Delay-Dependent H$_\infty$ Control of Linear Systems with Uncertain Input Delay Using State-Derivative Feedback

A. Shariati *. H. D. Taghirad*
B. Labibi**

* *Advanced Robotics and Automated Systems (ARAS), Faculty of Electrical and Computer Engineering, K. N. Toosi University of Technology, Tehran, Iran, (e-mail: Shariati@eetd.kntu.ac.ir).*
*** Faculty of Medicine, University of Toronto, Toronto, Canada*

Abstract: In some practical problems such as active vibration suppression systems, the state-derivative signals are easier to access than the state variables. This paper considers an H$_\infty$-based state-derivative feedback control problem for input-delayed systems. Applying this control law, the resulting closed-loop system turns into a specific time-delay system of neutral type. The significant specification of this neutral system is that its delayed term coefficients depend on the controller parameters. The time-delay is considered as uncertain time-invariant with a known constant bound. In this paper, the delay-dependent sufficient conditions for the existence of an H$_\infty$ state-derivative feedback controller are derived in terms of matrix inequalities. The resulting H$_\infty$ controller stabilizes the closed-loop neutral system and assures that the H$_\infty$-norm to be less than a prescribed level. An application example is presented to illustrate the effectiveness of the proposed method.

Keywords: H$_\infty$ Control; State-derivative feedback; Time delay; Neutral systems; Linear matrix inequality (LMI)

1. INTRODUCTION

It is well known from classical theory that derivative feedback is essential for achieving desired control objectives in many applications (Abdelaziz & Valášek, 2004). On the other hand, in many real case studies the state-derivative signals are easier to obtain than the original state signals. Representative examples for such case may be named as vibration suppression in mechanical systems, car wheel suspension systems, control of bridge cable vibration and the vibration control of landing gear components (Assuncão, Teixeira, Faria, Da Silva & Cardim, 2007). In these examples, usually the main sensors on board are accelerometers, whose signals may be used to reconstruct the velocities with high precision, however, precise displacements signals cannot be usually retrieved with that precision (Abdelaziz & Valášek, 2004). In such cases, the only accessible signals are the derivative of state variables. This practical limitation has motivated many researchers to focus on the design of state-derivative feedbacks for such applications. Abdelaziz and válašek (2004, 2005a) proposed a formula similar to Ackermann to solve the pole-placement problem for linear delay free SISO and MIMO systems using state-derivative feedback. Assoncão, et al. (2007) used this formulation to design a stabilizing state-derivative controller for systems that bounds the output peak as well as the state-derivative feedback. Moreover, the linear quadratic regulator control method has been applied to formulate state-derivative feedback control law within the framework of reciprocal state space by Duan et. al (2005). Recently, Faria et. al (2009) proposed necessary and sufficient conditions in terms of linear matrix inequality (LMI) for pole placement of linear systems using state-derivative feedback.

All these results are developed for delay free systems, whereas small delays may inevitably occur in practice. Vyhlídal et.al (2009) have investigated the impact of constant and small delays on the stability of feedback system when proportional-derivative state feedback is applied. It can be easily seen that the closed-loop system leads to a time-delay system of neutral type. Stability and stabilization of such systems is a problem of recurring interest in recent decades (Park, 2005, Zhang, et. al, 2007, Li, et. al, 2008). On the other hand, H$_\infty$ control of time-delay systems of retarded or neutral type is suitably used by many researchers in recent years (Chen, 2005, Souza, et. al, 2008, Suplin, et. al, 2006) to simultaneously satisfy stability and performance objectives. In this line of research, robust H$_\infty$ state feedback control of uncertain neutral system has been considered by Chen (2005), in which an optimization problem has been formulated through linear matrix inequality constraints in order to obtain an H$_\infty$ state feedback controller. Observer-based H$_\infty$ state feedback control for a class of uncertain neutral systems is another topic which has been considered by Lien (2005). H$_\infty$ output feedback control of neutral systems has also been the centre of attention in some papers (Baser, 2002, 2003). To design such H$_\infty$ controllers, bounded real lemma is certainly an effective tool to guarantee stability as well as desired performance. Xu et al. (2001) have used this lemma to design an H$_\infty$ state feedback and positive real control for a linear neutral delay system. Moreover, an H$_\infty$ output feedback controller has been designed in terms of three LMIs using bounded real lemma for a neutral system with multiple delays (Baser, 2002).

In order to better distinguish the contributions of this paper, let us introduce the general representation of linear input-delayed systems as follows:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{n} B_i u(t - h_i) + Ew(t) \tag{1}$$

while, the state-derivative controller is given as

$$u(t) = K\dot{x}(t) \tag{2}$$

Applying the control law (2) to the input-delayed system (1) leads immediately to a time delayed closed loop system of neutral type, which may be represented as follows:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{n} B_i K\dot{x}(t - h_i) + Ew(t) \tag{3}$$

Notice that Eq. (3) not only represents a time-delayed system of neutral type, but also the $\dot{x}(t-h_i)$'s coefficients depends on the controller parameters K. So far no design method for such state-derivative feedback has been reported in the literature for input time-delayed systems. Furthermore, as it is seen in Eq. (3), finding K in this problem not only introduces a new and theoretically challenging problem, but also could lead to a very effective method to obtain desired performance in practical active vibration suppression systems. The main contribution of this paper is the detail elaboration of this problem and H_∞-based state derivative feedback controller design in this closed-loop system in presence of uncertain bounded delay. As a main result, delay-dependent sufficient conditions are obtained in terms of some matrix inequalities for such problem. The paper is organized as follows: Problem formulation is introduced in Section 2, and in Section 3, H_∞ controller is designed. This is accomplished in terms of some matrix inequalities for the closed-loop time-delay system of neutral type. Illustrative examples are provided in section 4 to show the effectiveness of the proposed method for some case studies, and real application. Finally, the concluding remarks are given in Section 5.

2. PROBLEM FORMULATION

Consider the following time-delay system with input delay:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-\tau) + Ew(t) \\ z(t) &= Cx(t) + D_1 u(t-\tau) + D_2 w(t) \end{aligned} \tag{4}$$

where, $x\in\Re^n$ is the system state vector, w is the disturbance input of system and belongs to the Sobolev space $\mathcal{W}^{1,2}(0,\infty,\Re^p)\cap\mathcal{L}^2(0,\infty,\Re^p)$, $u\in\Re^m$ is the system input, $z\in\Re^q$ is the controlled system output, and τ is the input delay of the system which is assumed to be bounded $0<\tau\le\bar{\tau}$. The matrices $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$, $E\in\Re^{n\times p}$, $C\in\Re^{q\times n}$, $D_1\in\Re^{q\times m}$, $D_2\in\Re^{q\times p}$ are assumed to be known. Considering the control law (2), the state space equations of the closed-loop system are given by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BK\dot{x}(t-\tau) + Ew(t) \\ z(t) &= Cx(t) + D_1 K\dot{x}(t-\tau) + D_2 w(t) \\ x(t_0+\theta) &= \phi(\theta) \qquad \forall\theta\in[-\tau,0] \end{aligned} \tag{5}$$

Therefore, the resulting closed-loop system (5) is a time-delay system of neutral type, in which the coefficients of $\dot{x}(t-\tau)$ terms depend on the controller parameters. Let us state two useful lemmas which will be used further in the main result of the paper.

