

# A Heterogeneous Agent Network Model and Its Synchronization Analysis

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**Abstract:** This paper presents a heterogeneous agent network model as a simple representation to simultaneously characterize the time-invariant and time-varying topologies of various networked systems in communication and social networks, where two types of nodes: mobile agents, abstracted as random walkers in plane, and fixed agents interact with each other according to the neighborhood rule. The analytical results show that the mobile agent density determines synchronization of the considered heterogeneous network under the fast-switching constraint. In particular, a global synchronization behavior appears by introducing a proper mobile agent density. Numerical experiments verify the theoretical results above acquired.

*Keywords:* synchronization, agent network, switching topology.

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## 1. INTRODUCTION

Time synchronization is one of the fundamental problems in pervasive computing, since synchronized physical time are required in a lot of situations, such as sensor data fusion for detecting the same event by different sensors. Most existing literatures (Mills (1991), Sundararaman et al.(2005)) consider time synchronization by exchanging time information in wireless sensor networks (WSNs). Such networks are usually featured as static topology, i.e. all nodes are deployed in fixed locations. However, the methods of time synchronization might not be the optimal choice for a cyber physical system (CPS). CPS can be simply considered as a combination of WSNs and mobile nodes. Different from WSNs, the network topology changes in a CPS network as mobile nodes move. Therefore, it will be interesting to explore that how mobility of nodes can positively affect the methodologies for time synchronization.

Agent network model seems to be a good solution to investigate time synchronization in such communication systems. Generally speaking, each agent in the agent network is equipped with an identical oscillator, and switching topology is constructed via the change of neighbouring interactions. The mobile agent network, indeed, can be used to explore many problems such as clock synchronization in mobile robots (Chen et al. (2006)), swarming animals or the appearance of synchronized bulk oscillations (Danø et al. (2006)), consensus problem in multi-agent systems (Peng et al. (2009)) and so on, partially because of a good choice to capture jumps or switches of coupling evolutions. Frasca et al (2008) investigated fast-switching synchronization of such a moving agent network and pointed out that the density of mobile agents determines network synchronization. Shi et al (2010) developed the mobile agent network model by assigning different emission powers to each agent, which

further provides an insight into the collective behaviours of coupled agent systems (Wang et al.(2010a,b)).

Actually, there are two types of nodes according to the role of individuals in clock synchronization of the considered communication systems: fixed nodes (nodes in physical networks), mobile nodes (mobile sensors in wireless network). Apparently, the fixed nodes and the mobile nodes can not be equally treated in modeling and analyzing the system. Also notice that the networked systems can be simply decomposed into a (relative) static sub-network and a switching sub-network due to the heterogeneity of nodes. Such a heterogeneous property can be also found in many other systems, e.g., in social networks, lobby groups go about inducing voters whose attitudes are already interacted by a fixed relationship to elect a candidate or to give up an initial view, where the lobbies can be depicted by mobile agents, and static topology seems to be more suitable to characterize interactions between voters (Amblard and Deffuant (2004)); and in volleyball, the libero as a mobile agent influences the whole team cohesion, while other players share relatively fixed connections. Then there appears a question: Is synchronization of the heterogeneous network easier to achieve or not under the existence of mobile agents. There is no doubt, lobbies in social networks, mobile wireless sensors, or volleyball libero player, seem to work as the role of pinned nodes — guiding their neighbour nodes towards the desired objective — in synchronizing a complex network of coupled systems through pinning (Grigoriev et al. (1997), Li et al. (2004), Wang et al. (2009)). Furthermore, synchronization in these heterogeneous agent networks seems to be realized more easily compared with static networks. This paper is an attempt to explore synchronized behaviour based on a heterogeneous agent network. In this paper, a heterogeneous agent network model is presented to characterize a mixture feature of real-world network, where some agents with time-

invariant interactions are assigned to fixed positions in planar space, while others are considered as random walkers in the plane whose connections are dynamically established by the change of neighbouring agents. With fast-switching constraints, the synchronization problem of the agent network is then investigated. Particularly, the effect of mobile agents to synchronization of the heterogeneous network is discussed, which provides an insight into regulatory mechanisms and design of complex systems.

