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LMI Method to Design and Optimize FIR Loop Filters in Fixed WiMAX PLL

Hala J. El-Khozondar, Hatem ELaydi, Fady El-Batta

Electrical Engineering Department, Islamic University of Gaza, P. O. Box 108,
Palestinian Territory (Tel: 972-8-2860700 Ext. 1014;
email: hkhozondar@iugaza.edu.ps; helaydi@iugaza.edu.ps; batta1979@hotmail.com)

Abstract: Fixed WiMAX is the fourth generation of communication systems. Optimization of the performance of Fixed WiMAX system is an important issue in communication system. In this paper, we designed Finite Impulse Response (FIR), digital filters by Semi- Definite Programming (SDP) using SeDuMi (self-dual minimization) toolbox software. The stability is assured using LMI constraints. Design efficiency and performance of the proposed method are illustrated by simulations and comparisons to other design methods. The minimum length FIR filter algorithm was used to proof that the order of the FIR filter which was designed is optimal for our design specifications. The proposed method gave better result with regard to all specifications of control signal, where we had the faster system and more stable than the system with FIR filter which was designed using SDP method and simulated using toolbox software (CVX).

Keywords: Filter Design, control device, wimax, PLL, FIR.

1. INTRODUCTION

PLLs have been developed for many years. A PLL is an electronic control system that generates a signal that has a fixed relation to the phase of a "reference" signal (Hsieh & Huang,1996). An incomplete list of specific tasks accomplished by PLLs include carrier recovery, clock recovery, tracking filters, frequency and phase demodulation, phase modulation, clock synchronization, and frequency synthesis. A frequency synthesizer is an electronic system used for generating a range of frequencies from a single fixed time base or oscillator to high frequencies (Abramovich, 2002). The frequency synthesizer is used in Fixed and Mobile WiMAX communication system. Fig. 1 shows the basic form of a PLL which consists of three fundamental functional blocks namely (Abramovich, 2002)

- A Phase Detector (PD)
- A Loop Filter (LF)
- A voltage controlled oscillator (VCO).

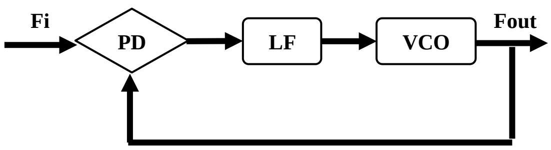

Fig. 1. A basic PLL Block with F_i and F_{out} are input and output frequencies respectively.

There are many different methods to design the loop filters in PLL. One of the main methods which used to design those filters is the LMI method as in (Chou, et al., 2006; Long, et al. 2007). Those designed filters were used in fractional-*N* frequency synthesizer in transceiver device for GPS communication systems. Digital filters can be designed using a minimax method based on SDP. FIR is an example of digital filters which has been designed using SDP as explained for example in Antoniou and Lu (2007). The stability of the filters was assured by using a single LMI constraint derived from the well-known Lyapunov theory (Boyd, et al. 1994). In this work, FIR is introduced as loop filter for PLL.

Antoniou (2005) designed the FIR filters using the weighted-Chebyshev method which is based on the Remez exchange algorithm. Al-Baroudi (1997) formulated the FIR filters design problem as a Linear Objective Optimization problem with LMI constraints, and solved it. FIR filter design via SDP and spectral factorization was presented by Wu, et al. (1996). The primal-dual path following algorithm was used to solve the SDP problem by Antoniou and Lu (2007) . In previous work (Al-Quqa, 2009), the optimal design of PLL Mobile WiMAX system is established. In which the digital filters (FIR) were designed using linear programming and convex programming (SDP). In another work by the authors IIR digital filter is designed for Fixed WiMAX using LMI method and gave better result than the system which have IIR filter which designed using LP method (Al-Batta, 2010). In this paper we introduced a new optimal design for FIR digital filters using LMI method. We will reformulate the filter design problem as SDP problem and solve it by self-dual minimization algorithm. We will employ multi-objective control technique to deal with the various design objectives such as:

1. Frequency range used for Fixed WiMAX system (3.5 - 5.8) GHz.
2. Stable system.
3. Small settling time.
4. Small overshoot.
5. Small raising time.

