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ADAPTIVE CONTROL STRATEGIES FOR A CLASS OF RECYCLED DEPOLLUTION BIOPROCESSES

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Abstract: *This paper presents the design and the analysis of some multivariable adaptive nonlinear control strategies for a class of depollution fermentation processes that are carried out in recycle bioreactors. The controller design is based on the input-output linearization technique. The resulted control methods are applied in depollution control problem in the case of the activated sludge process for which dynamical kinetics are strongly nonlinear and not exactly known. More precisely, the problem of adaptive controlling of two reactant concentrations with two control inputs is considered and is illustrated by the mentioned process. Simulation results are included to evaluate the performances of the designed controllers.*

Keywords: *Nonlinear control, Adaptive control, On-line estimation, Multivariable control, Activated sludge process.*

1. INTRODUCTION

In practice, the control of biotechnological processes is an important problem attracting wide attention. The main engineering motivation in applying control methods to such living processes is to improve operational stability and production efficiency. But the use of modern control for these bioprocesses is still low. Two known factors make biotechnological processes control particularly difficult. First, these processes exhibit large nonlinearities, strongly coupled variables and often poorly understood dynamics. Second, the real-time monitoring and on-line measurements of biological process variables, for example, biomass concentration and/or product concentrations, which are essential for control design, is hampered by the lack of cheap and reliable on-line sensors [1]. When biotechnology strategies are used in wastewater treatment, the two mentioned factors require an enhanced modelling effort, modern estimation strategies both for the unmeasured states and the bioprocess kinetics and advanced control strategies. The non-linearity of the bioprocesses and the uncertainty of kinetics impose the adaptive control strategy as a suitable approach [1], [8].

The difficulties encountered in the measurement of the state variables of the bioprocesses impose the use of the so-called “software sensors” [1]. Note that these software sensors are used not only for the estimation of the concentrations but also for the estimation of the kinetic parameters. The interest for the development of software sensors for bioreactor is proved by the big number of the publications and applications in this area [4], [8].

This paper presents the design and the analysis of some multivariable adaptive nonlinear control strategies capable of dealing with the model uncertainties in an adaptive way for a class of depollution fermentation processes that are carried out in recycle bioreactors. The controller is obtained via the input-output linearization technique [5]. The resulted control methods are applied in depollution control problem in the case of a complex biotechnological process, namely, the activated sludge process for which dynamical kinetics are strongly nonlinear and not exactly known. Although for this depollution bioprocess were reported some nonlinear and adaptive control algorithms [3], [10], in this paper, two new multivariable adaptive control algorithms are proposed and analyzed. More exactly, the problem of adaptive controlling of two reactant concentrations with two control inputs is considered and is illustrated by the mentioned process. The algorithm is based on the nonlinear structure of the model. The only information required about the process is the measurements of the state variables and its relative degree. It must be noted that if for the analyzed process some state variables are not accessible, these will be estimated by using an appropriate state observer.

The paper is organized as follows. In Section 2 it is presented the dynamical model of the activated sludge process. In Section 3 firstly it is suggested how to design a multivariable adaptive linearizing control scheme for a class of nonlinear squared systems. After that two new multivariable adaptive control strategies for mentioned process are presented. Simulations results presented in Section 4 illustrate the performances of the proposed control algorithms. Concluding remarks finished the paper.

2. DYNAMICAL MODEL OF ACTIVATED SLUDGE PROCESS

The activated sludge process is an aerobic process of biological wastewater treatment [1], [2]. It is usually operated in at least two interconnected tanks, Fig. 1: an aerator in which the biological degradation of the pollutants takes place and a sedimentation tank (settler) in which the liquid is clarified, that is the biomass is separated from the treated wastewater. Part of the settled biomass is fed back to the bioreactor, while the surplus biomass is removed from the process.

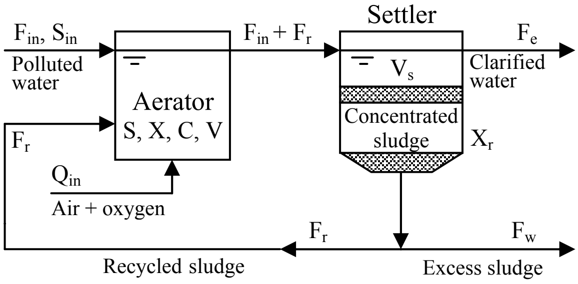

Fig.1. Schematic view of an activated sludge process

The reaction in the aerator may be described by a simple autocatalytic aerobic microbial growth that can be represented by the following scheme:

$$k_1 S + k_2 C \xrightarrow{\varphi} X \tag{1}$$

where S, X and C are respectively the pollutants, the biomass and the dissolved oxygen, φ is the reaction rate and k_1 and k_2 are the yield coefficients. It is worth noting that this reaction scheme is a simply qualitative relation and does not include stoichio-metric considerations.

