control assumed available at the previous sampling moment. The state and the control are then updated at the sampling moments providing thus a closed loop information required to accomplish stability and performance of the networked control system.

In the present paper, a different method is used to reconstruct the plant state during the *transfer time*, namely the time between the information exchanges. It is based on a hybrid Kalman filter incorporating both continuous-time and discrete-time components. Such hybrid structure often called in the literature *model with finite jumps* has received a considerable attention over the last decades since they allow an accurate representation of sampled data dynamic systems (see e.g. Lakshmikantham et al. 1989). Many useful results concerning the stability, optimal control and disturbance attenuation properties of systems with finite jumps are available (Sivashankar and Khargonekar 1994, Ichikawa and Katayama 1998). The communication network considered in present paper is approximated using the *Rice model* (Elia 2005, Primak et al. 2004). It is a representation of fading communication channels corrupted both with *multiplicative* and *additive white noise*. In (Petersen et al. 2000, Gershon et al. 2005) and in their references a detailed analysis of stochastic models with multiplicative noise and some of their applications may be found. In (Stoica and Yaesh 2008), a Kalman type filter for linear stochastic systems with multiplicative noise is designed. A hybrid model with multiplicative noise of networked control systems and its mixed H_2/H_∞ performance analysis may be found in (Stoica 2012).

The present paper is organised as follows: in the next section, a stochastic model of networked control system with fading communication channels is determined. In Section 3 a Kalman type filter with finite jumps is presented. It includes both a continuous-time and a discrete-time component and it allows reconstructing the state estimation between the sampling instants. In Section 4, a linear quadratic problem for stochastic systems with multiplicative noise is presented. A case study together with comparative results emphasizing the benefits of the hybrid Kalman filter, are presented in Section 5. The paper ends with some concluding remarks.

2. MATHEMATICAL MODEL OF THE NETWORKED CONTROL SYSTEM

The configuration of the networked control system considered in the present paper is represented in Fig. 1 in which the sampling period $h > 0$ is constant.

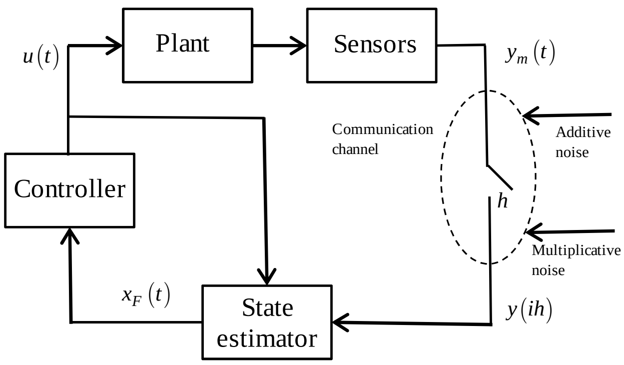

Fig. 1. Networked control system with fading communication channels.

The plant dynamics is assumed to be known and it is modelled including the measured outputs by the state-space equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y_m(t) &= Cx(t). \end{aligned} \tag{1}$$

The dynamics of the fading communication channel is approximated using the discrete-time Rice models (Elia 2005)

$$r(i) = \sum_{k=0}^{L} a_k(i) v(i-k) + n(i) \tag{2}$$

where $v(\cdot), r(\cdot) \in \mathbb{R}^{n_v}$ denote the transmitted and the received signals respectively, $n(\cdot)$ is an n_v-dimensional vector which elements are independent Gaussian white noises with zero mean and unit variance and $a_k(\cdot), k = 0,...,L$ are independent scalar random variables with known mean $\overline{a}_k$ and variance $\sigma_k{}^2$. The input signal $v(i)$ in equation (2) coincides in fact with the measurements $y_m(t_i)$ with $t_i = ih,\ i = 0,1,...$ If the delays induced by the communication channels are ignored (or considered much smaller than the sampling period), (2) becomes

$$r(i) = a_0(i) v(i) + n(i). \tag{3}$$

The following developments can be easily extended for the case when $L > 0$ but in order to simplify the reasoning in the present paper the case $L = 0$ is treated. Since $a_0(i) = \overline{a}_0 + w_d(i)$ where $w_d(i)$ is a sequence of scalar variables such that $E[w_d(i)] = 0$ and $E[w_d{}^2(i)] = \sigma_0{}^2 - \overline{a}_0{}^2$, where $E[\cdot]$ denotes the mean of the random variable $(\cdot)$, the above equation may be rewritten as