Trade-off among the conflicting objectives will be made using SDP in conjunction with appropriate adjustment of certain design parameters. In our system we used fractional-*N* frequency synthesizer to reduce the noise. FIR digital low-pass filter is used to eliminate the high frequency components resulting in more immunity to noise, where the VCO noise is neglected (Kozak & Friedman, 2004).

The structure of our paper is as follow:
Section 2 introduced the design of optimal FIR digital filters using SDP method. Section 3 displays the results using the toolbox, *SeDuMi* and compares our results with other. Finally, the conclusion is given in section 4.

2. DESIGN OF PLL FILTER

2.1 Review Stage

The Loop Filter, VCO (very stable crystal oscillator with a divide by *N*-programmable divider in the feedback loop) and PD together comprise a frequency synthesizer.

The programmable divider divides the output of the VCO by N and locks to the reference frequency generated by a crystal oscillator. We considered a real design problem of a frequency synthesizer loop filter with the general specifications as shown in Table 1 (AL-Batta, 2009). The design process is divided into several stages. We first present the overall block of frequency synthesizer, then select the integer value of N according to reference frequency and resolution.

Table 1: Design Specifications

DESIGN SPECIFICATIONS	
PARAMETER	**SPECIFICATION**
Frequency Range	3.5 GHz – 5.8 GHz
Resolution	240 KHz
Overshoot Less than	2%
Settling time Less	Less than 5µs

2.2 Fractional-N PLL block diagram

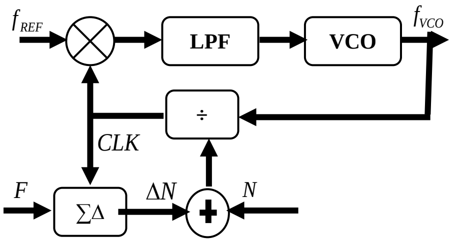

Fig. 2. Basic Fractional-N PLL Block Diagram.

where f_{REF} and f_{VCO} are reference and voltage control oscillator frequencies respectively. As shown in Fig. 2., the fractional-N PLL block diagram consists of

1. Phase/Frequency detector which is assumed to be XOR type.

2. Loop filter which is the objective of our design, is a low-pass filter (LPF).

3. Voltage Control Oscillator (VCO).

We begin the design with integer-N PLL: 240 kHz Reference pushes N from 14583 to 24167 (3500 / 0.240) to (5800 /0.240). As a result the loop filter cut off (less than 12.5 KHz) produces long settling time and VCO phase noise increased by

$$20*\log_{10}(N) \approx 89dB$$

To overcome the previous drawback, we use the Multi-Modulus Fractional PLL with these properties:
• Fractional value between *N* and *2N-1* (64-127).
• Sigma Delta Modulator (programmable resolution).
• Large reference (20 MHz) for good trade off with settling time.
• Reduced *N* impact on phase noise by 45 dB over integer *N*.

As a result, for *N* = (76 ~ 125), it can produce frequency range (1533MHz ~ 2533MHz), which can be up converted to (3500MHz ~ 5800MHz), for more information see AL-Batta (2009).