The rest of this paper is organized as follows: A heterogeneous agent network model is presented in Section 2. In Section 3, synchronization of the heterogeneous agent network is investigated under fast-switching constraints. Further discussions and illustrative examples for validation are given in Section 4. Conclusions are finally drawn in Section 5.

## 2. A HETEROGENEOUS AGENT NETWORK MODEL

Generally, a complex network consisting of  $l$  linearly and diffusively coupled nodes is described by

$$\dot{x}_i = f(x_i) - \sigma \sum_{j=1}^l G_{ij}^s H x_j, \quad i = 1, \dots, l \quad (1)$$

where  $x_i = (x_{i1}, x_{i2}, \dots, x_{in}) \in R^n$  is the state vector of node  $i$ ,  $f: R^n \rightarrow R^n$  is a nonlinear smooth vector-valued function, governing the dynamics of each isolated node,  $\sigma$  is the overall coupling strength,  $H \in R^{n \times n}$  is the inner linking matrix, and coupling matrix  $G^s = (G_{ij}^s) \in R^{l \times l}$  is a zero-row sum constant matrix, describing the network topology. If network (1) is connected in the sense of having no isolated clusters and edges signify the bidirectional ability, then  $G^s$  is symmetric and all its eigenvalues can be ranked as  $0 = \mu_1 < \mu_2 \leq \dots \leq \mu_l$ , where the eigenratio  $R^s = \mu_l / \mu_2$  is used to measure network synchronizability.

To obtain a heterogeneous agent network, one first assigns all the  $l$  nodes in network (1) to be fixed agents. For simplicity, we denote by  $N_l$  the set of fixed agents (circles in Fig.1), and all agents in  $N_l$  are uniformly distributed in a two-dimensional space of size  $L \times L$  with periodic boundary conditions. Moreover,  $m$  mobile agents are introduced to the plane (squares in Fig.1), each of which is considered as a random walker whose position and orientation are updated according to

$$\begin{cases} y_i(t + \Delta t) = y_i(t) + v_i(t) \Delta t \\ \theta_i(t + \Delta t) = \eta_i(t + \Delta t) \end{cases} \quad \forall i \in N_m, \quad (2)$$

where  $N_m$  is the set of mobile agents,  $y_i$  and  $\theta_i$  are the position and orientation angle of agent  $i$  at time  $t$ , respectively,  $\eta_i$ ,  $1 \leq i \leq m$ , are independent random variables chosen at each time unit with uniform probability from the interval  $[-\pi, \pi]$ ,  $v_i$  is the velocity of agent  $i$ , and  $\Delta t$  is the time unit. In the following, assume that each agent in the heterogeneous network is associated with a chaotic oscillator whose state

variable is characterized by  $x_i \in R^n$ . Then agent  $i$  evolves according to  $\dot{x}_i = f(x_i)$ . Moreover, consider the case of Rössler oscillators, where the state dynamics of each agent is given by

$$\begin{cases} \dot{x}_{i1} = -(x_{i2} + x_{i3}) \\ \dot{x}_{i2} = x_{i1} + a x_{i2} \\ \dot{x}_{i3} = b + x_{i3}(x_{i1} - c) \end{cases} \quad (3)$$

with  $x_i = (x_{i1}, x_{i2}, x_{i3})^T$ , and  $a = b = 0.2$ ,  $c = 7$ . It is obvious that, the topology of the heterogeneous agent network can be conveniently described by graph  $\varphi = \{N, E\}$ , where  $N = N_l \cup N_m$  is the node set (including  $l$  fixed agents and  $m$  mobile agents) and  $E = N \times N$  is the edge set of the graph, which is defined as follow: each mobile agent interacts at a given time with only those agents located within a neighbourhood of an interaction radius according to the rule of moving neighbourhood network. In detail, agents  $i$  and  $j$  are said to be adjacent if and only if