Lemma 1 (Moon et. al, 2001)**:** Assume $a(.)\in\Re^{n_a}$, $b(.)\in\Re^{n_b}$ and $N\in\Re^{n_a\times n_b}$ are defined on the interval Ω, then for any matrices $X\in\Re^{n_a\times n_b}$, $Y\in\Re^{n_a\times n_b}$ and $Z\in\Re^{n_a\times n_b}$, the following inequality holds:

$$-2\int_\Omega a^T(\alpha)Nb(\alpha)d\alpha \le \int_\Omega \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T-N^T & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha$$

where

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$$

Lemma 2 (Skogestad & Postlethwaite, 2005)**:** For a prescribed matrix, $M=\begin{bmatrix} A \\ B \end{bmatrix}$ or $M=\begin{bmatrix} A & B \end{bmatrix}$ we have the following inequality:

$$\max\{\bar{\sigma}(A),\bar{\sigma}(B)\} \le \bar{\sigma}(M) \le \sqrt{2}\max\{\bar{\sigma}(A),\bar{\sigma}(B)\}$$

Remark 1. It should be noted that the integral inequality in Lemma 1 can be extended to the similar inequalities with multiple integrals.

Remark 2. The results provided in this paper can be easily extended to systems with multiple input delays. The reason why we consider a single input delay system is to make our derivation clear and avoid complicated notations.

3. DELAY-DEPENDENT H_∞ CONTROL WITH UNCERTAIN DELAY

In this section, an H_∞ state-derivative feedback controller is derived not only to stabilize the closed-loop input delayed system, but also to achieve the minimum of an H_∞ norm bound of the closed-loop transfer matrix from the disturbance input to the controlled output. To this aim, we present a delay-dependent sufficient condition to design the proposed controller's gain with H_∞ performance for the closed-loop system (5) with uncertain delay.

Theorem 1. Given scalars $\bar{\tau}$ and m with $0<m<1$, the closed-loop system (5) for any time-delay τ satisfying $0<\tau\le\bar{\tau}$ is asymptotically stable and $\|T_{zw}\|_\infty<\gamma$, if there exist positive definite symmetric matrices L, T, H_1, H_2, $F\in\Re^{n\times n}$, negative definite symmetric matrix N and matrices $M\in\Re^{n\times n}$, $V\in\Re^{m\times n}$ satisfying matrix inequalities (6)~(8).

$$\begin{bmatrix} \Omega & -N & BV-\bar{\tau}N & 0 & E & 0 & LA^T & \bar{\tau}^2LA^TA^T & LA^TA^T & LC^T \\ * & -T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -LH_1^{-1}L-\bar{\tau}N & 0 & 0 & 0 & (BV)^T & \bar{\tau}^2(ABV)^T & (ABV)^T & (D_1V)^T \\ * & * & * & -LH_2^{-1}L & 0 & 0 & 0 & \bar{\tau}^2(BV)^T & (BV)^T & 0 \\ * & * & * & * & -m^2\gamma^2 I & 0 & E^T & \bar{\tau}^2E^TA^T & E^TA^T & D_2^T \\ * & * & * & * & * & -(1-m)^2\gamma^2 I & 0 & \bar{\tau}^2E^T & E^T & 0 \\ * & * & * & * & * & * & -H_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{\tau}^2F & 0 & 0 \\ * & * & * & * & * & * & * & * & -H_2 & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{6}$$

$$\begin{bmatrix} 2M & \bar{\tau}N \\ \bar{\tau}N^T & 2\bar{\tau}LF^{-1}L \end{bmatrix} > 0 \tag{7}$$

$$\begin{bmatrix} L^TL & V^TB^T \\ VB & I \end{bmatrix} > 0 \tag{8}$$

in which,

$$\Omega = LA^T + AL + \bar{\tau}M + (2-\bar{\tau})N + T$$

Moreover, H$_\infty$ state- derivative feedback control law is given by $u = VL^{-1}\dot{x}$.

First let us express the following useful lemmas which will be used in the proof of Theorem 1.

Lemma 3. Consider the neutral system (5) with $w \in \mathcal{W}^{1,2}(0,\infty,\Re^p) \cap \mathcal{L}^2(0,\infty,\Re^p)$. We define

$$d(t) = \begin{bmatrix} mw^T(t) & (1-m)\dot{w}^T(t) \end{bmatrix}^T$$

where m is a scalar real value and $0 < m < 1$. If $\| T_{zd} \|_\infty < \gamma$, then the inequality $\| T_{zw} \|_\infty < \gamma$ is satisfied.

Proof. Since $z(s) = T_{zw}(s)w(s)$ and $d(s) = \begin{bmatrix} mw(s) \\ (1-m)sw(s) \end{bmatrix}$, then we have

$$z(s) = \begin{bmatrix} T_{zw}(s) & T_{zw}(s)/s \end{bmatrix} d(s)$$

Therefore, $T_{zd}(s)$ can be rewritten as

$$T_{zd}(s) = \begin{bmatrix} T_{zw}(s) & T_{zw}(s)/s \end{bmatrix}$$

By Lemma 2 the following inequality holds as

$$\max\left\{ \|T_{zw}(s)\|_\infty \ , \ \|T_{zw}(s)/s\|_\infty \right\} \le \left\| \begin{bmatrix} T_{zw}(s) & T_{zw}(s)/s \end{bmatrix} \right\|_\infty$$

or

$$\max\left\{ \|T_{zw}(s)\|_\infty \ , \ \|T_{zw}(s)/s\|_\infty \right\} \le \|T_{zd}(s)\|_\infty$$

It can be easily concluded that if $\|T_{zd}(s)\|_\infty < \gamma$, then $\|T_{zw}(s)\|_\infty < \gamma$. This completes the proof. ■

Corollary 1. Consider the neutral system (5) and two following performance indices:

$$J_1(w) = \int_0^\infty \left(z^Tz - \gamma^2 w^Tw \right) d\tau \text{ and } J_2(w) = \int_0^\infty (z^Tz - \gamma^2 d^Td) d\tau .$$

where $d(t) = \begin{bmatrix} mw(t) \\ (1-m)\dot{w}(t) \end{bmatrix}$ and $w \in \mathcal{W}^{1,2}(0,\infty,\Re^p) \cap \mathcal{L}^2(0,\infty,\Re^p)$.