$$r(i) = (\overline{a}_0 + w_d(i)) v(i) + n(i),$$

from which taking into account that $v(i) = y_m(ih) = Cx(ih)$, it follows that the received measurement signal has the expression

$$y(ih) = r(i) = (C_0 + C_1 w_d(i)) x(ih) + n(i), \tag{4}$$

where by notation, $C_0 := \overline{a}_0 C$ and $C_1 := C$. One can see that by contrast with the classical filtering problems in which the measurement is corrupted only with additive white noise, the output (4) is also subject to the multiplicative (state-dependent) noise term $C_1 w_d(i) x(ih)$. In the next section a Kalman type filter for such case will be presented.

The representation including multiplicative noise terms naturally arises in the problem considered in this paper due to the model of the fading communication channel. There are many other applications in which such terms also appear in the state equation as in equation (5) below. Time-varying uncertainty or model errors depending on the process state are suitable to be represented by stochastic multiplicative noise terms (Gershon et al. 2005, Petersen et al. 2000). This is the reason for which in the following developments, for the sake of generality, the adopted plant model will include multiplicative noise terms both in the state and output equations.

3. A HYBRID KALMAN-TYPE FILTER FOR SYSTEMS WITH SAMPLED DATA

Consider the stochastic continuous-time system with state dependent noise

$$dx(t)=\left(A_0x(t)+Bu(t)\right)dt+A_1x(t)dw(t)+Gd\xi(t),\ t\geq 0 \quad (5)$$

and the discrete-time output $y(ih)\in\mathbb{R}^{n_y}$ measured at the instants $ih, i=0,1,...$

$$y(ih)=\left(C_0+C_1w_d(i)\right)x(ih)+G_d\xi_d(i),\ i=0,1,... \quad (6)$$

where $x(t)\in\mathbb{R}^n$ denotes the state vector, $u(t)\in\mathbb{R}^m$ is the control variable and $\{w(t)\}_{t\geq 0},\{\xi(t)\}_{t\geq 0},\{w_d(i)\}_{i\geq 0}$ and $\{\xi_d(i)\}_{i\geq 0}$ are stochastic processes on a given probability space $\{\Omega,\mathsf{F},\mathsf{P}\}$ satisfying the following usual assumptions:

(i) $\{w(t)\}_{t\geq 0}$ is scalar Wiener process with

$$E\left[w(t)\right]=0,\ E\left[\left(w(t)-w(s)\right)^2\right]=|t-s|,\ \forall\ t,s\geq 0\ ;$$

(ii) $\{\xi(t)\}_{t\geq 0}$ is an n_ξ-dimensional Wiener process with

$$E\left[\xi(t)\right]=0,\ E\left[\left(\xi(t)-\xi(s)\right)\left(\xi(t)-\xi(s)\right)^T\right]=I_{n_\xi}|t-s|$$

for all $t,s\geq 0$;

(iii) $\{w_d(i)\}_{i\geq 0}$ stands for a sequence of independent scalar random variables such that $E\left[w_d(i)\right]=0$ and $E\left[w_d{}^2(i)\right]=1$ for all $i\geq 0$;

(iv) $\{\xi_d(i)\}_{i\geq 0}$ denotes a sequence of m_{ξ_d}-dimensional random vectors with the properties that $E\left[\xi_d(i)\right]=0$ and $E\left[\xi_d(i)\xi_d{}^T(i)\right]=I_{m_{\xi_d}}$;

(v) The stochastic processes $\{w(t)\}_{t\geq 0},\{\xi(t)\}_{t\geq 0}$, and $\{w_d(i)\}_{i\geq 0}$, $\{\xi_d(i)\}_{i\geq 0}$ are mutually independent.

It is assumed that the system (5) is *exponentially stable in mean square*, namely there exist $\beta\geq 1$ and $\alpha>0$ such that $E\left[|x(t)|^2\right]\leq\beta e^{-\alpha t}|x_0|^2$ for any initial condition x_0 and for all $t\geq 0$.