$$\|y_i(t) - y_j(t)\| < r, \quad \forall i \in N_m, j \in N \quad (4)$$

at time  $t$ , where  $r$  is a parameter that defines the size of a neighbourhood,  $\|\cdot\|$  refers to an induced norm. And for any two fixed agents,  $i, j \in N_l$ , the connection between them is a constant, i.e.,  $G_{ij} = G_{ij}^s$ . In other words, the constant matrix  $G^s$  describes the static topology of the heterogeneous network. Hence, a heterogeneous agent network is constructed by combining fixed and mobile agents, chaotic oscillators and their dynamical laws, where the heterogeneous couplings include the time-invariant connections between fixed agents and the switching connections due to the moving of agents. Based on above assumptions, the heterogeneous network can be mathematically formulated as:

$$\dot{x}_i = f(x_i) - \sigma \sum_{j=1}^N G_{ij}(t) H x_j, \quad i \in N, \quad (5)$$

where the coupling matrix  $G(t) = (G_{ij}) \in R^{N \times N}$  is defined as follows: for non-diagonal entries,  $G(t) = G^s$  if  $i, j \in N_l$ , and  $G_{ij}(t) = G_{ij}^s = -1$  if  $i \in N_m$  or  $j \in N_m$  are adjacent at time  $t$ , otherwise  $G_{ij}(t) = G_{ij}^s = 0$ ; and the diagonal entries satisfy

$$G_{ii}(t) = - \sum_{j=1, j \neq i}^N G_{ij}(t). \quad (6)$$

Thus there exists a completely synchronized state in agent network (5), i.e.,  $x_1(t) = x_2(t) = \dots = x_N(t) = s(t)$ . Fig.1 shows an illustration of the heterogeneous agent network (5). From the perspective of network structure, the heterogeneous network (5) can be simply regarded as a mixture of a static sub-network and a switching sub-network, where the static sub-network is constructed by fixed agents in  $N_l$  with time-invariant topology  $G^s$ , and the switching sub-network is generated by the time-varying switching of mobile agents.

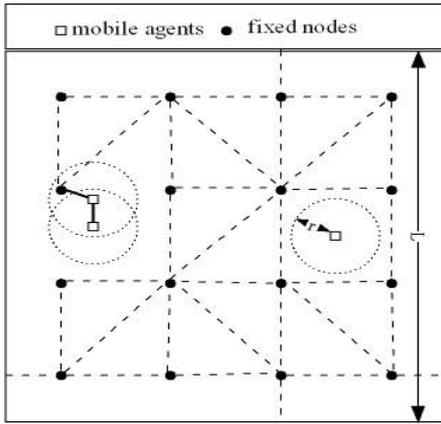


Fig. 1. A schematic illustration of the heterogeneous agent network

### 3. SYNCHRONIZATION ANALYSIS OF THE HETEROGENEOUS MODEL

This section investigates the synchronized behaviour of the heterogeneous network under the constraint of fast-switching.

Firstly, consider the average of  $G(t)$  for network (5). For an infinite sequence of contiguous time intervals  $[t_k, t_{k+1})$ ,  $k=0,1,\dots$  with  $t_0=0$  and  $t_{k+1}-t_k=\Delta t$ , where  $\Delta t$  is sufficiently small such that  $G(t)$  is a constant matrix for any  $t \in [t_k, t_{k+1})$ . Let  $G^k$  be the constant coupling matrix at  $k$ -th interval. Then  $G(t)$  can be expressed by

$$G(t) = \sum_i G^i \chi_{[t_k, t_{k+1})}^i(t), \quad (7)$$

where  $t=t_{k+1}$ , and  $\chi_{[t_k, t_{k+1})}^i(t)$  is the indicator function with support  $[t_k, t_{k+1})$ . From (7), the average of coupling matrix  $G(t)$  satisfying

$$\bar{G} = \sum_{i=1}^o p_i G^i, \quad (8)$$

where  $p_i$  is the probability that topological configuration  $i$  occurs,  $o$  is the number of possible configurations. Obviously,  $o$  is a finite number for a given heterogeneous network of finite order.