Since the inequalities $J_1(w) < 0$ and $J_2(w) < 0$ corresponds to H$_\infty$ constraints $\| T_{zw} \|_\infty < \gamma$ and $\| T_{zd} \|_\infty < \gamma$ respectively, then in order to satisfy the inequality $J_1(w) < 0$, it suffices to show that the condition $J_2(w) < 0$ is satisfied or

$$\int_0^\infty \left(z^Tz - \gamma^2 m^2 w^Tw - \gamma^2 (1-m)^2 \dot{w}^T\dot{w} \right) d\tau < 0 .$$

Proof of Theorem 1. Under the condition of the Theorem 1, we wish to find sufficient conditions that guarantee H$_\infty$ performance of the closed-loop system (5). Before applying Lyapunov method for stability of neutral system, we note that the difference operator $\mathcal{D}$: $C[-\bar{\tau}\ ,\ 0] \in \Re^n$ given by $\mathcal{D}x_t = x(t) - BKx(t-\tau)$ must be delay-independently stable with respect to delay. A sufficient condition for the stability of the operator $\mathcal{D}$ is given by the following inequality:

$$\|BK\| < 1 \tag{9}$$

which, $\|.\|$ denotes the Euclidean norm of the matrix BK and it is equal to the maximum singular value of BK. Hence, (9) is equivalent to the following inequality

$$\bar{\sigma}(BK) < 1 \ \text{ or } \ \sqrt{\lambda_{\max}\left((BK)^T BK \right)} < 1 \tag{10}$$

The above inequality can be rewritten as follows

$$(BK)^T BK < I \ \text{ or } \ I - (BK)^T BK > 0 \tag{11}$$

Let L be an $n \times n$ real symmetric positive definite matrix. By performing a congruence transformation to (11) by L and defining $V=KL$ together with some Schur complement operations, the LMI (8) is obtained. Now, we choose a Lyapunov-Krasovskii functional candidate for the system (5) as

$$V = V_1 + V_2 + V_3 \tag{12}$$

where

$$V_1 = x(t)^T Px(t) \tag{13}$$

$$V_2 = \int_{t-\tau}^{t} x^T(\alpha)Qx(\alpha)d\alpha + \int_{t-\tau}^{t} \dot{x}^T(\alpha)R_1\dot{x}(\alpha)d\alpha \tag{14}$$

$$\begin{aligned} V_3 &= \int_{-\tau}^{0}\int_{-\tau}^{\beta}\int_{t+\eta}^{t} \ddot{x}^T(\alpha)Z\ddot{x}(\alpha)d\alpha\, d\eta\, d\beta \\ &\quad + (1/2)\int_{t-\tau}^{t} \ddot{x}^T(\alpha)R_2\ddot{x}(\alpha)d\alpha, \end{aligned} \tag{15}$$

where P, Q, R_1, R_2 and Z are real symmetric positive definite matrices of appropriate dimensions. Differentiating V_1 with respect to t gives us

$$\dot{V}_1 = 2x^T(t)P\dot{x}(t) = 2x^T(t)P\{Ax(t) + BK\dot{x}(t-\tau) + Ew(t)\}$$

Let us introduce the following relation for the delayed derivative of the state:

$$\dot{x}(t-\tau) = \tau^{-1}\left[x(t) - x(t-\tau) - \int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}(\alpha)d\alpha d\beta\right] \tag{16}$$

Using (16), a new model transformation for the state space equation of system (5) is represented by following system with distributed delay:

$$\begin{aligned} \dot{x}(t) &= \left(A + \tau^{-1}BK\right)x(t) - \tau^{-1}BKx(t-\tau) \\ &\quad - \tau^{-1}BK\int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}(\alpha)d\alpha d\beta + Ew(t) \end{aligned} \tag{17}$$

Therefore, the time derivative of $V_1(x)$ along the trajectories of system (5) is given by

$$\begin{aligned} \dot{V}_1 &= 2x^T P\left(A + \tau^{-1}BK\right)x(t) - 2\tau^{-1}x^T PBKx(t-\tau) \\ &\quad - 2\tau^{-1}x^T PBK\int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}(\alpha)d\alpha\, d\beta + 2x^T(t)PEw(t) \end{aligned}$$

Considering remark 1 and defining a (α , β) = $x(t)$, *and* $b(\alpha,\beta) = \int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}(\alpha)d\alpha d\beta$, we provide the following inequalities

$$\begin{aligned} &-2x(t)PBK\int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}(\alpha)d\alpha\, d\beta \\ &\leq \int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \begin{bmatrix} x(t) \\ \ddot{x}(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - PBK \\ Y^T - (PBK)^T & \tau Z \end{bmatrix} \begin{bmatrix} x(t) \\ \ddot{x}(\alpha) \end{bmatrix} d\alpha\, d\beta \end{aligned} \tag{18}$$

$$\begin{bmatrix} X & Y \\ Y^T & \tau Z \end{bmatrix} > 0 \tag{19}$$

Finally, with the above conditions we obtain

$$\begin{aligned} \dot{V}_1 &\leq x^T\{A^T P + PA + \tau(X/2) + \tau^{-1}\left(Y + Y^T\right)\}x \\ &\quad + x^T(t)(PBK - Y)\dot{x}(t-\tau) + \dot{x}^T(t-\tau)(PBK - Y)^T x(t) \\ &\quad - \tau^{-1}x^T(t)Yx(t-\tau) - \tau^{-1}x^T(t-\tau)Y^T x(t) + 2x^T(t)PEw(t) \\ &\quad + \int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}^T(\alpha)Z\ddot{x}(\alpha)d\alpha d\beta \end{aligned}$$

It can be shown that the time derivative of V_2 and V_3 may be written in the forms of:

$$\begin{aligned} \dot{V}_2 &= x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) \\ &\quad + \dot{x}^T(t)R_1\dot{x}(t) - \dot{x}^T(t-\tau)R_1\dot{x}(t-\tau) \end{aligned} \tag{21}$$

$$\begin{aligned} \dot{V}_3 &= \ddot{x}^T(t)\left(\left(\tau^2/2\right)Z + R_2/2\right)\ddot{x}(t) - (1/2)\ddot{x}^T(t-\tau)R_2\ddot{x}(t-\tau) \\ &\quad - \int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}^T(\alpha)Z\ddot{x}(\alpha)d\alpha\, d\beta \end{aligned} \tag{22}$$

Therefore, we have

$$\dot{V}(t) = \sum_{i=1}^{3} \dot{V}_i \tag{23}$$

Consider (20) ~ (22) and define $\tau^{-1}Y = Y_1$, by the assumption of $Y = Y^T < 0$ and then adding and subtracting the terms $x^T(t)(\tau Y_1)x(t)$ and $\dot{x}^T(t-\tau)(\tau Y_1)^T\dot{x}(t-\tau)$ in (23), an upper bound for $\dot{V}$ is obtained as follows:

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{3}\dot{V}_i \leq x^T\{A^T P + PA + \overline{\tau}(X/2) + Y_1 + Y_1^T + Q\}x \\ &\quad + 2x^T(t)PBK\dot{x}(t-\tau) + 2x^T(t)PEw(t) \\ &\quad + \left(x^T(t) + \dot{x}(t-\tau)\right)^T(-\overline{\tau}Y_1)\left(x^T(t) + \dot{x}(t-\tau)\right) \\ &\quad - x^T(t)Y_1x(t-\tau) - x^T(t-\tau)Y_1x(t) \\ &\quad - x^T(t-\tau)Qx(t-\tau) + \dot{x}^T(t)R_1\dot{x}(t) - \dot{x}^T(t-\tau)R_1\dot{x}(t-\tau) \\ &\quad + \ddot{x}^T(t)\left(\left(\overline{\tau}^2/2\right)Z + R_2/2\right)\ddot{x}(t) \\ &\quad - (1/2)\ddot{x}^T(t-\tau)R_2\ddot{x}(t-\tau) \\ &\quad + x^T(t)(\tau Y_1)x(t) + \dot{x}^T(t-\tau)(\tau Y_1)^T\dot{x}(t-\tau) \end{aligned}$$

Assume zero initial condition, i.e. $\phi(t) = 0$, $\forall t \in [-\tau, 0]$ we have $V(q(t))|_{t=0} = 0$. For a prescribed $\gamma > 0$ and scalar $0 < m < 1$, consider the following performance index:

$$J_{zd}(w) = \int_0^\infty \left(z^T z - \gamma^2 d^T d\right)d\tau \tag{25}$$

in which. $d(t) = \begin{bmatrix} mw(t) \\ (1-m)\dot{w}(t) \end{bmatrix}$,

therefore, the performance index (25) can be rewritten as

$$J_{zd}(w) = \int_0^\infty \left(z^T z - m^2\gamma^2 w^T w - (1-m)^2\gamma^2\dot{w}^T\dot{w}\right)d\tau$$

since $V(t)|_{t=0} = 0$ and $V(t)|_{t\to\infty} \geq 0$, we obtain

$$\begin{aligned} J_{zd}(w) &= \int_0^\infty \left(z^T z - m^2\gamma^2 w^T w - (1-m)^2\gamma^2\dot{w}^T\dot{w} + \dot{V}(t)\right)d\tau \\ &\qquad + V(t)\big|_{t=0} - V(t)\big|_{t\to\infty} \\ &\leq \int_0^\infty \left(z^T z - m^2\gamma^2 w^T w - (1-m)^2\gamma^2\dot{w}^T\dot{w} + \dot{V}(t)\right)d\tau \end{aligned} \tag{20}$$

hence, the following inequality is obtained:

$$
\begin{aligned}
J_{zd}(w) \le \int_0^\infty \{ & x^T C^T C x + 2x^T C^T D_1 K\dot{x}(t-\tau) + 2x^T C^T D_2 w \\
& + \dot{x}^T(t-\tau) K^T D_1^T D_1 K \dot{x}(t-\tau) + 2\dot{x}^T(t-\tau) K^T D_1^T D_2 w \\
& + w^T D_2^T D_2 w - m^2\gamma^2 w^T w - (1-m)^2 \gamma^2 \dot{w}^T \dot{w} + \dot{V}(t) \} d\tau
\end{aligned} \quad (26)
$$

Considering (24), $0 < \tau \le \overline{\tau}$ and substituting

$$
\begin{aligned}
\ddot{x}(t) &= (d/dt)\big(Ax(t) + BK\dot{x}(t-\tau) + Ew(t)\big) \\
&= A\dot{x}(t) + BK\ddot{x}(t-\tau) + E\dot{w}(t)
\end{aligned}
$$

and

$$
\dot{x}(t) = Ax(t) + BK\dot{x}(t-\tau) + Ew(t)
$$

then, a new upper bound for (26) is obtained as

$$
\begin{aligned}
J_{zd} \le \int_0^\infty \{ & \zeta^T \Pi \zeta + x^T(t)(\tau Y_1)x(t) \\
& + \dot{x}^T(t-\tau)(\tau Y_1)^T \dot{x}(t-\tau) \} d\tau
\end{aligned} \quad (27)
$$

with defined

$$
\zeta = \begin{bmatrix} x(t) & x(t-\tau) & \dot{x}(t-\tau) & \ddot{x}(t-\tau) & w(t) & \dot{w}(t) \end{bmatrix}
$$

and $\Pi = [\Sigma_{ij}]$ where $\Sigma_{ij} = \Sigma_{ij}^T$ and $i,j = 1,2,\ldots 6$.

in which,

$$
\begin{aligned}
\Sigma_{11} &= A^T P + PA + \overline{\tau}(X/2) + (2-\overline{\tau})Y_1 + Q + C^T C \\
&\quad + A^T R_1 A + A^T A^T \Upsilon A A \\
\Sigma_{12} &= -Y_1 \qquad \Sigma_{13} = PBK - \overline{\tau} Y_1 + A^T R_1 BK \\
&\qquad\qquad\qquad + A^T A^T \Upsilon ABK + C^T D_1 K \\
\Sigma_{14} &= A^T A^T \Upsilon BK \qquad \Sigma_{15} = PE + A^T R_1 E + A^T A^T \Upsilon AE + C^T D_2 \\
\Sigma_{16} &= A^T A^T \Upsilon E \qquad \Sigma_{22} = -Q \\
\Sigma_{23} &= \Sigma_{24} = \Sigma_{25} = \Sigma_{26} = 0 \\
\Sigma_{33} &= -R_1 - \overline{\tau} Y_1 + (BK)^T R_1 BK + (ABK)^T \Upsilon (ABK) + K^T D_1^T D_1 K \\
\Sigma_{34} &= (ABK)^T \Upsilon BK \\
\Sigma_{35} &= (BK)^T R_1 E + (ABK)^T \Upsilon AE + K^T D_1^T D_2 \\
\Sigma_{36} &= (ABK)^T \Upsilon E \qquad \Sigma_{44} = -R_2/2 + (BK)^T \Upsilon BK \\
\Sigma_{45} &= (BK)^T \Upsilon AE \qquad \Sigma_{46} = (BK)^T \Upsilon E \\
\Sigma_{55} &= E^T R_1 E + E^T A^T \Upsilon AE + D_2^T D_2 - m^2\gamma^2 I \\
\Sigma_{56} &= E^T A^T \Upsilon E \qquad \Sigma_{66} = E^T \Upsilon E - (1-m)^2 \gamma^2 I
\end{aligned}
$$

and $\Upsilon = (1/2)(\overline{\tau}^2 Z + R_2)$.

