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A New Design Method of Mismatched Smith Predictor (optimisation approach)

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Abstract: This paper presents a new design method of mismatched Smith predictor for delayed systems. It concerns an optimal controller parameters tuning and a system modelling. The method is based on the resolution of a multi-objective optimisation problem which consists in finding the parameters of a controller and a system model that minimises two objective functions: the modelling and the tracking errors. As the two objectives functions are minimised simultaneously, therefore both the controller and the model parameters act together for a fast convergence of the optimal problem. The second advantage of the method is that both the controller and the system model structures can be chosen by the user according to the real plant dynamic. The method is applied to several typical delayed processes and the results are compared with those of other methods in order to illustrate its efficiency.

Keywords: Delay systems, Optimal control, PID tuning.

1. INTRODUCTION

The control of dead time systems still the subject of many researches until today, this is due to the fact that many industrial processes (especially petrochemical ones) exhibit an inherent time delay. The well-known Smith predictor method is successfully applied on the delayed systems in the case of perfect match modelling of the actual (real) process. Unfortunately in the mismatched case, the method suffer from a sensitivity problem which can lead to instability and/or very poor closed loop performances (Zhang and Xu (2001)). Recently, the mismatched Smith predictor case has been the focus of many important research works, such (Zhang and Xu (2001), Wang et al. (2000), Zhang et al. (2002), Lee et al. (1999), Sourdille and O'Dwyer (1999) and Kristiansson and B. Lennartson (2001)) In (Zhang and Xu (2001)), an analytical design method of the controller is proposed, but over its simplicity, the efficiency of this method depends strongly on the good choice of a tuning parameter (scalar called λ). Another strategy given by (Wang et al. (2000)), (Zhang et al. (2002)) proposes a new control scheme, using a simple primary controller and a deliberately mismatched model in order to enhance the performances of the actual (real) system. The mentioned methods are organised in two steps: the first one is the process modelling and the second one is the design of a primary controller based on the free part of the model obtained in the first step. This paper will propose a controller designing method, based on the Smith predictor control scheme of (Wang et al. (2000), via the resolution of a multi-objective optimisation problem. The idea is to find the parameters of both the model and the controller in the same time that minimises the modelling and the tracking errors. These two objective functions are chosen because in the Smith predictor classical method, the controller is designed in the inner loop for the free delay process model, so more the modelling errors are small, better will be the performances for the outer loop including the real plant. In the other hand, the minimisation of the tracking error is the main goal of any control law. The first advantage of the method presented in this paper, is that the two objectives functions are minimised simultaneously so therefore, both the controller and the model parameters act together to find an optimal solution (the best performances). The second important advantage, is that the controller and the model structures are flexible and can be imposed by the user; this allows more possibilities to solve the optimisation problem and therefore to reach the desired feedback performances.

2. OVERVIEW OF THE SMITH PREDICTOR CONTROL SCHEME

The so-called Smith predictor (Smith (1959)), is the most commonly used controller design method for the systems with time delay; it is based on the following control scheme:

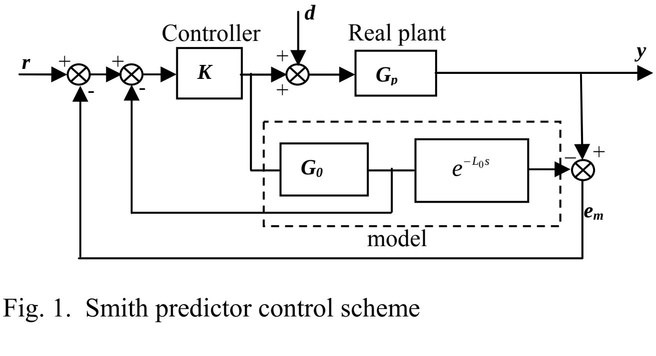

Fig. 1. Smith predictor control scheme

With: r is the reference signal, d is the disturbance signal and e_m is the modelling error.

As one can see, the primary controller ***K*** is designed based on the nominal delay free part G_0 of the plant model: $G_0 e^{-L_0 s}$, this prevents all difficulties due to the delay term (especially the non-minimum phase effect).