2.3 FIR Low Pass Filter Design by LMI

In this section, we designed FIR filter using a minimax method based on SDP. In fact, the SDP approach can be used to design FIR filters with arbitrary amplitude and phase responses including certain types of filters that cannot be designed with other methods such as low-delay FIR filters with approximately constant passband group delay (Antoniou & Lu, 2007). Consider an FIR filter of order N characterized by the general transfer function

$$H(z) = \sum_{n=0}^{N} b_n z^{-n} \tag{2.1}$$

The frequency response of such a filter can be expressed as

$$H(\omega) = \sum_{n=0}^{N} b_n e^{-jn\omega} = b^T c(\omega) - jb^T s(\omega). \tag{2.2}$$

$$c(\omega) = [1 \quad \cos\omega \cdots \cos N\omega]^T, \tag{2.3}$$

$$s(\omega) = [0 \quad \sin\omega \cdots \sin N\omega]^T, \tag{2.4}$$

$$b = [b_0 \;\; b_1 ....b_N]^T \quad . \tag{2.5}$$

Let $H_d(\omega)$ be the desired frequency response and assume a normalized sampling frequency of 2π. In a minimax design, we need to find a coefficient vector b that solves the optimization problem

$$\text{minimize} \quad \delta \tag{2.6}$$

subject to $W^2(\omega)\left|H(\omega)-H_d(\omega)\right|^2 \le \delta$ for $\omega \in \Omega$ (2.7)

We can write

$$W^2(\omega)\left|H(\omega)-H_d(\omega)\right|^2 = W^2(\omega)\left\{\left[b^T c(\omega)-H_r(\omega)\right]^2+\left[b^T s(\omega)+H_i(\omega)\right]^2\right\}$$

$$=\alpha_1^2(\omega)+\alpha_2^2(\omega) \quad (2.8)$$

where

$$\alpha_1(\omega)=b^T c_w(\omega)-H_{rw}(\omega) \quad (2.9)$$

$$\alpha_2(\omega)=b^T s_w(\omega)+H_{iw}(\omega) \quad (2.10)$$

$$c_w=W(\omega)c(\omega)\,,\ s_w=W(\omega)s(\omega) \quad (2.11)$$

$$H_{rw}(\omega)=W(\omega)H_r(\omega)\,,\ H_{iw}=W(\omega)H_i(\omega) \quad (2.12)$$

Using (2.8), the constraint in (2.7) becomes

$$\delta-\alpha_1^2(\omega)-\alpha_2^2(\omega)\ge 0 \qquad \text{for} \quad \omega\in\Omega \quad (2.13)$$

It can be shown that the inequality in (2.2) holds if and only if

$$D(\omega)=\begin{bmatrix}\delta & \alpha_1(\omega) & \alpha_2(\omega)\\ \alpha_1(\omega) & 1 & 0\\ \alpha_2(\omega) & 0 & 1\end{bmatrix}\ge 0 \qquad \text{for} \quad \omega\in\Omega \quad (2.14)$$

$D(\omega)$ is positive definite for the frequencies of interest (Antoniou & Lu, 2007). If we write

$$x=\begin{bmatrix}\delta\\ b\end{bmatrix}=\begin{bmatrix}\delta\\ b_0\\ b_1\\ \vdots\\ b_N\end{bmatrix},\ c=\begin{bmatrix}1\\ 0\\ \vdots\\ 0\end{bmatrix} \quad (2.15)$$

Then matrix $D(\omega)$ is *affine* with respect to x in Eq.2.15. If $S=\{\omega_i : 1\le i\le M\}\subset\Omega$ is a set of frequencies which is sufficiently dense on Ω, then a discretized version of (2.8) is given by

$$F(x)\ge 0 \quad (2.16)$$

Where $F(x)=diag\ \{D(\omega_1), D(\omega_2), \cdots,\ D(\omega_M)\}$ (2.17)

$$=\begin{bmatrix}D(\omega_1) & 0\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots 0 & & \\ 0 & D(\omega_2) & 0\cdots\cdots\cdots\cdots\cdots 0 & \\ \vdots & 0 & D(\omega_3)\cdots\cdots\cdots 0 & \\ \vdots & \vdots & \vdots \quad \ddots & \\ 0 & 0 & 0\cdots\cdots\cdots D(\omega_M)\end{bmatrix}$$

$D(\omega)$ is found in Eq. 2.14.