It is often assumed that the settler work perfectly, i.e. there is no biomass in the overflow of the settler. Then, the dynamics of the plant (aerator + settler) is described by the following mass balance equations:

$$\frac{dS}{dt} = -k_1 \mu X - \frac{F_{in} + F_r}{V} S + \frac{F_{in}}{V} S_{in} \tag{2a}$$

$$\frac{dC}{dt} = -k_2 \mu X - \frac{F_{in} + F_r}{V} C + Q_{in} \tag{2b}$$

$$\frac{dX}{dt} = \mu X - \frac{F_{in} + F_r}{V} X + \frac{F_r}{V} X_r \tag{2c}$$

$$\frac{dX_r}{dt} = \frac{F_{in} + F_r}{V_s} X - \frac{F_r + F_w}{V_s} X_r \tag{2d}$$

where S_{in} is the concentration of influent substrate (*g/l*), Q_{in} is the oxygen feed rate (*g/lh*), X_r is the concentration of the recycled biomass (*g/l*), F_{in}, F_r and F_w are the influent, recycle and waste flow rates (*l/h*), respectively, V and V_s the aerator and settler volumes (l), respectively, and $\mu(\cdot)$ is the specific growth rate (h^{-1}) of reaction φ.

If we define by $\xi = [S\ C\ X\ X_r]^T$ the state vector of (2), $\varphi = \mu(\cdot)X$ the reaction rate, $F = [D_{in}S_{in}\ Q_{in}\ 0\ 0]^T$ the feed rate vector, $Q = [0\ 0\ 0\ 0]^T$ the gaseous outflow rate vector and $K = [-k_1\ -k_2\ 1\ 0]^T$ the yield coefficient matrix, then the dynamical model (2) can be compactly written as:

$$\dot{\xi} = K\varphi(\xi) - D\xi + F - Q \tag{3}$$

This model describes in fact the dynamics of a large class of bioprocesses carried out in stirred tank bioreactors and is referred as *general dynamical state-space model* of this class of bioprocesses [1]. In (3), D stand for the dilution rate matrix and is given by:

$$D = \begin{bmatrix} D_1 & 0 & 0 & 0 \\ 0 & D_1 & 0 & 0 \\ 0 & 0 & D_1 & -D_r \\ 0 & 0 & -D_2 & -D_3 \end{bmatrix} \tag{4}$$

whose entries are defined as:

$$D_{in} = \frac{F_{in}}{V},\quad D_r = \frac{F_r}{V},\quad D_1 = D_{in} + D_r,$$
$$D_2 = \frac{F_{in} + F_r}{V_s},\quad D_3 = \frac{F_r + F_w}{V_s} \tag{5}$$

The oxygen feed rate Q_{in} is usually set equal to the liquid-gas oxygen transfer rate:

$$Q_{in} = k_L a \cdot (C_s - C) \tag{6}$$

where $k_L a$ is the oxygen mass transfer coefficient and C_s the saturation constant. In the following, we shall consider that $k_L a$ is a linear function of the air flow rate W [3]:

$$k_L a = a_0 W,\quad a_0 > 0 \tag{7}$$

The most difficult task for the construction of the dynamical model (3) is the modelling of the reaction kinetic φ. The form of kinetics is complex, nonlinear and in many cases partial or completely unknown. A realistic assumption is that a reaction can take place only if all reactants are presented in the bioreactor. Therefore, the reaction rates are necessarily zero whenever the concentration of one of reactants is zero.

3. CONTROL STRATEGIES

3.1. Problem statement

For the class of bioprocesses described by general dynamical model (3) we consider the problem of controlling $p \geq 2$ outputs, $p < n = \dim(\xi)$, which are linear combinations of the process components ξ by using p control inputs under the following assumptions:

(i) the p control inputs are either feed rates or dilution rates;
(ii) the reaction rates φ_k are time-varying and unknown;
(iii) the matrix K is known;
(iv) the vectors F and Q and the matrix D are known either by measurement or by user's choice;
(v) the process is minimum phase and the relative degree of all the p equations are equal to one.

Let be $y \in \Re^p$ the output vector defined as

$$y = C\xi \tag{8}$$

where C is a $(p \times n)$- matrix of known constants. By using (3) or an appropriately reduced order model formulation, the dynamic of output y can be written as follows:

$$\dot{y} = f(K, \varphi, \xi) + g(\xi)u = A(\xi) + \Phi^T(\xi)\theta + B(\xi)u \tag{9}$$

where $u \in \Re^p$ is the control input vector, $A(\xi) \in \Re^p$ is a state dependent vector and $B(\xi) \in \Re^{p \times p}$ is a quadratic state dependent matrix. θ is the unknown parameter vector ($\dim(\theta) = q$) which contains the kinetics and/or yield coefficients that have been assumed to be unknown and $\Phi^T(\xi)$ is the regressor matrix ($\dim(\Phi^T) = p \times q$). Note that if all the p equations in (9) have relative degree equal to one (as it has been assumed in (v)), then the matrix $B(\xi)$ is nonsingular.