Recalling the evolution of network (5), the connections between any fixed agents are time-invariant, i.e.,  $\bar{G}_{ij} = G_{ij}^s, \forall i, j \in N_i$ ; and other cases,  $\bar{G}_{ij} = \sum_{k=1}^o p_k G_{ij}^k$ , if  $i \in N_m$ ; or  $j \in N_m$ .

Without loss of generality, let node  $i$  be a mobile agent. Then no matter agent  $j$  is a fixed or a mobile one, the probability that agent  $j$  is within the interaction radius  $r$  of agent  $i$  is equal to  $p = \pi r^2 / L^2$ . Therefore, the non-diagonal entries of for network (5) satisfy

$$\bar{G}_{ij} = \begin{cases} G_{ij}^s & \text{if } i, j \in N_i \\ -p & \text{otherwise} \end{cases} \quad (9)$$

and  $\bar{G}$  is of the following form

$$\bar{G} = \begin{pmatrix} G^s + mpI_l & -p\theta & \cdots & -p\theta \\ -p\theta^T & (N-1)p & \ddots & -p \\ \vdots & \vdots & \ddots & \vdots \\ -p\theta^T & -p & \cdots & (N-1)p \end{pmatrix}, \quad (10)$$

where  $p$  is the probability that two agents are neighbours,  $I_l$  is an  $l \times l$  identity matrix,  $\theta = (1, 1, \dots, 1)^T \in R^l$ . By elementary transformation, the eigenvalues of the average Laplacian  $\bar{G}$  are calculated as:

$$\lambda_i = \{0, \mu_j + mp, \underbrace{pN, \dots, pN}_m \mid j = 2, \dots, l\}. \quad (11)$$

It has been shown by Stilwell et al. (2006) that, if the time average of coupling matrix  $G(t)$ , defined as  $\bar{G}$ , admits a stable synchronization manifold and if the switching among all the possible network configurations is fast enough, then the time-varying network (5) will synchronize, where  $T$  is a positive constant. Following this result, synchronization of switching network (5) can be investigated by the network reading

$$\dot{x}_i = f(x_i) - \sigma \sum_{j=1}^N \bar{G}_{ij}(t) H x_j. \quad (12)$$

Let  $e_i$  be the variation of the  $i$ -th node and  $e = (e_1^T, e_2^T, \dots, e_N^T)^T$  be the collection of variations. Then linearizing network (12) at  $x_i = s$  yields

$$\dot{e} = [I_N \otimes J_f(s) - \sigma \bar{G} \otimes H] e, \quad (13)$$

where  $J_f$  is the Jacobian of the function  $f$  evaluated at  $s(t)$  and  $\otimes$  stands for the Kronecker product. Furthermore, the linear stability of the synchronized state  $s(t)$  for network (12) can be studied by diagonalizing the variational equations of network (13) into  $N$  blocks of the form

$$\dot{\xi}_i = (J_f - \sigma \lambda_i H) \xi_i, \quad (14)$$

where  $\xi_i = (U \otimes I_n) e_i \in R^n$ , and  $U \in R^{N \times N}$  is a unitary matrix such that