and the minimization problem in (2.6) can be converted into the optimization problem

minimize $c^T x$ (2.18)

subject to $F(x)\ge 0$ (2.19)

where

$$c=\begin{bmatrix}1\\ 0\\ \vdots\\ 0\end{bmatrix} \quad (2.20)$$

The problem in (2.18) is an SDP problem. We have shown that FIR design with LMI constraints which is minimizing the value of the squared weighted error δ between the designed FIR low pass filter and the desired low pass loop filter can be cast as SDP feasibility problems. In fact, many extensions of the problem can be handled by simply adding a cost function and/or LMI constraints to our SDP formulation. The other problem is finding the optimal order of FIR filter which gives the desired specifications.

2.4 Minimum-length FIR design

The length of an FIR filter is a quasi-convex function of its coefficients (Wu, et al., 1996). Hence, the problem of finding the minimum-length FIR filter given magnitude upper and lower bounds as following:

minimize G (2.21)

subject to $L(\omega_i)\le\left|H_N(\omega_i)\right|\le U(\omega_i)\,,\quad i=1,\cdots,M$ (2.22)

where $G = N+1$ is the length of an FIR filter, $L(\omega)$ is lower magnitude bound and $U(\omega)$ is upper magnitude bound. The problem (2.21) is quasi-convex and can be solved using bisection on N. We will use the theorem stated in Appendix A to solve (2.21) then, we can formulate the SDP feasibility problem as:

Find $r\in R^n$ *and* $P=P^T\in R^{(G-1)\times(G-1)}$ (2.23)

subject to $L^2(\omega_i)\le R(\omega_i)\le U^2(\omega_i)\,,\quad \omega_i\in\Omega$ (2.24)

$$\begin{bmatrix}P-A^TPA & C^T-A^TPB\\ C-B^TPA & D+D^T-B^TPB\end{bmatrix}\ge 0 \quad (2.25)$$

with (A;B;C;D) are given [see Appendix A.] and $r=b$ is autocorrelation coefficients in Eq. 2.1. The SDP feasibility problem (2.23) can be cast as an ordinary SDP and solved efficiently. Each iteration of the bisection in (2.21) involves solving an SDP feasibility problem in (2.23).

3. ANALYSIS AND COMPARISON

The design process is divided into two steps presented as follows: The first step is to design FIR digital low pass filter using SDP algorithm. The second step is to simulate the designed filters, discuss the results, and compare them with others.

3.1 Design FIR Low Pass Filter

To design FIR we implemented the SDP algorithm with the following parameters:
Order of filter *(n) = 18;* where the order is implemented by the minimum-length FIR design algorithm (AL-Batta, 2009).
Stopband frequency $(w_{stop}) = 0.2\pi\ rad/s$.
Maximum peak passband error *(δ) = 0.023;*
We begin the design process by using SeDuMi toolbox to solve the SDP problem in (2.18).
Using MATLAB code, the FIR filter magnitude response is shown in Fig. 3. The designed FIR filter specifications are as follow:

• The maximum pass band ripple = - 0.3167 dB with $w_{pass} = 0.006\pi\ rad/s$.
• Stop band attenuation below - 44.2459 dB with $w_{stop} = 0.2\pi\ rad/s$.

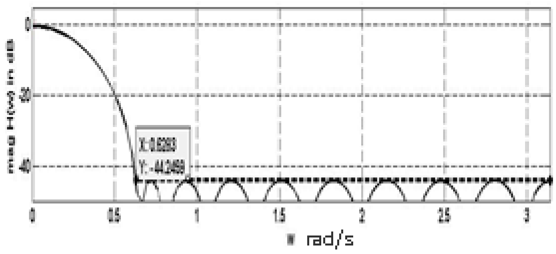

Fig. 3. FIR Filter Magnitude Response

3.2 Simulation

To simulate the designed filters, we used MATLAB simulink and SeDuMi to construct simulation module shown in Fig. 4. When we simulate the obtained FIR filter with Fixed WiMAX simulation block diagram shown in Fig. 4, the simulation work properly and the correct output frequency has been achieved.