Since $Y_1<0$, by assuming $w \in \mathcal{W}^{1,2}(0,\infty,\Re^p) \cap \mathcal{L}^2(0,\infty,\Re^p)$ and $\Pi<0$ this implies that $J_{zd}<0$, and therefore, $\|T_{zd}\|_\infty < \gamma$. This condition is the H_∞ performance index, in which a good performance is ensured by a small value of γ. By Lemma 3, the inequality $\|T_{zd}\|_\infty < \gamma$ guarantees that $\|T_{zw}\|_\infty < \gamma$ is satisfied. Using Schur complement, the condition $\Pi<0$ is equivalent to the following matrix inequality:

$$
\left[\begin{array}{c|c} \Xi_{11} & \Xi_{12} \\ \hline \Xi_{12}^T & \Xi_{22} \end{array}\right] < 0 \quad (28)
$$

with

$$
\begin{bmatrix} X & \tau Y_1 \\ \tau Y_1^T & \tau Z \end{bmatrix} > 0 \quad (29)
$$

and

$$
\Xi_{11} =
\begin{bmatrix}
\Omega_1 & -Y_1 & PBK - \overline{\tau} Y_1 & 0 & PE & 0 \\
* & -Q & 0 & 0 & 0 & 0 \\
* & * & -R_1 - \overline{\tau} Y_1 & 0 & 0 & 0 \\
* & * & * & -(1/2)R_2 & 0 & 0 \\
* & * & * & * & -m^2\gamma^2 I & 0 \\
* & * & * & * & * & -(1-m)^2\gamma^2 I
\end{bmatrix}
$$

$$
\Xi_{22} = -diag\left(R_1^{-1}, 2\overline{\tau}^2 Z^{-1}, 2R_2^{-1}, I\right)
$$

$$
\Xi_{12} = \begin{bmatrix} \Delta_1 & \overline{\tau}^2 \Delta_2 & \Delta_2 & \Delta_3 \end{bmatrix}
$$

where

$$
\begin{aligned}
\Omega_1 &= A^T P + PA + (\overline{\tau}/2)X + (2-\overline{\tau})Y_1 + Q \\
\Delta_1 &= \begin{bmatrix} A & 0 & BK & 0 & E & 0 \end{bmatrix}^T \\
\Delta_2 &= \begin{bmatrix} AA & 0 & ABK & BK & AE & E \end{bmatrix}^T \\
\Delta_3 &= \begin{bmatrix} C & 0 & D_1 K & 0 & D_2 & 0 \end{bmatrix}^T
\end{aligned}
$$

Denote $P^{-1}, 2Z^{-1}, R_1^{-1}, 2R_2^{-1}$ as L, F, H_1 and H_2 respectively, by performing a congruence transformation to (28) through diag(L,L,L,L,I,I,I,I,I,I) together with introducing the variables $M = L(X/2)L$, $N = LY_1L$, $T = LQL$, $V = KL$, the matrix inequality (6) is derived. Furthermore, by performing a congruence transformation to (29) through diag (L,L), we can obtain

$$
\begin{bmatrix} LXL & \tau L Y_1 L \\ \tau L Y_1^T L & \tau LZL \end{bmatrix} > 0
$$

Using Schur complement, we have

$$
LXL - (\tau L Y_1 L)(\tau LZL)^{-1}(\tau L Y_1^T L) > 0
$$

Substituting $M = L(X/2)L$, $N = LY_1L$ and $F = 2Z^{-1}$, the following matrix inequality is derived.

$$
2M - (\tau N)(2\tau L F^{-1} L)^{-1}(\tau N^T) > 0 \quad (30)
$$

On the other hand we have

$$
2M - (\tau N)(2\tau L F^{-1} L)^{-1}(\tau N)^T \ge 2M - (\overline{\tau} N)(2\overline{\tau} L F^{-1} L)^{-1}(\overline{\tau} N)^T \quad (31)
$$

Therefore, satisfying the inequality (32) guarantees the inequality (26) to be satisfied.

$$\Upsilon_1 = \begin{bmatrix} \Omega & -N & BV-\bar{\tau}N & 0 & E & 0 & LA^T & \bar{\tau}^2 LA^T A^T & LA^T A^T & LC^T \\ * & -T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -U_1-\bar{\tau}N & 0 & 0 & 0 & (BV)^T & \bar{\tau}^2(ABV)^T & (ABV)^T & (D_1V)^T \\ * & * & * & -U_2 & 0 & 0 & 0 & \bar{\tau}^2(BV)^T & (BV)^T & 0 \\ * & * & * & * & -m^2\gamma^2 I & 0 & E^T & \bar{\tau}^2 E^T A^T & E^T A^T & D_2^T \\ * & * & * & * & * & -(1-m)^2\gamma^2 I & 0 & \bar{\tau}^2 E^T & E^T & 0 \\ * & * & * & * & * & * & -H_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{\tau}^2 F & 0 & 0 \\ * & * & * & * & * & * & * & * & -H_2 & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (35)$$

$$2M - (\bar{\tau}N)\left(2\bar{\tau}LF^{-1}L\right)^{-1}(\bar{\tau}N)^T > 0 \quad (32)$$

Applying Schur complement, the matrix inequality (7) is obtained, and this completes the proof. ■

Remark 3. It should be noted that in practical problems, the disturbance signals are usually differentiable therefore; in the proof of the Theorem 2 the constraint $w \in \mathcal{W}^{1,2}(0,\infty,\Re^p) \cap \mathcal{L}^2(0,\infty,\Re^p)$ is not a severe limitation.

Remark 4. Theorem 1 presents delay-dependent sufficient conditions which guarantee both asymptotic stability and H_∞ performance of the closed-loop system (5) with uncertain input-delay τ in terms of some matrix inequalities. It is worth mentioning that, these conditions are the first sufficient conditions given in the literature for designing an H_∞ state-derivative feedback for an input-delay system.

Remark 5. It should be noted that the resulting conditions for the state-derivative feedback in Theorem 1 are not resulting to LMI's due to the terms $LH_1^{-1}L$ and $LH_2^{-1}L$ in (6), $LF^{-1}L$ in (7) and L^TL in (8). A remedy to solve this problem is to follow Moon's algorithm presented in (Moon, et. al, 2001) to cast this non-convex optimization problem into a nonlinear minimization problem with LMI conditions. To elaborate on this method first, define new variables U_i, Λ and S such that $L^TL > \Lambda$, $LH_i^{-1}L \geq U_i$ and $LF^{-1}L \geq S$. Then replace (7)~(8) by

$$\begin{bmatrix} 2M & \bar{\tau}N \\ \bar{\tau}N^T & 2\bar{\tau}S \end{bmatrix} > 0, \quad LF^{-1}L \geq S \quad (33)$$

$$\begin{bmatrix} \Lambda & V^TB^T \\ VB & I \end{bmatrix} > 0, \quad L^TL \geq \Lambda \quad (34)$$

Since the condition $LH_i^{-1}L \geq U_i$ is equivalent to $L^{-1}H_iL^{-1} \leq U_i^{-1}$, using Schur complement we obtain

$$\begin{bmatrix} U_i^{-1} & L^{-1} \\ L^{-1} & H_i^{-1} \end{bmatrix} \geq 0 \quad \forall i = 1,2$$