For the process (9), the *control objective* is to make outputs y track specified trajectories denoted $y^* \in \Re^p$. However, the problem is very difficult or even impossible to be solved since the vector θ is assumed to be completely unknown.

▪ *Exactly feedback linearizing control.* Firstly, we consider the ideal case where maximum prior knowledge concerning the process is available, that is the vectors $A(\xi)$ and θ in (9) are assumed completely known and all the state variables are available for on-line measurements. Assume now that for closed loop system (process + controller) we wish to have the following first order linear stable dynamical behavior:

$$\frac{d}{dt}(y^* - y) + \Lambda \cdot (y^* - y) = 0 \qquad (10)$$

with $\Lambda = diag\{\lambda_i\}, \lambda_i > 0,\ i = 1,..., p$. Then, from (9) and (10) we obtain the following *multivariable decoupling feedback linearizing control law*:

$$u = B(\xi)^{-1}\left[\Lambda(y^* - y) - A(\xi) - \Phi^T(\xi)\,\theta + \dot{y}^*\right] \qquad (11)$$

The control law (11) leads to the following linear error model:

$$\dot{e}_t = -\Lambda e_t \qquad (12)$$

where $e_t = y^* - y$ represents the tracking error. It is clear that if $\lambda_i > 0, i = 1,..., p$, the error model (12) has an exponential stable point at $e_t = 0$. Since the order of the system (12) is $p < n$, then $n - p$ other system states are associated with the zero dynamics [5]. These correspond to unobservable modes. It has been shown [11], [12] that if the zero dynamics are asymptotically stable, then the system (3) will be feedback stabilizable by an input-output linearization law of the form (11). A nonlinear system with (un)stable zero dynamics is said to be (non)minimum phase. Usually, we can obtain conditions on the stability of the zero dynamics from the zeros of the linear tangent model transfer function derived at some equilibrium points [3].

▪ *Adaptive control*. If the parameter vector θ in (9) is assumed to be unknown, in the linearizing control law (11) it will be replaced by its corresponding on-line estimate, yielding the following adaptive control law:

$$u = B(\xi)^{-1}\left[\Lambda(y^* - y) - A(\xi) - \Phi^T(\xi)\,\hat{\theta} + \dot{y}^*\right] \qquad (13)$$

Since in (9) the unknown parameter appears linearly, the on-line estimate of θ can be performed by using appropriately techniques. It must be noted also that if in (11) appear state variables that are not accessible, these will be replaced either by some auxiliary variables or by their estimates performed by using an appropriately state observer.

3.2. Adaptive control strategies of activated sludge process

For the activated sludge process, the main *control objective* is to maintain the wastewater degradation at a desired level despite load variations and substrate concentration variations. As in any aerobic fermentation process, proper aeration is crucial to process efficiency; at a result an adequate control of dissolved oxygen concentration in aerator is very important.

Then the controlled variables are concentrations of pollutant S and dissolved oxygen C inside the aerator, that is $y = [S \;\; C]^T$. Regarding the input control variables we will analyze two cases.

▪ *Case 1*. Consider the case when the manipulated variables are the influent flow F_{in} and the air flow rate W that is $u = [F_{in} \;\; W]^T$, while the recycle flow rate F_r is maintained constant.

▪ *Case 2*. A more realistic case is that when both the flow rate and concentration of influent substrate are variable, the manipulated variables being now the recycle flow rate F_r and the air flow rate W, that is $u = [F_r \;\; W]^T$.

Consequently, in both cases, we have a multivariable control problem with two inputs and two outputs. So, in Case 1: $u_1 = [F_{in} \;\; W]^T$ and $y = [S \;\; C]^T$, while in Case 2: $u_2 = [F_r \; W]^T$ and $y = [S \; C]^T$, respectively.

Note. In the following we will use the indexes 1 and 2 for those elements (vectors and matrices) which have different structures for these two cases.

From (2) it can be seen that the relative degrees of both controlled variables S and C, respectively are equal to one. Then, the expressions of $A(\xi), \Phi^T(\xi)$, θ and $B(\xi)$ in (9) corresponding to the two analyzed cases are readily obtained from model (2), (4) as:

$$A_1(\xi) = \begin{bmatrix} -D_r S \\ -D_r C \end{bmatrix},\ \Phi^T(\xi) = \begin{bmatrix} -k_1 \\ -k_2 \end{bmatrix},\ \theta = \mu X,$$

$$B_1(\xi) = \begin{bmatrix} (S_{in} - S)/V & 0 \\ -C/V & a_0(C_s - C) \end{bmatrix} \tag{14}$$

and

$$A_2(\xi) = \begin{bmatrix} D_{in}(S_{in} - S) \\ -D_{in}C \end{bmatrix},\ \Phi^T(\xi) = \begin{bmatrix} -k_1 \\ -k_2 \end{bmatrix},$$

$$\theta = \mu X,\ B_2(\xi) = \begin{bmatrix} -S/V & 0 \\ -C/V & a_0(C_s - C) \end{bmatrix} \tag{15}$$

$B_1(\xi)$ is nonsingular and so invertible as long as $S_{in} - S$ and $C_s - C$ are different from zero, and $B_2(\xi)$ is nonsingular and so invertible as long as S and $C_s - C$ are different from zero.