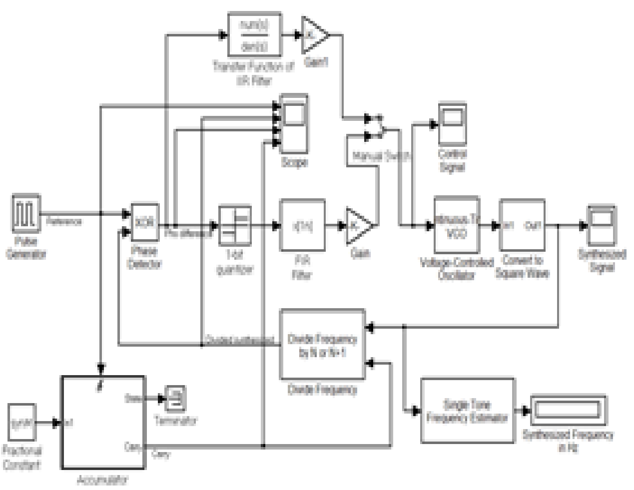

Fig. 4. PLL Frequency Synthesizer Simulation Model

The control signal using this FIR filter is shown in Fig. 5 which explores that:

• Overshoot is eliminated.
• The settling time = 0.22μs.
• The rise time =0.125μs.

We have shown that, FIR design with minimizing the value of the squared weighted error δ between the designed FIR low pass filter and the desired low pass loop filter (ideal case).

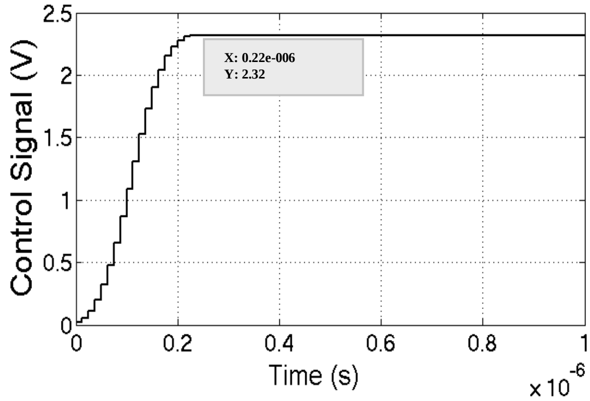

Fig. 5. The Control Signal of VCO input using Designed FIR Filter

As a result, we conclude that designing FIR digital filter by using SeDuMi toolbox to solve the SDP problem produced a very fast stable system compare with method in Al-Quqa (2009). Table 1 summarizes these differences. In previous work (Al-Batta, 2010), IIR digital filter is designed for Fixed WiMAX using LMI method and Table 2 summarizes the differences between the frequency response in FIR & IIR digital filters.

Table 1. Comparison in frequency and control signal responses of FIR digital filters design

Frequency response of FIR digital filters design		
PARAMETER	**FIR Filter design (SDP by CVX)**	**FIR Filter design (SDP by SeDuMi)**
Filter order	19	19
Passband frequency(rad/s)	0.006π	0.006π
Stopband frequency(rad/s)	0.2π	0.2π
maximum pass band ripple (dB)	0.4	0.3167
Stop band attenuation (dB)	44.5	44.2459
Control signal response of FIR digital filters design		
Settling Time (μs)	0.4998	≈ 0.22
Rise Time (μs)	≈ 0.25	0.125
Maximum Overshoot (%)	zero	zero

Table 2. Comparison in frequency and control signal responses of FIR & IIR digital filters design

Frequency response of digital filters design		
PARAMETER	**IIR Filter design (SDP)**	**FIR Filter design (SDP)**
Filter order	3	19
Passband frequency(rad/s)	0.006π	0.006π
Stopband frequency(rad/s)	0.7π	0.2π
maximum pass band ripple (dB)	0.0179	0.3167
Stop band attenuation (dB)	43.41	44.2459
Control signal response of digital filters design		
Settling Time (μs)	≈ 0.25	≈ 0.22
Rise Time (μs)	0.1	0.125
Maximum Overshoot (%)	zero	zero