Similarly, the conditions (33) and (34) are equivalent to

$$\begin{bmatrix} 2M & \bar{\tau}N \\ \bar{\tau}N^T & 2\bar{\tau}S \end{bmatrix} > 0, \quad \begin{bmatrix} S^{-1} & L^{-1} \\ L^{-1} & F^{-1} \end{bmatrix} \geq 0$$

$$\begin{bmatrix} \Lambda & V^TB^T \\ VB & I \end{bmatrix} > 0, \quad \begin{bmatrix} \Lambda^{-1} & L^{-1} \\ L^{-1} & I \end{bmatrix} > 0$$

Then, by introducing new variables T_1, O, G_i, I_i, and J, the original conditions (6) ~ (8) can be represented as

$$\Upsilon_1 < 0$$

$$\begin{bmatrix} 2M & \bar{\tau}N \\ \bar{\tau}N^T & 2\bar{\tau}S \end{bmatrix} > 0 \quad \begin{bmatrix} \Lambda & V^TB^T \\ VB & I \end{bmatrix} > 0$$

$$\begin{bmatrix} G_i & J \\ J & I_i \end{bmatrix} \geq 0 \quad \begin{bmatrix} T_1 & J \\ J & O \end{bmatrix} \geq 0 \quad \begin{bmatrix} \nabla & J \\ J & I \end{bmatrix} > 0$$

$$G_i = U_i^{-1}, \quad I_i = H_i^{-1}, \quad T_1 = S^{-1}$$

$$O = F^{-1}, \quad J = L^{-1}, \quad \nabla = \Lambda^{-1} \quad \forall i = 1,2$$

Υ_1 is as defined in (35) at the top of this page. Now, using a cone complementary problem (Ghaoui, et. al, 1997), the following nonlinear minimization problem involving LMI conditions is suggested to be solved instead of the original non-convex feasibility problem of Theorem 1.

Minimize $Tr\,(\,LJ + \nabla\Lambda + OF + T_1S + \sum_{i=1}^{2}(G_iU_i + I_iH_i)\,)$ *subject to*

$$\begin{gathered} \Upsilon_1 < 0 \\ \begin{bmatrix} 2M & \bar{\tau}N \\ \bar{\tau}N^T & 2\bar{\tau}S \end{bmatrix} > 0 \\ \begin{bmatrix} G_i & J \\ J & I_i \end{bmatrix} \geq 0 \quad \begin{bmatrix} T_1 & J \\ J & O \end{bmatrix} \geq 0 \quad \begin{bmatrix} G_i & I \\ I & U_i \end{bmatrix} \geq 0 \\ \begin{bmatrix} T_1 & I \\ I & S \end{bmatrix} \geq 0 \quad \begin{bmatrix} O & I \\ I & F \end{bmatrix} \geq 0 \quad \begin{bmatrix} I_i & I \\ I & H_i \end{bmatrix} \geq 0 \\ \begin{bmatrix} L & I \\ I & J \end{bmatrix} \geq 0 \quad \forall i = 1,2 \end{gathered} \quad (36)$$

If the solution of this minimization problem is $8n$ (n is the system dimension which leads to the dimension of $n \times n$ for the matrices L, J, Δ, ∇, O, F, T_1, S, G_i, U_i, I_i, H_i), that is, $Tr(LJ + \nabla\Lambda + OF + T_1S + \sum_{i=1}^{2}(G_iU_i + I_iH_i)) = 8n$, from Theorem 1 it can be concluded that the time-delay system (4) with state-derivative feedback control law (2) is asymptotically stable with a noise attenuation level of γ. Although it is still not always possible to find the global

$$\begin{bmatrix} \Omega & -Y_1 & PA_d-\bar{\tau}Y_1 & 0 & PE & 0 & A^TR_1 & \bar{\tau}^2A^TA^TZ & A^TA^TR_2 & C^T \\ * & -Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -R_1-\bar{\tau}Y_1 & 0 & 0 & 0 & A_d^TR_1 & \bar{\tau}^2(AA_d)^TZ & (AA_d)^TR_2 & D_d^T \\ * & * & * & -(1/2)R_2 & 0 & 0 & 0 & \bar{\tau}^2A_d^TZ & A_d^TR_2 & 0 \\ * & * & * & * & -m^2\gamma^2I & 0 & E^TR_1 & \bar{\tau}^2E^TA^TZ & E^TA^TR_2 & D_2^T \\ * & * & * & * & * & -(1-m)^2\gamma^2I & 0 & \bar{\tau}^2E^TZ & E^TR_2 & 0 \\ * & * & * & * & * & * & -R_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -2\bar{\tau}^2Z & 0 & 0 \\ * & * & * & * & * & * & * & * & -2R_2 & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix}<0 \tag{37}$$

optimal solution, the proposed nonlinear minimization problem is easier to solve than the original non-convex feasibility problem. In fact, we can find a suboptimal maximal delay using an iterative algorithm almost similar to the algorithm presented in (Moon, et. al, 2001). Since it is numerically quite difficult to obtain the optimal solution such that $Tr(LJ+\nabla\Lambda+OF+T_1S+\sum_{i=1}^{2}(G_iU_i+I_iH_i))$ is exactly equal to $8n$, the conditions (6) ~ (8) are used as a stopping criterion in the algorithm.

Remark 6. Theorem 1 can be modified to manage bounded real lemma (BRL) problem, leading to the following corollary.

Corollary 2. Consider the time-delay system (5), set A_d=BK and D_d=D_1K. By the assumption of $\|A_d\|<1$, for prescribed scalars $\gamma>0$, $0<m<1$ and given A_d and D_d, the cost function $J_1(w)<0$ for all non-zero $w\in\mathcal{W}^{1,2}(0,\infty,\Re^p)\cap\mathcal{L}^2(0,\infty,\Re^p)$ and all uncertain bounded delays satisfying $0<\tau\le\bar{\tau}$, if there exist positive definite symmetric matrices P, Z, R_1, R_2, Q $\in\Re^{n\times n}$, negative definite symmetric matrix $Y\in\Re^{n\times n}$ and the matrix $X\in\Re^{n\times n}$ satisfying linear matrix inequalities (37) ~ (38).

$$\begin{bmatrix} X & \bar{\tau}Y_1 \\ \bar{\tau}Y_1^T & \bar{\tau}Z \end{bmatrix}>0 \tag{38}$$

in which,

$$\Phi=A^TP+PA+\bar{\tau}(1/2)X+(2-\bar{\tau})Y_1+Q$$

Remark 7. It should be noted that due to known matrices A_d and D_d in corollary 2, the above BRL with LMI conditions are obtained. A significant advantage of the resulting bounded real lemma representation in this corollary is its efficiency in analysis and design of the H∞ controllers with $\dot{x}(t-h_i)$'s coefficients depending on the controller parameters. To see this advantage, consider the LMI (37). Y_1 is a matrix variable which appears in matrix elements (1,1) and (1,3). It means that Y_1 affects on the negative definiteness of (1,1) as well as the value of A_d. Hence, despite the BRLs proposed in the literature (Fridman & Shaked, 2002, Jiang, & Han, 2005, Xu, et. al, 2006), Y_1=0 is not the best solution for LMI (37) in the feasibility region and consequently, A_d is not forced to be chosen zero when depending on the controller parameters.