$$U^T \bar{G} U = \text{diag}(\lambda_1, \dots, \lambda_N). \quad (15)$$

Apparently, the synchronized state  $s(t)$  is stable if the largest Lyapunov exponent of system (15) is negative. According to the master stability function approach (Pecora and Carroll (1998)), the stability of the synchronized state  $s(t)$  depends on the choice of the dynamical function  $f$ , the inter-coupling matrix  $H$  and the average coupling matrix  $\bar{G}$ . For the coupled Rössler oscillator with  $H = \text{diag}(1, 0, 0)$ , there is a single interval  $(\gamma_1, \gamma_2)$ , in which the largest Lyapunov exponent is negative, where  $\gamma_1$  and  $\gamma_2$  are constants. Therefore, synchronization of network (5) can be guaranteed by

$$\frac{\gamma_1}{\sigma} < \lambda_2 < \lambda_N < \frac{\gamma_2}{\sigma}, \quad (16)$$

where  $\lambda_2$  and  $\lambda_N$  are the second smallest and largest eigenvalues of matrix  $\bar{G}$ , respectively. Based on (11) and (16), the synchronization condition for the heterogeneous network is thus derived. For a special case (this case can be realized by choosing  $H=I_n$  or  $H=\text{diag}(0,0,1)$ ), synchronization of network (5), no matter what the coupling matrix  $G^s$  is, can be ensured by introducing a certain number of mobile agents. If define  $\rho_l=l/L^2$  as fixed agent density and  $\rho_m=m/L^2$  as the mobile agent density, then there exists a critical density of mobile agents

$$\rho_m^c = \frac{\gamma_2}{\kappa\sigma} - \min\{\rho_l, \frac{\mu_2}{\kappa}\} \quad (17)$$

for network synchronization: synchronous motion appears if and only if  $\rho_m > \rho_m^c$  with  $\kappa=\pi r^2$ . In this situation, it helps to enhance synchronizability by introducing mobile agents to the static network (1).

#### 4. DISCUSSIONS AND NUMERICAL SIMULATIONS

The existence of mobile agents affects the eigenvalues of  $\bar{G}$ , which further plays an important role in synchronizing heterogeneous network (5). This section discusses this issue from the subsequent two aspects.

##### 4.1 From the Perspective of Synchronizability

It is noted that (16) is fulfilled for some values of  $\sigma$  when the eigenratio  $R$  satisfies the following inequality:

$$R = \frac{\lambda_N}{\lambda_2} < \frac{\gamma_2}{\gamma_1} \quad (18)$$

It is obvious that the eigenratio  $R$  depends on the average matrix  $\bar{G}$ . For a smaller  $R$ , the condition in (18) is easier to satisfy and synchronization is easier to achieve. Then, similarly to the definition in static network, one can characterize the synchronizability of network (5) with  $R$ . Meanwhile, a comparison of synchronizability between the heterogeneous network (5) and its static sub-network (1) can be made according to their corresponding eigenratios. Table 1 shows three cases of  $\rho_l$  for a given static sub-network, where  $\rho = \rho_l + \rho_m$  is the density of the whole network. From the table, the eigenratio  $R$ , no matter what values of  $\rho_l$ , exhibits a monotonic decreasing tendency with respect to  $\rho_m$ . That is to say, synchronizability of heterogeneous network (5) increases as the mobile agent density. Compared with the static network (1), the heterogeneous network (5) shows a better synchronizability if  $R < R^s$ . By solving this inequality, one obtains  $\rho_m > \rho_m^c$ , where  $\rho_m^c$  is a critical value of  $\rho_m$  satisfying

$$\rho_m^c = \frac{R^s}{R^s - 1} \max\{0, \frac{\mu_2}{\kappa\rho_l} - 1, \frac{1}{R^s} - \frac{\mu_2}{\kappa\rho_l}\} \quad (19)$$

Namely, a smaller probably means network (5) is harder to achieve synchronization from the point of view of the interval width in (16), while synchronization is probably easier to realize by assigning a larger mobile agent density.