5. CONCLUSIONS

Phase locked loop is an interesting topic for the research, because of its usage in many applications (electrical - control – communication …etc). In this work, we introduced a novel loop filter design method for PLL loop filter taking into consideration various design objectives; i.e., small settling time, small overshoot and meeting Fixed WiMAX requirements. The SDP based method for the design of the digital FIR low pass loop filter was used under LMI constraint. This constraint was minimizing the value of the squared weighted error δ between the designed FIR low pass filter and the desired low pass loop filter (ideal case). The minimum length FIR filter algorithm was used to proof that the order of the FIR filter which was designed is optimal for our design specifications. MATLAB was used to run SDP formulation of the design problem's equation and solved it using SeDuMi toolbox (Self-Dual Minimization). Then, the resultant FIR filter was used in designing fractional-*N* synthesizer as a loop filter to derive the desired Fixed WiMAX frequency. Simulations showed that the FIR filter which was designed using SDP method and simulated using toolbox software (SeDuMi) minimizing the maximum passband ripple of FIR filter in dB (0.3167 dB) was lower than the method in Al-Quqa (2009) (0.4 dB). The proposed method gave better result with regard to all specifications of control signal, where we had the faster system and more stable than the system with FIR filter which was designed using SDP method and simulated using toolbox software (CVX).

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Appendix A

Theorem 1:
Given a discrete-time linear system (*A;B;C;D*), *A* stable, (*A;B;C*) minimal and $D + D^T \geq 0$ the transfer function $H(z) = C(zI - A)^{-1}B + D$ satisfies

$$H(\omega) + H^*(\omega) \geq 0 \text{ for all } \omega \in [0,2\pi] \qquad (A.1)$$

if and only if there exists real symmetric matrix *P* such that the matrix inequality

$$\begin{bmatrix} P - A^T PA & C^T - A^T PB \\ C - B^T PA & D + D^T - B^T PB \end{bmatrix} \geq 0 \qquad (A.2)$$

is satisfied. The theorem proof is found in Antoniou and Lu (2007). In order to apply Theorem 1, we would like to define $(A;B;C;D)$ in terms of r such that

$$C(zI-A)^{-1}B+D = \frac{1}{2}r(0)+r(1)z^{-1}+\cdots+r(G-1)z^{-(G-1)}, \tag{A.3}$$

where

$$r(g)=\sum_{k=-\infty}^{\infty}h(k)\,h(k+g), \tag{A.4}$$

where we take $h(k) = 0$ for $k < 0$ or $k > G - 1$. The sequence $r(g)$ is symmetric around $g = 0$. Note that the Fourier transform of $r(g)$, for $g \le -G$ or $g \ge G$ and $r(0) \ge 0$

$$R(\omega)=\sum_{g=-\infty}^{\infty}r(g)\,e^{-j\omega g}=\left|H(\omega)\right|^{2}, \tag{A.5}$$

is the power spectrum of $h(g)$ and we use r as our design variables. An obvious choice is the controllability canonical form:

$$A=\begin{bmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & & 0 \end{bmatrix}, \qquad B=\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{A.6}$$

$$C=\begin{bmatrix} r(1) & r(2) & \cdots & r(G-1) \end{bmatrix}, \qquad D=\frac{1}{2}r(0).$$

It can be easily checked that $(A;B;C;D)$ given by (A.6) satisfies (A.2) and all the hypotheses of Theorem 1. Therefore, the existence of r and symmetric P that satisfy the matrix inequality (A.2) is the necessary and sufficient condition for $R(\omega)\ge 0$ for all $\omega \in [0,\pi]$, by Theorem 1. Note that (A.2) depends affinely on r and P.