For this process we consider that the specific growth rate μ is a Monod-type model, i.e. [10]:

$$\mu(S,C) = \mu_{\max} \frac{S}{K_S + S} \cdot \frac{C}{K_C + C} \tag{16}$$

In the ideal case when the process is completely known, it can be shown, after long but rather straightforward calculations applied to linear approximations of the model (2) that in the both cases, the process (2) is minimum phase. Under these conditions, the exactly linearizing controller (11) with $y^* = [S^*\ C^*]^T$ is particularized as follows:

$$u_1 = B_1(\xi)^{-1}\left[\Lambda(y^* - y) - A_1(\xi) - \Phi^T(\xi)\theta + \dot{y}^*\right]$$

i.e.

$$\begin{bmatrix} F_{in} \\ W \end{bmatrix} = \begin{bmatrix} (S_{in} - S)/V & 0 \\ -C/V & a_0(C_s - C) \end{bmatrix}^{-1} \times$$
$$\times\left(\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\begin{bmatrix} S^* - S \\ C^* - C \end{bmatrix} + \begin{bmatrix} D_r S \\ D_r C \end{bmatrix} + \begin{bmatrix} k_1\mu X \\ k_2\mu X \end{bmatrix}\right) \tag{17}$$

in the Case 1 and,

$$u_2 = B_2(\xi)^{-1}\left[\Lambda(y^* - y) - A_2(\xi) - \Phi^T(\xi)\theta + \dot{y}^*\right]$$

i.e.

$$\begin{bmatrix} F_r \\ W \end{bmatrix} = \begin{bmatrix} -S/V & 0 \\ -C/V & a_0(C_s - C) \end{bmatrix}^{-1}\left(\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\times\right.$$
$$\left.\times\begin{bmatrix} S^* - S \\ C^* - C \end{bmatrix} - \begin{bmatrix} D_{in}(S_{in} - S) \\ -D_{in}C \end{bmatrix} + \begin{bmatrix} k_1\mu X \\ k_2\mu X \end{bmatrix}\right) \tag{18}$$

in the Case 2.

Now we shall develop two adaptive control algorithms under the following realistic conditions: the specific growth rate $\mu(\cdot)$ is time-varying and unknown, the concentrations of biomasses X and X_r are not accessible, and the only measurements available on-line are: the output pollution level, the dissolved oxygen concentration and the influent substrate concentration S_{in}. Under these conditions an *adaptive controller* is obtained as follows.

The unmeasured variables X and X_r can be estimated by using an asymptotic state observer [2], [10]. For that, let us define the auxiliary variables z_1, z_2 and z_3 as follows:

$$z_1 = k_1 X + S,\ z_2 = k_2 X + C,\ z_3 = X_r \tag{19}$$

whose dynamics derived from model (2), (4) are independent of the unknown kinetics:

$$\frac{d}{dt}\begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_3 \end{bmatrix} = \begin{bmatrix} -D_1 & 0 & k_1 D_r \\ 0 & -D_1 & k_2 D_r \\ 0 & D_2/k_2 & -D_3 \end{bmatrix}\begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_3 \end{bmatrix} + \begin{bmatrix} D_{in}S_{in} \\ a_0 W(C_s - C) \\ -D_2 C/k_2 \end{bmatrix} \tag{20}$$

Then, from (19) and (20) the estimates of X and X_r are given by:

$$\hat{X} = (\hat{z}_2 - C)/k_2,\ \hat{X}_r = \hat{z}_3 \tag{21}$$

The stability of the state observer (20), (21) depends on the sability of the matrix:

$$\begin{bmatrix} -D_1 & 0 & k_1 D_r \\ 0 & -D_1 & k_2 D_r \\ 0 & D_2/k_2 & -D_3 \end{bmatrix} \tag{22}$$

It can be straightforward shown that if the waste flow rate $F_w \neq 0$, all the three eigenvalues of the matrix (22) have their real parts strictly negative. It can be concluded that the state observer (20), (21) is asymptoticaly stable.

The unknown specific growth rate μ in (16) can be rewritten as follows:

$$\mu(S,C) = \theta \cdot SC \tag{23}$$

where θ is an unknown positive function of process components which will be on-line estimated by using an appropriately parameter estimator.