**Table 1. Three cases of  $\rho_l$**

Cases	$\lambda_2$	$\lambda_N$	$R$
$\kappa\rho_l < \mu_2$	$\kappa\rho$	$\rho_l + \kappa\rho_m$	$(\mu_1 + \kappa\rho_m) / (\kappa\rho)$
$\mu_2 \leq \kappa\rho_l \leq \mu_1$	$\mu_2 + \kappa\rho_m$	$\mu_1 + \kappa\rho_m$	$(\mu_1 + \kappa\rho_m) / (\mu_2 + \kappa\rho_m)$
$\kappa\rho_l > \mu_1$	$\mu_2 + \kappa\rho_m$	$\kappa\rho$	$\kappa\rho / (\mu_2 + \kappa\rho_m)$

To validate the theoretical findings, consider the static network  $G^s$  to be the case of a Barabási-Albert (BA) network (Barabási, and Albert (1999)), where the degree distribution follows a power law. Numerical simulations of two cases (a)  $\kappa\rho_l < \mu_2$ ; (b)  $\mu_2 < \kappa\rho_l < \mu_1$  are reported in Fig.2, where time unit  $\Delta t=10^{-3}$ , velocity  $v=10^3$ ,  $\mu_2=1.366$ ,  $\mu_1=31.21$ ,  $\rho_l=0.3$  for case a and  $\rho_l=2.0$  for case b. It is worth noting that, the average matrix  $\bar{G}$  in simulations is approximated by  $\bar{G} \approx \int_0^t G(\tau) / t d\tau$ , where  $t$  is selected to be  $10^3$ s in simulations. As illustrated in Fig.2,  $R$  monotonically decreases as  $\rho_m$ . Besides,  $R > R^s$  holds for any density of mobile agents in case a, and there indeed exists a critical density of  $\rho_m$  such that  $R > R^s$  in case b. All these agree quite well with the theoretical results.

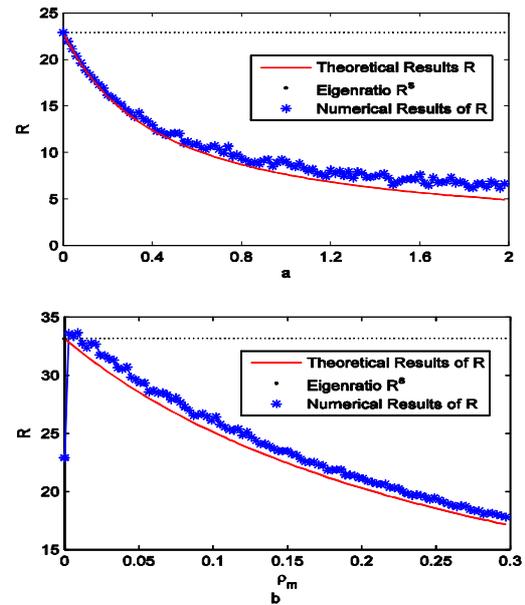


Fig. 2. Eigenratio  $R$  vs. mobile agent density  $\rho_m$ .

##### 4.2 From the Perspective of Synchronization Realization

Though a larger mobile agent density  $\rho_m$  means a better synchronizability of network (5), it is likely to lead to the largest eigenvalue of  $\bar{G}$  over the upper bound in (16). And synchronization is lost with a large mobile agent density for a particular heterogeneous network. According to (16) and Table.1, an upper bound of  $\rho_m$  is given by

$$\rho_m < \rho_m^u = \frac{\gamma_2}{\sigma\kappa} - \max\{\rho_l, \frac{\mu_1}{\kappa}\} \quad (20)$$

An obvious result is that, no matter what value of  $\rho_m$  is, the heterogeneous network (5) is not synchronizable about synchronized state in the condition of  $\mu_l > \gamma_2 / \sigma$ , where the condition implies a non-synchronized motion of static network (1). It is not difficult to see the existence of mobile agents fails to synchronize the considered heterogeneous network. As a result, we favour the introduction of mobile agents for those static networks (1) whose eigencoupling is located in the negative region of the master stability function. In the following, assume that  $\sigma\mu_l < \gamma_2$  holds.