Given the remote signal

$$z(t)=C_zx(t),\ t\geq 0$$

the filtering problem consists in determining a filter $\mathscr{F}$ with the state-space realization

$$\begin{aligned}&dx_F(t)=\left(A_Fx_F(t)+B_Fu(t)\right)dt,\ ih<t\leq(i+1)h\\&x_F\left(ih^+\right)=A_{F_d}x_F(ih)+B_{F_d}y(ih),\ i=0,1,...\\&z_F(t)=C_Fx_F(t),\end{aligned} \quad (7)$$

where $x\left(ih^+\right)$ is the initial condition of $x_F(t)$ on the interval $\left(ih,(i+1)h\right]$, such that the cost function

$$J(\mathscr{F}):=\lim_{\tau\to\infty}\frac{1}{\tau}\int_{t_0}^{t_0+\tau}E\left[|z(t)-z_F(t)|^2\right]dt$$

is minimized.

Remark 1. From the structure (7) of the filter $\mathscr{F}$ one can see that it provides a time-varying estimation of $z(t)$ by contrast with case when a classical discrete-time Kalman filter is used to get a constant piecewise estimation of $z(t)$ between the sampling moments.

The solution of the above filtering problem is given by the following result. Its proof for the slightly modified case, when the control $u(t)\equiv 0$, is given in (Dragan and Stoica 2012).

Theorem 1. The matrices arising in the state-space representation (7) of the optimal filter $\mathscr{F}$ have the expressions:

$$\begin{aligned}&A_F=A_0,\ A_{F_d}=I_n+KC_0,\\&B_F=B,\ B_{F_d}=-K,\\&C_F=C_z\end{aligned} \quad (8)$$

with

$$K:=-YC_0{}^T\left(C_0YC_0{}^T+C_1P_cC_1{}^T+G_dG_d{}^T\right)^{-1}, \quad (9)$$

where Y denotes the stabilizing solution of the filtering Riccati-type equation

$$Y=\tilde{A}Y\tilde{A}^T-\tilde{A}YC_0{}^T\left(C_0YC_0{}^T+\tilde{R}\right)^{-1}C_0Y\tilde{A}^T+\tilde{M} \quad (10)$$

in which the following notations have been used:

$$\tilde{A} := e^{A_0 h},\ \tilde{R} := C_1 P_c C_1^T + G_d G_d^T,$$
$$\tilde{M} := \int_0^h e^{A_0 s}\left(A_1 P_c A_1^T + GG^T\right)e^{A_0^T s}ds \tag{11}$$

and P_c stands for the solution of the Lyapunov-type equation

$$A_0 P_c + P_c A_0^T + A_1 P_c A_1^T + GG^T = 0. \tag{12}$$

Remark 2. In (Dragan and Stoica 2012) it is proved that the filter (7) with the expressions (8), (9) is optimal with respect to all other filters, no matter their order is. The assumption that the given system (5) is exponentially stable in mean square is required due to the presence of the multiplicative (state-dependent) noise terms. As it is well-known such assumption is not necessary in the classical case when these terms are missing. The algebraic equation (12) is a particular form of Lyapunov-type equation arising in the stability analysis and control of stochastic systems with state-dependent noise (see for instance, (Petersen et al. 2000) and its references). In the absence of the multiplicative noise it simply reduces to a classical Lyapunov equation from the continuous-time case.

4. OPTIMAL LINEAR QUADRATIC CONTROL FOR STOCHASTIC SYSTEMS WITH STATE-DEPENDENT NOISE

Consider the stochastic continuous-time system with multiplicative noise

$$dx(t) = \left(A_0 x(t) + Bu(t)\right)dt + A_1 x(t)dw(t), \tag{13}$$

where $w(t)$ stands for as a Wiener process as defined in Section 3 and where $u(t)$ is a control variable designed such that the system (13) is exponentially stable in mean square and the cost function

$$J(u) = E\left[\int_0^\infty \left(x^T(t)Qx(t) + u^T(t)Ru(t)\right)dt\right] \tag{14}$$

is minimised, the weighting matrices $Q \geq 0$ and $R > 0$ being given. The solution of this problem is given by (see e.g. (Willems and Willems 1983, Dragan et al. 2006) and their references)

$$u(t) = Fx(t), \tag{15}$$

where $F := -R^{-1}B^T X$, X denoting the *stabilizing solution* of the algebraic Riccati equation

$$A_0^T X + XA_0 + A_1^T XA_1 - XBR^{-1}B^T X + Q = 0. \tag{16}$$

Recall that a solution X of (16) is stabilizing if the system

$$dx(t) = \left(A_0 - BR^{-1}B^T X\right)x(t)dt + A_1 dw(t)$$

is exponentially stable in mean square.