Let us write out the transfer function from the output y to the input r:

$$\frac{y}{r} = \left(\frac{K(s)G_p(s)}{1 + K(s)(G_0(s) - G_0(s)e^{-L_0 s} + G_p(s))} \right) \tag{1}$$

We denote that the characteristic equation contains the modelling error term:

$$e_m(s) = G_0(s) - G_0(s)e^{-L_0 s} \tag{2}$$

This means that a minor modelling error (perfect match case), leads to a delay free characteristic equation which avoid all difficulties that the delay term can introduce. Unfortunately, a perfect model of the real plant is rarely obtained; this is for what the smith predictor method suffers from the poor robustness performances.

3. THE PROPOSED METHOD DESCRIPTION

In order to resolve the robustness problem of the Smith predictor method, we use the following modified smith predictor control scheme introduced by (Wang et al. (2000)):

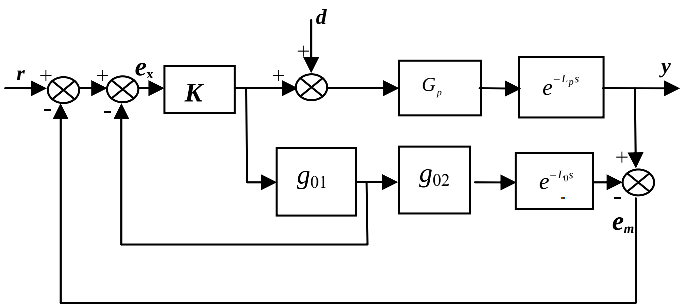

Fig. 2. Modified Smith predictor control scheme

With: e_x is the tracking error.

As we can see in this figure, the model delay free part G_o is partitioned in two parts as follow:

$$G_0 = g_{01} g_{02} \tag{3}$$

With g_{01} is chosen as a first order transfer function:

$$g_{01} = \frac{K_0}{T_0 s + 1} \tag{4}$$

This choice is particularly interesting, because the controller will be designed based on a first order transfer function model which considerably facilitates the design procedure. Reminds g_{02} which can be chosen as follow:

$$g_{02}(s) = \frac{1}{(T_0 s + 1)^{n-1}} \tag{5}$$

With: n is the order of the transfer function of the free part of the model G_o, it is generally chosen n = 2 (most of real processes can be modelled by a second order plus delay transfer function).

3.1 Optimisation problem formulation

The main goal of any controller design procedure is to minimise as possible the tracking error. As the controller is designed based on the first order part of the model transfer function $g_{01}(s)$ (figure.2), let us write out the controller input signal in the frequency plan:

$$e_x(s) = \frac{1}{1 + K(s)g_{01}(s)} \tag{6}$$

If we suppose that the modelling error e_m is minimal (optimal modelling), then for a unit step input r, $e_x(s)$ is given by:

$$e_x(s) = \frac{1}{s}\left(\frac{1}{1 + K(s)g_{01}(s)} \right) \tag{7}$$

The problem is then: how to find the optimal controller parameters that minimises the tracking error, this is a frequency domain nonlinear optimisation problem. The resolution of this optimisation problem can be much more efficient, if we find a model of transfer function that the frequency dynamics meets as closely as possible the real process ones, because as we said before: a minimal modelling error considerably simplify the closed loop transfer function ($e_m \approx 0$ see figure.2), helping in the end to find the optimal controller parameters that minimise the tracking error.

In order to illustrate the method, let us use a filtered PID controller structure given by:

$$K(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \tag{8}$$

The idea is then to find simultaneously the optimal model transfer function parameters (K_0, T_0 and the delay L_0) and the PID controller parameters (K_p, T_i, T_d, the filter coefficient is taken by default N = 10), which minimises both the modelling error $e_m(s)$ and the tracking errors $e_x(s)$. Actually, it's like if we had transformed the mismatch Smith predictor problem in a perfect match one, enhancing of course the closed loop transfer function stability and performances.

We note that we can use other transfer function models (if the given ones are not appropriate for the system), but we must always put the first part $g_{01}(s)$ as a first order transfer function, we can also choose any controller structures (PID, PI or PD... or others), if the PID is not appropriate for the system. This allows more flexibility and gives more possibilities to the controller designer.