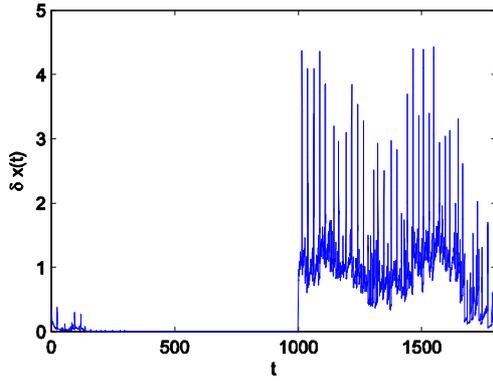


Fig. 3. Evolution of synchronization error  $\delta x$ .

Similarly, one can derive a lower bound of mobile agent density from (16), i.e.,

$$\rho_m > \rho_m^l = \frac{\gamma_1}{\sigma\kappa} - \min\{\rho_l, \frac{\mu_2}{\kappa}\}. \quad (21)$$

Now, some mobile agents are introduced to such a particular static network in the case of  $\sigma\mu_2 < \gamma_1$ . It is obvious that synchronization of static network (1) can be ensured by  $\sigma\mu_2 < \gamma_1$  due to the bounded synchronization region. However, there is a transition from the synchronized behaviour to non-synchronized behaviour when adding several even only one mobile agent. Fig.3 reports the corresponding numerical simulation, where five mobile agents are added to the static network at time  $t=10^3$ , the synchronization error is defined as

$$\delta x(t) = \left( \sum_{i=2}^N \|x_i - x_1\| \right) / N, \quad (22)$$

the static network  $G^s$  obeys the scale-free distribution of the BA model with  $l=80$ ,  $\mu_2=1.45$ ,  $\mu_1=25.73$ , the fixed agent density is  $\rho_l=0.2$ , the overall coupling strength is  $\sigma=0.17$ ,  $\kappa=\pi$ . From the figure,  $\delta x=0$  as  $m=0$ , which indicates the original static network (5) is synchronizable; while the mobile agents are introduced to network, synchronization is lost since the second smallest eigenvalue of  $\bar{G}$  jumps from  $\mu_2$  to a very small value  $\kappa\rho$ . It is shown from Fig.3 that adding mobile agents to network (5), sometimes, are against realization of synchronization. Of course, a proper density of mobile agent is prone to network synchronization. The main results are shown in Fig.4, where all parameters are the same as Fig.3, and synchronization index  $\langle \delta x \rangle = \langle \delta x(t) \rangle$ , which is averaged over 100 realizations during a long enough time in the steady state from  $T$  to  $T+\Delta T$ . In simulations, let  $T=500$  and  $\Delta T=100$ .

As explained in Fig.4, the considered network achieves synchronization again when  $\rho_m > 0.2$ . Also notice that a synchronized motion disappears as  $\rho_m > 0.5$  due to the bounded synchronization region.

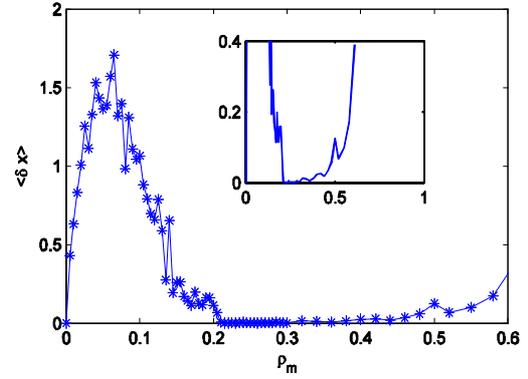


Fig. 4. Synchronization index  $\langle \delta x \rangle$  vs. mobile agent density  $\rho_m$ .