In ((Dragan et al. 2006), Chapter 4, p. 142) an iterative procedure to solve (16) is presented together with the proof of its convergence towards the stabilizing solution (see also (Guo, 2001)):

Step 1. Set $k = 0$; determine a gain F_0 such that the system

$$dx(t) = \left(A_0 + B_0 F_0\right)x(t)dt + A_1 dw(t)$$

is exponentially stable in mean square and compute a matrix $X_0 > 0$ such that

$$\left(A_0 + BF_0\right)^T X_0 + X_0\left(A_0 + BF_0\right) + F_0^T RF_0 + Q + \varepsilon I \leq 0$$

for a fixed $\varepsilon > 0$;

Step 2. Set $k = k+1$ and solve the Lyapunov equation

$$\left(A_0 + BF_{k-1}\right)^T X_k + X_k\left(A_0 + BF_{k-1}\right) + Q_k = 0,$$

where by definition

$$Q_k = Q + \frac{\varepsilon}{k+1}I_n + F_k^T RF_k.$$

Set $F_k = -R^{-1}B^T X_k$;

Step 3. If $\left\|X_k - X_{k-1}\right\| < \nu$ with $\nu > 0$ arbitrarily small, then set $X = X_k$ and STOP. Otherwise go to Step 2.

5. A NUMERICAL EXAMPLE

Assume that the matrices in the model (5) and (6) are

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & -1 \end{bmatrix},\ A_1 = 0_{3\times 3},\ B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ G = 0.01 \cdot I_3,$$
$$C_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ C_1 = \bar{a}_0 C_0,\ G_d = 0.1 \cdot I_2.$$

In all simulations discussed below it is assumed that the process is stabilized using a state feedback control law which optimal gain was determined as solution of a linear quadratic problem for which the weightings $Q = I_3$ and $R = 10$ have been chosen in (14).

The following two cases have been considered.

Case A: $\bar{a}_0 = 0$. This case corresponds to the ideal situation when the multiplicative noise term in the received information is missing, namely $C_1 = 0$. The measurements are corrupted in this case only by additive white noise. In Fig. 2a, b, c are shown the time responses of the estimation errors and of the control when a *hybrid Kalman filter* of form (7) with the matrices given by (8)-(12) is implemented and when a *classical discrete-time Kalman filter* was used in the configuration of the networked control system. The classical Kalman filter was designed for the discretized form of (1) (a discrete-time representation approximating a continuous-time system may be found in (Gershon et al. 2005) (Appendix A9, p.212). In the case of the classical Kalman filter, the control signal is piecewise between the sampling moments. In Fig. 2 the time responses have been determined for the sampling period $h = 0.2\,\text{sec}$.

In Fig. 3, the corresponding time responses have been plotted for the case when the sampling period is $h = 1.2\,\text{sec}$. On can see (Fig. 3a) that in this case the estimation errors provided by the hybrid Kalman filter still tend to vanish by contrast with the ones obtained when a classical Kalman filter is used (Fig. 3b).

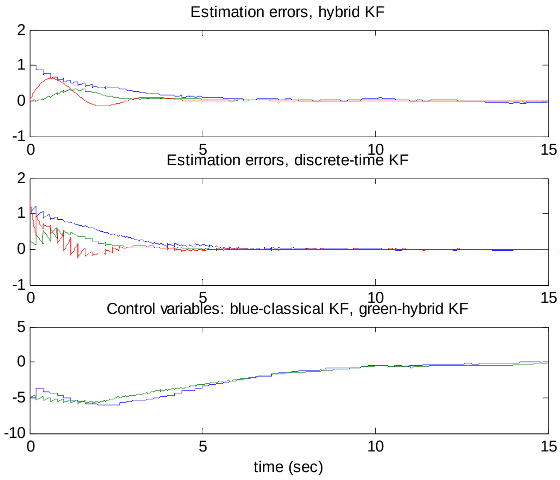

Fig. 2. Estimation errors and control time responses for $\bar{a}_0 = 0$, $h = 0.2\,\text{sec}$.