The optimisation problem described below is a multi-objective frequency domain optimisation problem (Eckart et al. (2003)), (Marler and Arora (2004)). Mathematically, it can be assessed like following:

$$\min_{K_0,T_0,L_0,K_p,T_i,T_d} \begin{cases} e_m(jw) \\ e_x(jw) \end{cases} \tag{9}$$

This approach guaranties a minimal tracking error and also the robustness of the closed loop transfer function, thanks to the minimal modelling error.

The optimisation problem (9) can be solved by different software's as MATLAB, SCILAB etc.

3.2 Generalisation of the method

As we said before, the method can be generalised for any delayed systems, using any controller structures, we have just to adapt the system model according to the design methodology described above, i.e.: decomposition of the system model in two parts, with the first one in the first order transfer function form). Thus the optimisation problem (9) can be generalised as follow:

$$\min_{(system\ \ parameters),(controller\ \ parameters)} \begin{cases} e_m(jw) \\ e_x(jw) \end{cases} \tag{10}$$

This type of optimal problems can be resolved using recent software's packages, like the optimisation toolbox of MATLAB®.

4. ILLUSTRATIVE EXAMPLES

In this section, the proposed method will be applied to three typical delayed processes, using a PID controller structure. The optimisation toolbox (Optimisation toolbox (2001)), (especially minimax routine) of MATLAB® software, is used to find both the optimal model and the controller parameters, by resolving the optimisation problem (9). The simulation results will be compared with those of Wang's method (Wang et al. (2000)).

The three chosen delayed processes presents particular difficulties for the control problem, indeed:

The first process is represented by a five order plus delay transfer function given by:

$$G_1(s) = \frac{1}{(s+1)^5} e^{-4s} \tag{11}$$

The second one is represented by an unstable zero five order plus delay transfer function, given by:

$$G_2(s) = \frac{-s+1}{(s+1)^5} e^{-2s} \tag{12}$$

The third one is a multiple lag process, given by:

$$G_3(s) = \frac{1}{(s+1)^{10}} \tag{13}$$

Note that Wang's method employs a second order plus delay systems models, respectively given by:

$$G_{01}(s) = \frac{1}{(1.64s+0.999)^2} e^{-5.79s} \tag{14}$$

$$G_{02}(s) = \frac{1}{(1.46s+0.999)^2} e^{-5.07s} \tag{15}$$

$$G_{03}(s) = \frac{1}{(2.43s+0.995)^2} e^{-5.39s} \tag{16}$$

And the controllers are respectively given by:

$$K_{w1}(s) = \frac{5.03s+3.06}{s(s+2.48)} \tag{17}$$

$$K_{w2}(s) = \frac{5.85s+4}{s(s+2.83)} \tag{18}$$

$$K_{w3}(s) = \frac{6.25s+2.56}{s(s+2.29)} \tag{19}$$

As explained before, the application of the proposed method gives an optimal second order plus delay models, respectively given by:

$$G_{m01}(s) = \frac{1}{(1.606s+1)^2} e^{-5.86s} \tag{20}$$

$$G_{m02}(s) = \frac{1}{(1.718s+1)^2} e^{-4.62s} \tag{21}$$

$$G_{m03}(s) = \frac{1}{(1.25s+1)^2} e^{-5.00s} \tag{22}$$

The obtained optimal PID controllers are respectively given by:

$$K_{pid1}(s) = \frac{1.649s^2 + 16s + 10}{s(0.1s + 1)} \tag{23}$$

$$K_{pid2}(s) = \frac{16.49s^2 + 35.18s + 17.82}{s(0.01s + 1)} \tag{24}$$

$$K_{pid3}(s) = \frac{31s^2 + 10.13s + 1.25}{s(0.1s + 1)} \tag{25}$$

4.1. Time responses results

4.1.1 Step time and disturbance rejection

The following figures, shows the step time responses and the disturbance rejections of 20% of a unit step setpoint (for the first and the second systems, introduced at time: t = 40 sec, for the third one at time: t = 50 sec), with the proposed method compared to Wang's one:

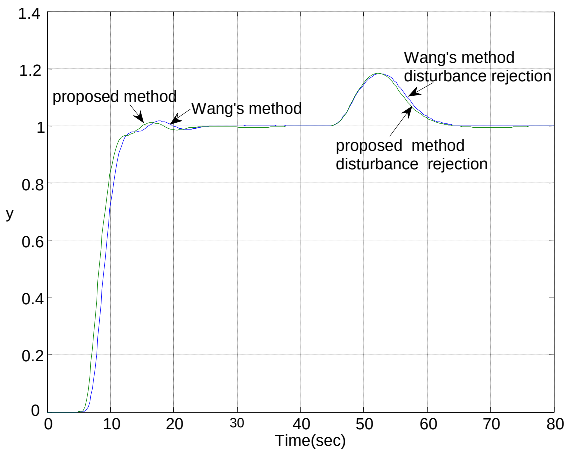

Fig. 3. Step time response and disturbance rejection of the first system with the two methods

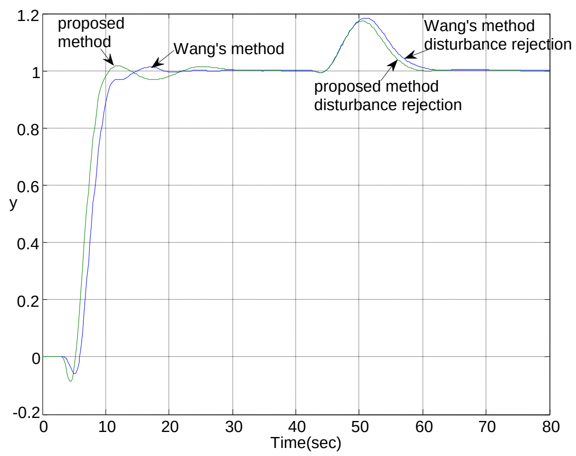

Fig. 4. Step time response and disturbance rejection of the second system with the two methods

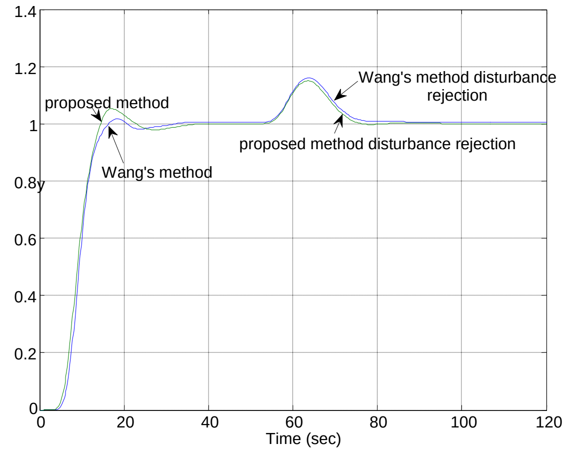

Fig. 5. Step time response and disturbance rejection of the Third system with the two methods

4.1.2 Tracking and modelling errors dynamics

The following figures shows the time domain dynamics behaviour of the tracking error (e_x) and the modelling error (e_m), for the three systems with the proposed method compared to the Wang's one