Using (20), (21) and (23), for the Case 1, the *adaptive* version of the controller (17) is given by:

$$\begin{bmatrix} F_{in} \\ W \end{bmatrix} = \begin{bmatrix} (S_{in} - S)/V & 0 \\ -C/V & a_0(C_s - C) \end{bmatrix}^{-1}\left(\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\times\right.$$
$$\left.\times\begin{bmatrix} S^* - S \\ C^* - C \end{bmatrix} + \begin{bmatrix} D_r S \\ D_r C \end{bmatrix} + \begin{bmatrix} SC(\hat{z}_1 - S)\hat{\theta} \\ SC(\hat{z}_2 - C)\hat{\theta} \end{bmatrix}\right) \tag{24}$$

Using also (20), (21) and (23), for the Case 2, the *adaptive* version of the controller (18) is given by:

$$\begin{bmatrix} F_r \\ W \end{bmatrix} = \begin{bmatrix} -S/V & 0 \\ -C/V & a_0(C_s - C) \end{bmatrix}^{-1} \left(\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} S^* - S \\ C^* - C \end{bmatrix} - \begin{bmatrix} D_{in}(S_{in} - S) \\ -D_{in}C \end{bmatrix} + \begin{bmatrix} SC(\hat{z}_1 - S)\hat{\theta} \\ SC(\hat{z}_2 - C)\hat{\theta} \end{bmatrix} \right) \tag{25}$$

Both in (24) and (25) the variables $\hat{z}_1$ and $\hat{z}_2$ are calculated via equations (20) and $\hat{\theta}$ is updated by using an observer-based parameter estimator [8] that here is particularized as follows:

$$\dot{\hat{S}} = -SC(\hat{z}_1 - S)\hat{\theta} + D_{in}(S_{in} - S) - D_r S + \omega_1(S - \hat{S}) \tag{26a}$$

$$\dot{\hat{C}} = -SC(\hat{z}_2 - C)\hat{\theta} - D_{in}C - D_r C + a_0 W(C_s - C) + \omega_2(C - \hat{C}) \tag{26b}$$

$$\dot{\hat{\theta}} = -\gamma_1 SC(\hat{z}_1 - S)(S - \hat{S}) - \gamma_2 SC(\hat{z}_2 - C)(C - \hat{C}) \tag{26c}$$

where $\omega_1, \omega_2, \gamma_1$ and γ_2 are positive design parameters to control the stability and convergence of the estimator.

To prove the stability and convergence properties of the parameter estimator (26a)-(26c) we define the estimation errors as

$$\tilde{S} = S - \hat{S}\,;\ \tilde{C} = C - \hat{C} \tag{27}$$

and the tracking error as

$$\tilde{\theta} = \theta - \hat{\theta} \tag{28}$$

It is easy to show that the errors (27) and (28) verify the following time varying linear system:

$$\dot{\tilde{e}}(t) = \Omega\tilde{e}(t) + KH(\hat{z})\tilde{\theta}(t) \tag{29a}$$

$$\dot{\tilde{\theta}}(t) = -KH(\hat{z})\Gamma\tilde{e}(t) + \dot{\theta}(t) \tag{29b}$$

where

$$\tilde{e} = \begin{bmatrix} \tilde{S} \\ \tilde{C} \end{bmatrix},\ \Omega = \begin{bmatrix} -\omega_1 & 0 \\ 0 & -\omega_2 \end{bmatrix},\ \Gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix},$$

$$KH(\hat{z}) = \begin{bmatrix} -SC(\hat{z}_1 - S) & 0 \\ 0 & -SC(\hat{z}_2 - C) \end{bmatrix} \tag{30}$$

Lemma 1. If Ω is a $(n \times n)$ - Hurwitz matrice, Γ is a $(n \times n)$ - positive definite and symmetric matrice so that $\Omega^T\Gamma + \Gamma\Omega = -P$, where P is a $(n \times n)$ - positive definite and symmetric matrice, $\dot{\theta}(t) = 0$, that is $\theta(t)$ is a very slowly time variable function, and if $\|KH(\hat{z})\|$ and $\left\|\frac{d}{dt}(KH(\hat{z}))\right\|$ are uniformly bounded, and the regressor matrice $KH(\hat{z})$ is persistently exciting, then for the system (29) the point $(\tilde{e}, \tilde{\theta})$ is an exponential stable steady point.

Definition 1. The $(n \times r)$ - dimensional matrice $KH(\hat{z})$ whose entries are bounded and time derivable functions is persistently exciting, if there exist the positive constants α and β, so that

$$\int_t^{t+\beta} (KH(\hat{z}))(KH(\hat{z}))^T d\tau \ge \alpha I_n > 0, \forall t \ge 0 \tag{31}$$