Moreover, from the above figures it follows that when the sampling period increases, the amplitude of the control signal obtained when a classical Kalman filter is implemented, is significantly larger than the one provided by the hybrid Kalman filter.

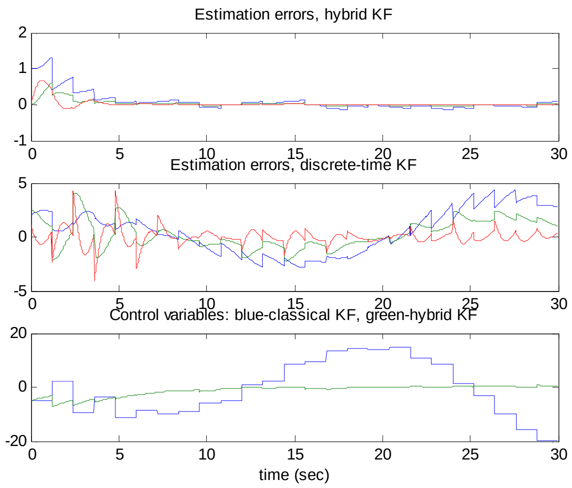

Fig. 3. Estimation errors and control time responses for $\bar{a}_0 = 0$, $h = 1.2\,\text{sec}$.

Case B: $\bar{a}_0 \neq 0$. In this case the multiplicative noise term in (4) is nonzero. It corresponds to the case of Rice model of zero order for the fading communication channel. In Fig. 4, the time responses obtained for $\bar{a}_0 = 0.2$ and $h = 0.7\,\text{sec}$ for the hybrid Kalman filter and for the classical discrete-time Kalman filter (designed ignoring the multiplicative terms) are shown.

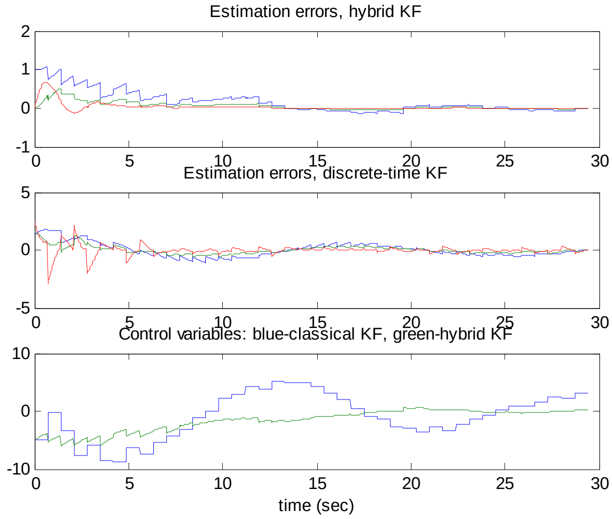

Fig. 4. Estimation errors and control time responses for $\bar{a}_0 = 0.2$, $h = 0.7\,\text{sec}$.

In Fig. 5, the time responses obtained for the same sampling period $h = 0.7\,\text{sec}$ but for $\bar{a}_0 = 0.5$ are shown. Comparing them with the ones from Fig. 4 one can notice the significant influence of the multiplicative noise over the estimation performance. In both cases, the obtained state feedback gain in (15) is $F = \begin{bmatrix} -4.0248 & -1.0237 & -0.7741 \end{bmatrix}$.