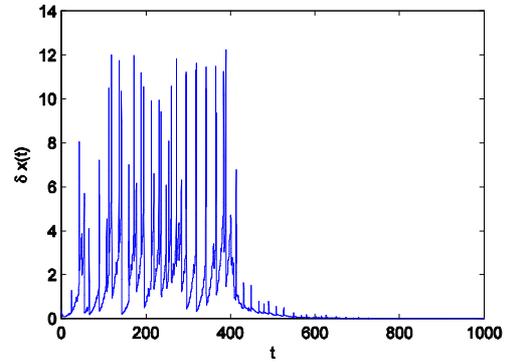


Fig. 5. Evolution of synchronization error  $\delta x$ .

Under the other case  $\sigma\mu_2 < \gamma_1$ , it has been shown by master stability function approach that static network (1) cannot synchronize. However, synchronization can be easily realized by introducing some, even only one mobile agent to network (1) according to (21). A numerical example is given in Fig.5 to validate the analytical result, where the static network  $G^s$  is unconnected graph with  $l=150$ ,  $\mu_2$  is assigned to be zero,  $\mu_l=32.18$ , the fixed agent density is 3.0, the coupling strength is  $\sigma=0.12$ ,  $\kappa=\pi$ . Since  $G^s$  is an unconnected graph, then a global synchronization of static network (1) cannot be accessed due to the isolated clusters in  $G^s$ . It is observed from Fig.5 that adding several mobile agents (in simulations, five mobile agents are introduced to network at  $t=400$ s) can guarantee network synchronization. The role of mobile agents works as a bridge which creates connections among different isolated clusters. For more mobile agents introduced to the considered static network, simulations are performed in Fig.6. From the figure,  $\langle \delta x \rangle$  is very close to zero if  $\rho_m \in [0.32, 1.74]$ , which is consistent with the interval  $[0.31, 1.75]$  obtained by (19)-(20). In addition, the transition from a non-synchronized behaviour to a synchronized behaviour is sharper than the transition from a synchronized behaviour to

a non-synchronized behaviour, which is quite different from the result of mobile agent networks.

From above discussions, there does exist a bounded region of mobile agent density: synchronization of network (5) is ensured if and only if  $\rho_m \in (\rho_m^l, \rho_m^u)$ . For a particular static network (1) with given  $G^s$  and  $\rho_m$ , a too large or a too small mobile agent density will prevent heterogeneous network (5) from achieving synchronization.

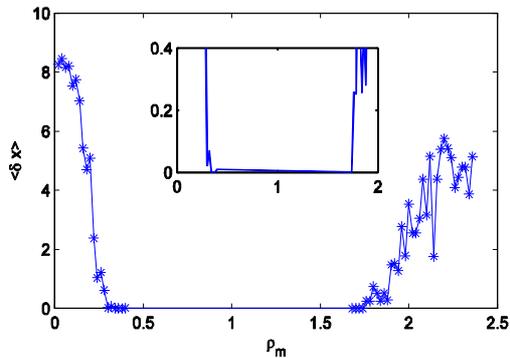


Fig. 6. Synchronization index  $\langle \delta x \rangle$  vs. mobile agent density  $\rho_m$ .

## 5. CONCLUSIONS

In this paper, a heterogeneous agent network is proposed to capture a mixture feature of time-invariant and time-varying topologies existing in many real-world complex networks. Under the constraint of fast-switching, it has been theoretically and numerically shown that synchronization of the heterogeneous network depends on the mobile agent density, the fixed agent density and the spectrum of time-invariant sub-network. For a given heterogeneous network, synchronization motion can be established if mobile agent density of the network lies in a bounded interval, in which its two end-points are determined by the fixed agent density and the static topology. It is worth noting that, compared with the static sub-network, synchronizability can be enhanced when a proper density of mobile agents is introduced to the heterogeneous network. All these results may provide some insights for the future theoretical investigations and practical engineering designs.

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