Proof of the Lemma 1. First, we show that:

$$\lim_{t\to\infty} \|\tilde{e}(t)\| = 0\,,\ \forall \tilde{e}(0) \tag{32}$$

Consider the following candidate Lyapunov function:

$$V = \tilde{e}^T \Gamma \tilde{e} + \tilde{\theta}^T \tilde{\theta} \tag{33}$$

The time derivative of V along the systems (29a), (29b), in which is used the Lyapunov equation defined in Lemma 1, will be:

$$\begin{aligned} \dot{V} &= \tilde{e}^T(\Omega^T\Gamma + \Gamma\Omega)\tilde{e} + \tilde{\theta}^T(KH(\hat{z}))^T\Gamma\tilde{e} \\ &+ \tilde{e}^T\Gamma(KH(\hat{z}))\tilde{\theta} - \tilde{e}^T\Gamma(KH(\hat{z}))\tilde{\theta} \\ &- \tilde{\theta}^T(KH(\hat{z}))^T\Gamma\tilde{e} = -\tilde{e}^T P \tilde{e} \le 0 \end{aligned} \tag{34}$$

Using the Basic Lyapunov Theorem [12], from (34) one obtains that for the system (29a), (29b) the point $(\tilde{e}, \tilde{\theta})$ is an uniformly stable steady point. Then, from (33) and (34) one derives that $\|\tilde{e}(t)\|$ and $\|\tilde{\theta}(t)\|$ are uniformly bounded for any $t \ge 0$. Now, since $\|KH(\hat{z})\|$ is uniformly bounded it follows that $\|\dot{\tilde{e}}(t)\|$ is uniformly bounded also.

Since $V(t)$ is an uniformly bounded decreasing function for any $t \ge 0$, then from (34) one obtains that:

$$\lim_{t\to\infty} \int_0^t \tilde{e}^T(\tau) P \tilde{e}(\tau) d\tau = -\lim_{t\to\infty} \int_0^t \dot{V}(\tau) d\tau = V(0) - V(\infty) < \infty \tag{35}$$

From (35) one obtains that [7]:

$\lim_{t\to\infty}\|\tilde{e}(t)\| = 0$, $\forall \tilde{e}(0)$.

To prove that $\lim_{t\to\infty}\|\tilde{\theta}(t)\| = 0$, $\forall \tilde{\theta}(0)$ one can follow the Proposition 5.1 from [8].

Finally, one can derive that $(\tilde{e}, \tilde{\theta})$ is an uniformly asymptotic stable steady point of the system (29a), (29b). Since this system is a linear one, it follows that the point $(\tilde{e}, \tilde{\theta})$ is an exponential stable steady point.

A bloc diagram of the multivariable adaptive system, in the Case 2, is shown in Fig. 2.

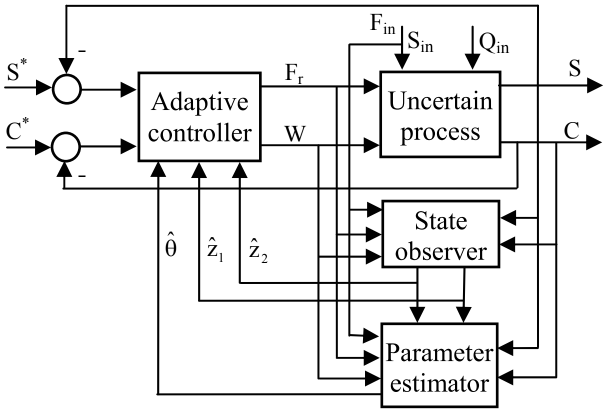

Fig. 2. A bloc diagram of the multivariable adaptive system

4. SIMULATION RESULTS

The performances of the two above multivariable adaptive controllers have been tested by performing extensive simulation experiments.

The numerical values of the kinetic parameters are [10]: $\mu^0_{max} = 0.2\ h^{-1}$, $K_S = 75\ mg/l$, $K_C = 2\ mg/l$, and the model parameters are: $k_1 = 1.2$, $k_2 = 0.565$, $a_0 = 0.017\ m^{-3}$, $C_s = 10\ mg/l$, $V = 380\ m^3$, $V_s = 256\ m^3$.

The gains and the tuning parameters of the two adaptive controllers have been set to the following values: $\lambda_1 = 25$, $\lambda_2 = 7.5$, $\omega_1 = \omega_2 = -50$, $\gamma_1 = \gamma_2 = 2.0\text{e-}6$.

The auxiliary variables z_1, z_2, z_3 and the estimated parameter θ have been initialized as follows: $\hat{z}_1 = 9.6\ g/l$, $\hat{z}_2 = 4.5\ g/l$, $\hat{z}_3 = 90\ g/l$, $\hat{\theta} = 7.5\text{e-}4$.

- *Case 1*. The system's behavior was analyzed assuming that the pollutant concentration S_{in} acts as a perturbation of the form presented in Fig. 2, and the kinetic coefficient μ_{max} is time-varying as:

$$\mu_{max}(t) = \mu^0_{max}(1 - 0.05\sin(\pi t/11)) \qquad (36)$$

The behavior of closed-loop system using multivariable adaptive controller (24) by comparison to the exactly linearizing law (17) for $F_w = 0.25\ m^3/h$ and $F_r = 8\ m^3/h$ is presented in Figs. 4a-4d. Fig. 4e shows the evolution of actual time-varying specific growth rate θ and its estimate $\hat{\theta}$, respectively.