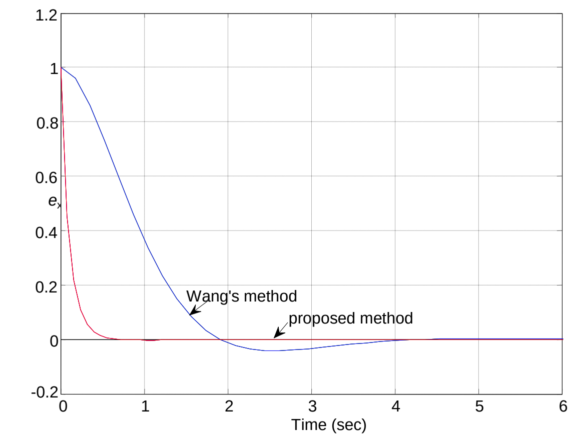

Fig. 6. Tracking error dynamics of the first system with both methods

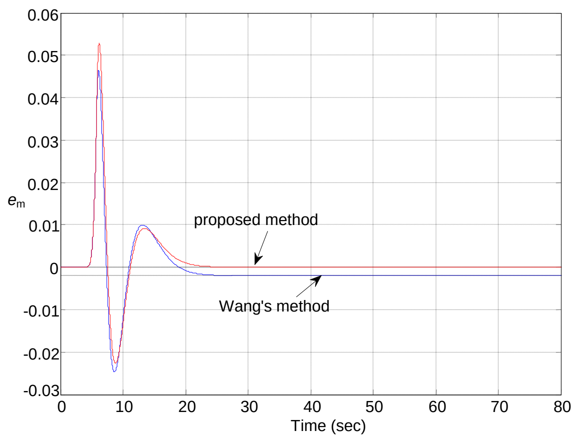

Fig.7 . Modelling error dynamics of the first system with both methods

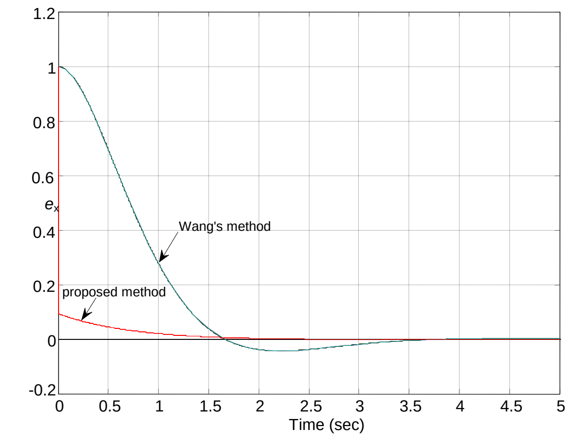

Fig.8 . Tracking error dynamics of the second system with both methods

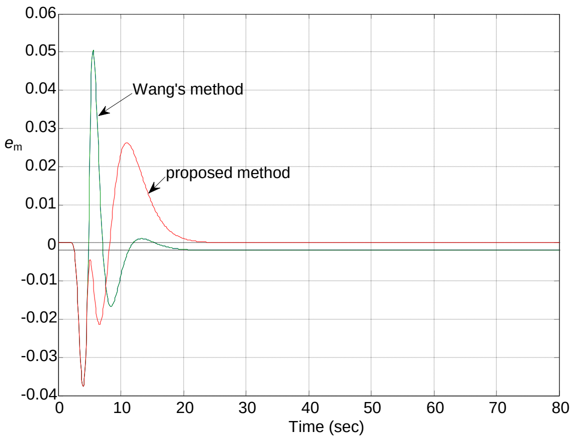

Fig. 9. Modelling error dynamics of the second system with both methods

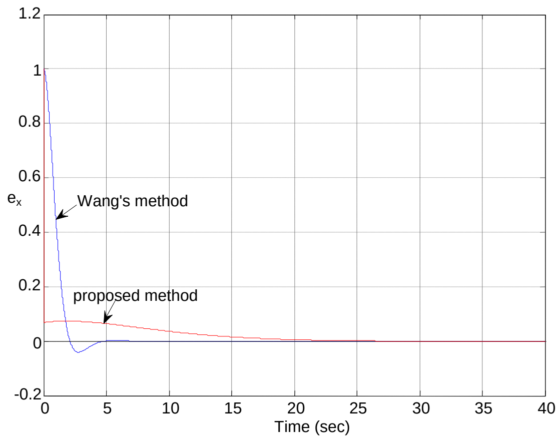

Fig. 10. Tracking error dynamics of the third system with both methods

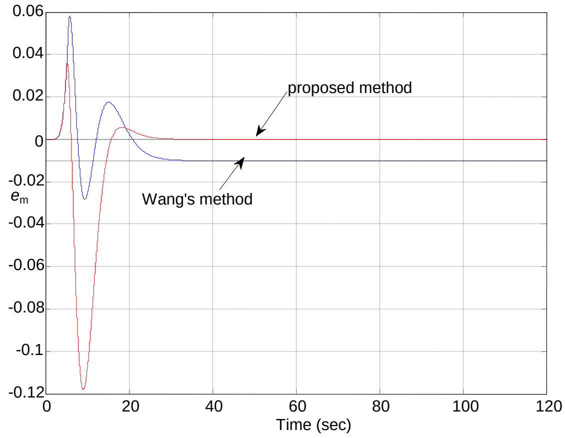

Fig. 11. Modelling error dynamics of the third system with both methods

The preceding figures shows a better step time responses (faster rise time and disturbance rejection) and a smaller tracking and modelling errors for the proposed method in comparison with Wang's method, this confirm the relevance of the proposed approach.