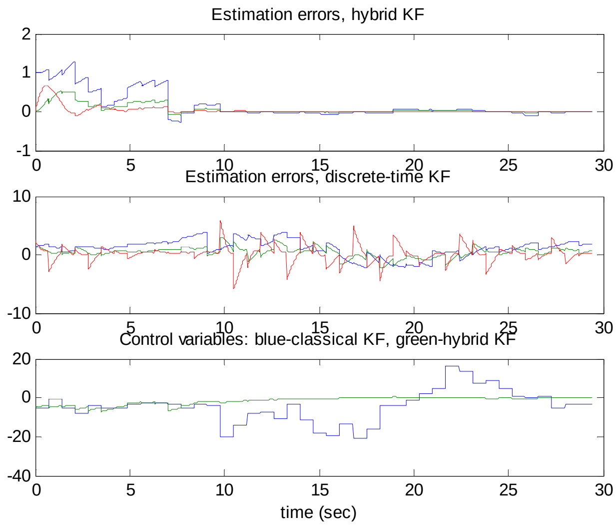

Fig. 5. Estimation errors and control time responses for $\bar{a}_0 = 0.5$, $h = 0.7\,\text{sec}$.

In Fig. 6, the controllers configurations are illustrated for the case of hybrid Kalman filter and for the classical discrete-time Kalman filter, respectively. For the latest, a zero-order-hold element (ZOH) is required in order to determine a piecewise time-constant control for the continuous-time plant. All computations and simulations have been performed using the MATLAB package.

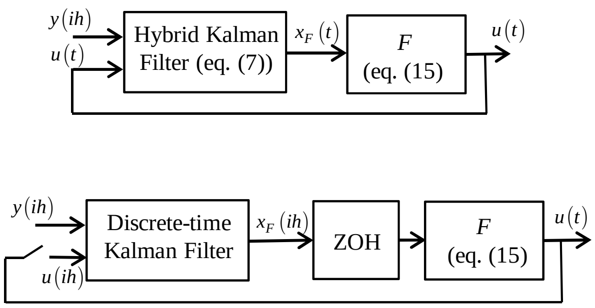

Fig. 6. Controllers configurations. Above: using a hybrid Kalman filter; below: using a discrete-time Kalman filter.

Finally, the control performance was evaluated in terms of the value of the cost function (14). The results are summarized in the tables below. The index value J_1 represents the value of (14) determined when a hybrid Kalman filter is used in the networked control system and J_2 stands for the value obtained when a classical Kalman filter is used together with a piecewise constant control.

Table 1. Case A: $\bar{a}_0 = 0$

h(sec)	J_1	J_2
0.2	$1.3524 \cdot 10^3$	$1.4891 \cdot 10^3$
1.2	$1.6000 \cdot 10^3$	$3.0363 \cdot 10^4$

Table 2. Case B: $\bar{a}_0 \neq 0$, $h = 0.7\,\mathrm{sec}$

$\bar{a}_0$	J_1	J_2
0.2	$1.7580 \cdot 10^3$	$4.7373 \cdot 10^3$
0.5	$1.9734 \cdot 10^3$	$4.0097 \cdot 10^4$

The above numerical results reveal the following aspects:

• in the absence of the multiplicative noise, for small values of the sampling period h both Kalman filters provide similar control performances;

• increasing of the sampling period does not significantly influence the control performance when a hybrid Kalman filter is used in the configuration of the networked control system; the performance severely deteriorates when a classical Kalman filter is used for large sampling periods;

• the multiplicative noise strongly influences the control performance when a classical Kalman filter is used; this performance is less sensitive when a hybrid Kalman filter is implemented in the networked control system.

6. CONCLUDING REMARKS

The paper investigates some aspects concerning the effects of the sampling period increase over the performances of a class of networked control systems with fading communication channels. From practical point of view this is useful mainly in applications where a small sampling period of the measured data is not allowed due to limited real time processing capacity or to the large number of tasks carried out simultaneously by the controller processor. It is shown that using a Ricean model of the fading communication channels, a stochastic model with multiplicative noise of the networked system is obtained. Moreover, due to discrete-time data transmission this model has a hybrid structure including both a continuous-time dynamics and a discrete-time one. The attention is focused on the state estimation between the sampling instants. The implementation of the hybrid filter (7) -(10) requires as in any other Kalman filtering problem, the knowledge of the plant model. The continuous-time estimations provided by the first equation (7) are corrected using the measurements at the sampling moments. The comparative analysis of the numerical results reveals some major benefits of using a hybrid Kalman type filter which dynamics includes both a continuous-time and a discrete-time component, over the classical Kalman filter implemented with zero order holders. Among them, one mentions the estimation performances and the reduction of the control effort. In the future research these results will be developed for Markovian models of the networked control systems with fading communication channels.

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