From graphics in Figs. 4a-4d it can be seen that the behavior of adaptive system is good, being very close to the behavior of closed loop system in the ideal case when the process model in completely known. Note also the regulation properties and ability of the controller to maintain the controlled outputs S and C close to their desired values (low level for S^*) despite the time varying parameters and the process uncertainties.

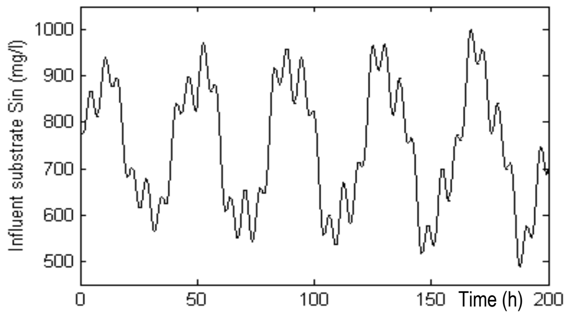

Fig. 3. Variation of perturbation S_{in}

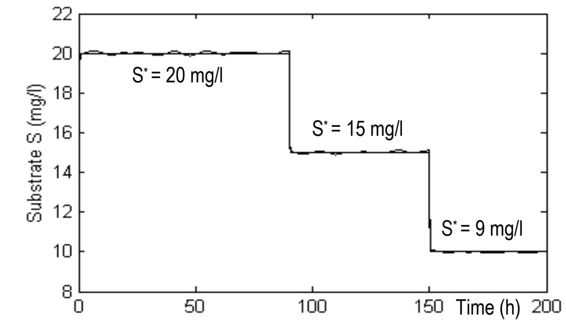

Fig. 4a. Controlled output S

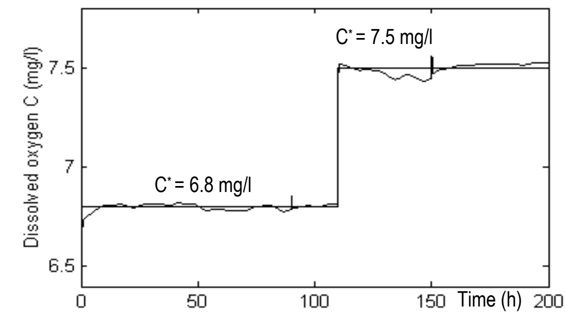

Fig. 4b. Controlled output C

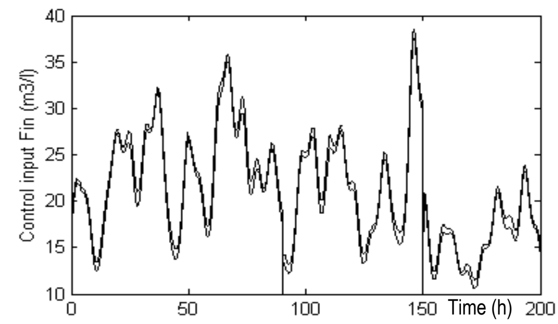

Fig. 4c. Control input F_{in}

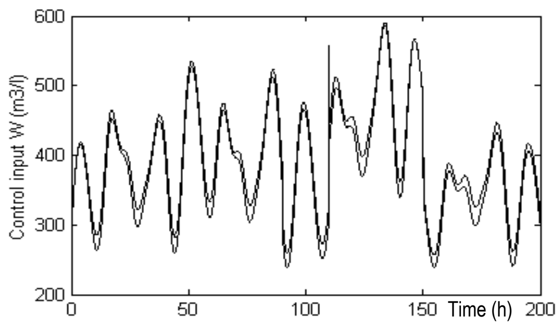

Fig. 4d. Control input W

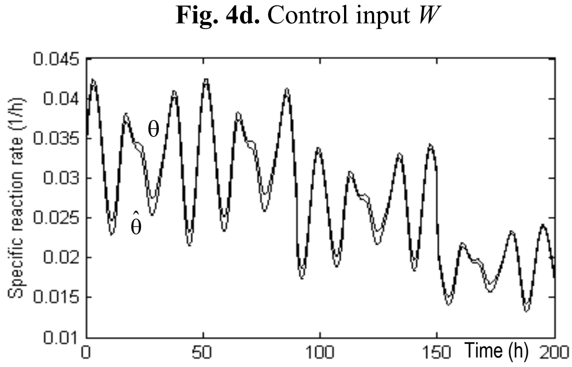

Fig. 4e. Actual and estimated parameter $\hat{\theta}$

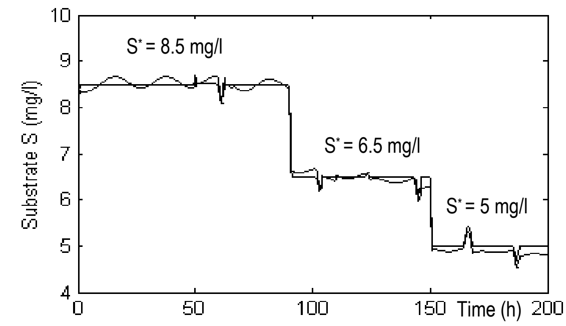

Fig. 5a. Controlled output S

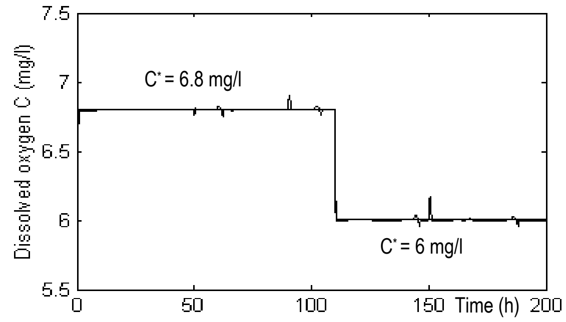

Fig. 5b. Controlled output C

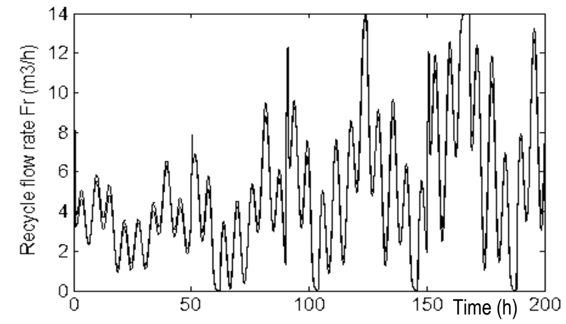

Fig. 5c. Control input F_r

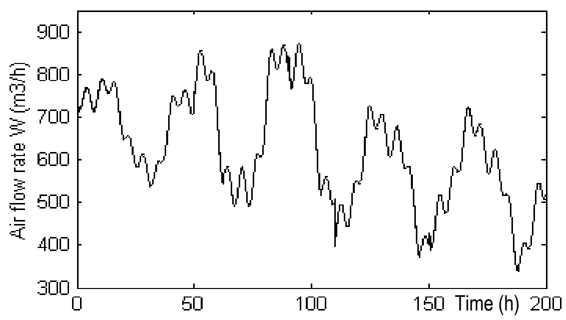

Fig. 5d. Control input W

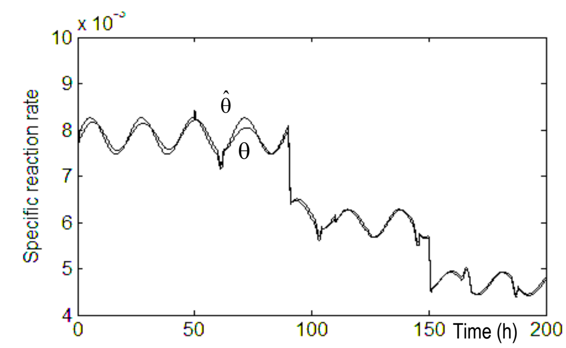

Fig. 5e. Actual and estimated parameter $\hat{\theta}$

One can observe a good behaviour both of state observer (20), (21) and parameter estimator (26).

- *Case 2*. The behavior of closed-loop system using multivariable adaptive controller (25) by comparison to the exactly linearizing law (18) is presented in Figs. 5a-5d. The system's behavior was analyzed assuming the same conditions both for perturbation S_{in} and the kinetic coefficient $\mu_{\max}$ as in the Case 1.