5. ROBUSTNESS ENHANCEMENT

Further of the perturbation rejection presented below, we will introduce some robustness margins, respectively: the gain margin, the phase margin, the modulus margin and the delay margin. This robustness margins allows us to measure the robustness enhancement of each method. The following figure presents a classical feedback system loop:

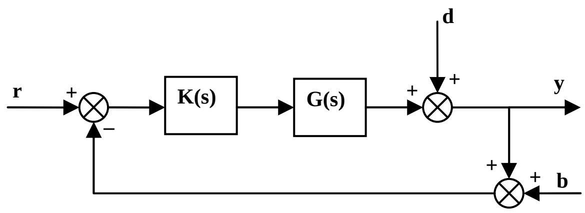

Fig. 12. Classical feedback loop

With: b is the perturbation signal introduced by the measuring sensor.
Define the robustness margins as (Ogata, (2010)), (De Larminat, (1996)), (Chen, (1993)):

5.1. The gain margin

The gain margin is the measure of how much the open loop gain can change before system become unstable.
Its value (in decibels), is given by:

$$M_g(db) = -20\log\left|G(jw_p)K(jw_p)\right| \tag{26}$$

Where: w_p is the phase crossover frequency: the frequency at which the phase angle of the open-loop transfer function equals –180°.

5.2. The phase margin

The phase margin is the measure of how much the open loop phase can change before system become unstable.
Its value (in degrees), is given by:

$$M_p(\deg) = \Psi + 180^o \tag{27}$$

Where: Ψ is the angle of the open-loop transfer function at the gain crossover frequency (the frequency at which the magnitude of the open loop transfer function, is unity).

5.3. The Modulus margin

The modulus margin (Litrico and Georges (1999)) is a useful robustness indicator of the closed loop system, it is equal to the inverse of the Hinfinity norm of the system output sensibility, i.e.: the minimum distance between the open loop Nyquist plot and the critical point (-1,180^0).

$$M_m = \frac{1}{\left\|S_y\right\|_\infty} \tag{28}$$

With:

$$S_y = \frac{1}{1+GK} \tag{29}$$

5.4. The Delay margin

The delay margin is the maximum time delay τ which allows the closed loop of the disturbed process to remain stable, it is given by:

$$M_d = \frac{Q}{w_p} \tag{30}$$

Where: Q is the phase margin of the open loop system and w_p is the phase crossover frequency.

5.5. Robustness margin for Smith predictor scheme

As the robustness margins are measured using the open loop transfer function (let us call it: $L = K.G$), we can obtain it from the closed loop transfer function given by the equation (1), let us call it T:

$$T = \frac{y}{r} = \frac{L}{1+L} \Rightarrow L = \frac{T}{1-T} \tag{31}$$

The following tables show the robustness margins of the three systems with the two methods:

Table.1 Robustness margins (Wang's method)

	M_g(db)	M_p (deg)	M_m	M_d
$G_1(s)$	0.917	91.92	7.87	0.067
$G_2(s)$	2.08	61.97	6.53	0.063
$G_3(s)$	2.68	63.1	8.59	0.07

Table.2. Robustness margin (Proposed method)

	M_g(db)	M_p (deg)	M_m	M_d
$G_1(s)$	1.15	156.44	8.542	0.043
$G_2(s)$	2.217	61.25	7.47	0.044
$G_3(s)$	0.6954	63.62	8.22	0.034

We denote that the robustness margins are globally satisfying (substantially the same for both methods), however the proposed method gives better modulus margins.

6. CONCLUSION

A new control method of delayed systems based on a modified Smith predictor control scheme is presented in this paper; the approach can be applied at any delayed processes with any controller structures. The method is founded on the resolution of a multi-bjective optimisation problem. The simulation results for different typical delayed systems shows a good closed loop performances and robustness compared to those of other methods.

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