4. CASE STUDY

In this section we provide the following example with the focus on the H∞ control issue related with the application of the state-derivative feedback.

4.1 *Vibration Suppression of a Platform*

The system under study is an active vibration suppression system presented by Vyhlídal, et. al (2009), in which, the state-derivative feedback has been used to control the system. The state space equations are represented by the following equations:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\\ \dot{x}_4(t)\end{bmatrix}=\begin{bmatrix}0&0&1&0\\0&0&0&1\\-k_1c_1&-k_2c_2&-b_1c_1&b_2c_2\\-k_1c_2&-k_2c_1&-b_1c_2&b_2c_1\end{bmatrix}\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\\x_4(t)\end{bmatrix}+\begin{bmatrix}0&0\\0&0\\c_1&c_2\\c_2&c_1\end{bmatrix}u(t-\tau)+\begin{bmatrix}0&0\\0&0\\k_1c_1+k_2c_2&b_1c_1+b_2c_2\\k_1c_2+k_2c_1&b_1c_2+b_2c_1\end{bmatrix}w(t) \tag{39}$$

$$z(t)=\begin{bmatrix}1&0&0&0\\0&1&0&0\end{bmatrix}\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\\x_4(t)\end{bmatrix}$$

where $w(t)=\begin{bmatrix}x_s & \dot{x}_s\end{bmatrix}^T$, $u(t)=\begin{bmatrix}u_1 & u_2\end{bmatrix}^T$, $c_1=1/m+L^2/I$, $c_2=1/m-L^2/I$. the parameters m and I represent the mass and moment of, k_1 and k_2 are the spring constants and $2L$ is the distance between two supporting points. Furthermore, x_1 and x_2 are the mass displacement from both sides and u_1 and u_2 are the control inputs. The model parameters are given as m=10kg, I=1kg m^2, L=1 m, k_1=500 N/m, k_2=700 N/m, b_1=10 N s/m and b_2= 20 N s/m.

Vyhlídal et. al (2009) presented a stability analysis condition for systems with derivative state feedback controller in presence of small delays. No controller synthesis method is obtained in Vyhlídal et. al (2009). They presented a state feedback controller gain for the system (39) which satisfies their proposed stability analysis condition as

$$K = \begin{bmatrix} 19.64 & 5.899 & 0.220 & -0.210 \\ 8.367 & 49.0 & -0.150 & 0.120 \end{bmatrix} \quad (40)$$

They showed that feedback system with state-derivative feedback controller gain obtained by Abdelaziz & Valášek (2005b) is extremely sensitive with respect to negligible delay. Moreover, the robust stability of the closed-loop system against small delays with state-derivative feedback gain (40) is achieved applying a first order filter to (2). As we will discuss in this section, the stability of the feedback system with the above controller is lost, if a large delay occurs. To illustrate the result of the present paper which possessed both analysis and synthesis of state-derivative controller, we use LMI toolbox in MATLAB and obtain the following state derivative controller gain for the closed-loop system which is a suboptimal solution of the nonlinear minimization problem mentioned in Remark 5 as follows

$$K = \begin{bmatrix} 0.910 & -1.12 & -0.147 & -0.19 \\ -0.735 & 0.35 & -0.26 & -0.14 \end{bmatrix} \quad (41)$$

This solution guarantees the stability of the closed-loop system with $\bar{\tau} = 0.7$ and γ=7.6. To have a good comparison between the controller gains (40) and what is obtained in here (41), first set $\bar{\tau} = 0.05$. The transient responses of the closed-loop systems with controllers (40) and (41) from an initial state $x(0)$=[-0.01 0.02 -0.02 0.01]^T are shown in Figs. 1 and 2 respectively.

As it can be seen in the Figs. 1 and 2, the feedback system with state derivative controller gains (40) and (41) are both stable, although our controller provides a better transient response compared to that of the controller gain (40). Now repeat the simulation with $\bar{\tau} = 0.7$ to see the impact of large delays on the stability of the closed-loop systems with controller gains (40) and (41). Figs. 3 and 4 show the transient responses of the feedback systems with $\bar{\tau} = 0.7$ from the same initial states as previous simulation. As it can be seen, the stability of the closed-loop systems with controller gain (40) is lost due to high frequency oscillations, whereas our designed state derivative controller (41) provides fast and well damp response for the closed-loop system even in presence of the large delay $\bar{\tau} = 0.7$, though it is low gain. Furthermore, although both state-derivative controllers with gains (40) and (41) satisfy the condition proposed in Vyhlídal et. al (2009), but this condition is only an analysis condition and it is limited to stability of the closed-loop systems with small delays as well, whereas by our proposed method we attain the ability of both analysis and synthesis of stabilizing and H_∞ state-derivative controller in presence of sufficiently large delay.

5. CONCLUSIONS

H_∞ control of a time-delay system with uncertain input delay was elaborated in this paper. The resulting closed-loop system with the state-derivative feedback control law is a particular system of neutral type. In this system, the coefficients of the neutral terms depend on the control law parameters. Since state-derivative feedback is a good remedy in practice, the proposed control strategy is of great practical significance as well as theoretical accomplishment. We used Lyapunov theory to derive a new set of delay-dependent sufficient conditions in presence of uncertain bounded delay. These sufficient conditions were derived for the existence of an H_∞ state-derivative feedback controller for the closed loop system in terms of some matrix inequalities. Moreover, a case study is presented to illustrate the effectiveness of our method. Simulation results showed that our designed state-derivative feedback controller provides fast and well-damped response in presence of large delays, whereas applying the existing state-derivative feedback controllers in the literature is unable to guarantee the stability of the feedback system in such conditions.

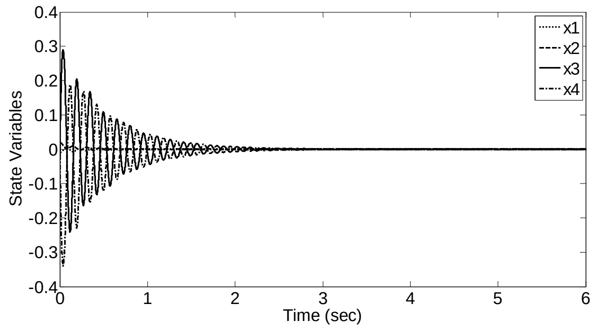

Fig. 1. Response of the closed-loop system with $\bar{\tau} = 0.05$ and state derivative feedback gain (40)

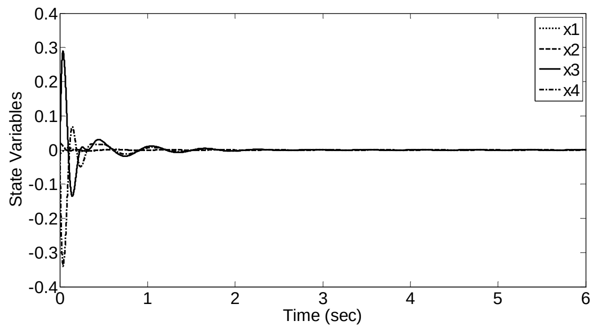

Fig. 2. Response of the closed-loop system with $\bar{\tau} = 0.05$ and state derivative feedback gain (41)