Much more, in this case, the influent flow rate F_{in} (m^3/h) is also time varying as:

$$F_{in}(t) = 40 \cdot (1 - 0.025\sin(\pi t/80) + 0.025\cos(\pi t/10)) \tag{37}$$

The graphics in Fig. 5e show the evolution of actual time-varying specific growth rate θ and its estimate $\hat{\theta}$, respectively.

The graphics in Figs. 5a-5d show a good behavior of this closed loop adaptive system by comparison to the behavior of closed loop system when the control law is exactly one.

Note the ability of the adaptive controller (25) to maintain the pollutant (controlled output) S at a very low level ($S^* =$ 5 mg/l) despite the very high load variations, both for S_{in} and F_{in}, and the time variation of the process parameters.

One can observe also a good behavior both of the proposed state observer (20), (21) and parameter estimator (26).

5. CONCLUSIONS

Two multivariable adaptive nonlinear control strategies have been designed and analyzed for a class of depollution fermentation processes that are carried out in recycle bioreactors. The approach has been illustrated on the activated sludge process. The stability and the convergence properties of the proposed parameter estimator were proved through Lyapunov's method.

Since, in most situations, the kinetic parameters are uncertain and time varying and the process non-linearities are not exactly known and, much more, not all the state variables are on-line measurable, it can be concluded that adaptive controllers are the only viable alternative.

The simulation results of applications of the two designed multivariable adaptive controllers confirm the efficiency of the two control schemes, especially in the second case.

ACKNOWLEDGMENT

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