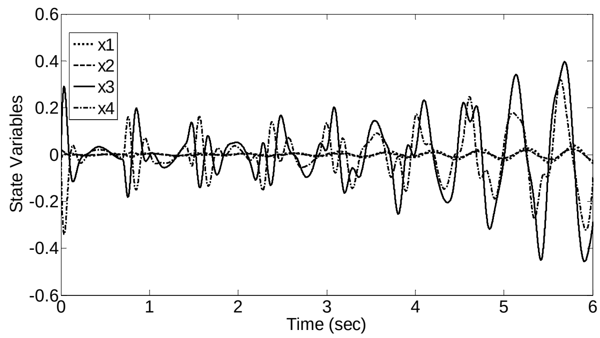

Fig. 3. Response of the closed-loop system with $\bar{\tau} = 0.7$ and state derivative feedback gain (40)

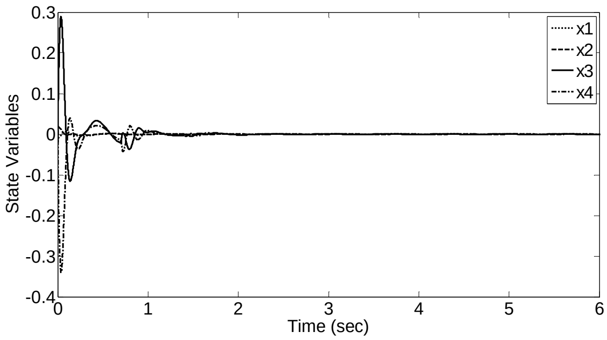

Fig. 4. Response of the closed-loop system with $\overline{\tau} = 0.7$ and state derivative feedback gain (41)

REFERENCES

Abdelaziz, T.H.S. and Valášek, M. (2004). Pole-placement for SISO linear systems by state-derivative feedback. *IEE Proceedings of Control Theory Application*, volume (151), pp. 377–385.

Abdelaziz, T.H.S. and Valášek, M. (2005a). Direct algorithm for pole placement by state-derivative feedback for multi-input linear systems-nonsingular case. *Kybernetika*, volume (41), pp. 637-660.

Abdelaziz, T.H.S. and Valášek, M. (2005b). State derivative feedback by LQR for linear time-invariant system. *In Proceeding of 16th IFAC World Congress, Prague, Czch Republic*.

Assuncão, E., Teixeira, M.C.M., F.A., Faria, Da Silva, N.A.P. and Cardim, R. (2007). Robust state-derivative feedback LMI-based designs for multivariable linear systems. *International Journal of Control*, volume (80), pp. 1260-1270.

Baser, U. (2002). The H_∞ control problem for neutral systems with multiple delays. *In: Proceedings of 15th IFAC World Congress, Barcelona, Spain*, pp. 20-26.

Baser, U. (2003). Output feedback H_∞ control problem for linear neutral systems: delay independent case. *Journal of Dynamic Systems, Measurement and Control*, volume (125), pp. 177-185.

Chen, J.D. (2005). LMI-based robust H_∞ control of neutral systems with state and input delays. *Journal of Optimization Theory and Applications*, volume (126), pp. 553-570.

Duan, Y.F., Ni, Y.Q. and Ko, J.M. (2005). State-derivative feedback control of cable vibration using semiactive magnetorheological dampers. *Computer-Aided Civil and Infrastructure Engineering*, volume (20), pp. 431-449.

Faria, F.A., Assuncão, E, Teixeira, M., Cardim, R. and da Silva, N.A.P. (2009). Robust state-derivative pole placement LMI-based designs for linear systems. *International Journal of Control*, volume (82), pp. 1-12.

Fridman E. and Shaked, U. (2002). A Descriptor System Approach to H_∞ Control of Linear Time-Delay Systems, *IEEE Transaction on Automatic Control*, volume (47), pp. 253-270.

Ghaoui, L.El. Oustry, F. and Rami, M.A. (1997). A cone complementarity linearization algorithm for static output-feedback and related problems, *IEEE Transaction on Automatic Control*, volume (42), pp. 1171-1176.

Gu, K. (2000). An integral inequality in the stability of time-delay systems. *Proceeding of the 39th IEEE Conference on Decision and Control, Sydney, Australia*, pp. 2805-2810.

Jiang, X. and Han, Q.L. (2005). On H_∞ Control for Linear Systems with Interval Time-Varying Delay, *Automatica*, volume (41), pp. 2099-2106.

Li, Y., Xu, S., Zhang, B. and Chu, Y. (2008). Robust stabilization and H_∞ control for uncertain fuzzy neutral systems with mixed time delays. *Fuzzy Sets and Systems*, volume (159), pp. 2730-48.

Lien, C.H. (2005). H_∞ observer-based control for a class of uncertain neutral time-delay systems via LMI optimization approach. *Journal of Optimization Theory and Applications*, volume (127), pp. 129-144.

Moon, Y.S., Park, P., Kwon, W.H. and Lee Y.S. (2001). Delay dependent robust stabilization of uncertain state delayed systems. *International Journal of Control*, volume (74), pp. 1447-1455.

Park, J.H., (2005). LMI optimization approach to asymptotic stability of uncertain neutral delay differential equation with time-varying coefficients. *Applied. Mathematics and. Computation*, volume (160), pp. 355–61.

Skogestad, S. and Postlethwaite, I. (2005). *Multivariable Feedback Control: Analysis and Design*, John Willy & Sons, England.

Souza, F., Palhares, R. and Leite, V. Improved robust H_∞ control for neutral systems via discretised Lyapunov-Krasovskii functional. *International Journal of Control*, volume (81), pp. 1462-74.

Suplin, V., Fridman, E. and Shaked, U. (2006). H_∞ control of linear uncertain time-delay systems-a projection approach. *IEEE Transactions on Automatic Control*, volume (51), pp. 680-685.

Vyhlídal, T., Michiels, W., Zítek P. and McGahan, P. (2009). Stability impact of small delays in proportional–derivative state feedback, *Control Engineering Practice*, volume (17), pp. 382-393.

Xu, S., Lam, J. and Yang, C. (2001). H_∞ and Positive-Real Control for Linear Neutral Delay Systems. *IEEE Transaction on Automatic Control*, volume 46, pp. 1321-1326.

Xu, S., Lam J. and Zou, Y. (2006). New Results on Delay-dependent Robust H_∞ Control for Systems with Time-Varying Delays. *Automatica*, volume (42), pp. 343-348.

Zhang, Z., Wang, W. and Yang, B. (2007). Delay and its time-derivative dependent robust stability of neutral control system, *Applied Mathematics. and Computation*, volume (187), pp. 1326–32.