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BACK TO SOME FUNDAMENTAL PROBLEMS OF SYSTEM THEORY

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***Abstract**: The paper focuses on two basic problems for the control engineer: the separability, as a fundamental assumption to frame correctly the system connections and the concept of proportional transfer element used as a model for many theoretical developments. Some fundamental connections and the concepts of the input and output signals are analysed, as well as some aspects regarding the control and the load disturbance signals. The problem of proportional elements and proportional feedbacks is developed both from theoretical and practical point of view, with its main advantages and disadvantages. A lot of examples give a deep understanding on the discussed aspects.*

***Keywords**: system theory, system connection, separability, proportional transfer element, proportional feedback.*

1. INTRODUCTION

The highest level of approach used today in control system theory is not to exclude the necessity of a deep study of the system theory basis. The mathematics used in control system theory has to be applied to credible models and all operations have to rely on some phenomenological support. As the phenomenological support is usually very hard to be identified accurately, some conceptual reductions are performed in order to obtain approximation models. Due to the engineer's responsibility to carry out any practical approach, he must perform such assumptions taking into account all the types of consequences that can appear. The level where we can find conceptual reduction in control system theory is located at its fundamentals. A physical introspection into the basic types of elements of processes (sources, storage elements, transformers, converters and sinks) is given in [1] and [6].

This paper focuses on two basic problems: the separability, as a fundamental assumption to frame correctly the system connection and the concept of proportional transfer element used as a model for many theoretical developments.

2. SEPARABILITY–FUNDAMENTAL ASSUMPTION TO FRAME THE SYSTEM CONNECTION

Statement of the problem

A system can be considered as an ensemble of interconnected objects, with a well-determined task, interacting with the environment. The task is related to the orientation of the system, i.e. the establishing the input and the output. The flow channels corresponding to internal connections of the system are specified by the so-called „structure of the system". The fundamental assumption regarding the interaction between system and environment is that the environment doesn't react at the interaction with system, or simpler, the environment is both an ideal source (at the input of the system) and an ideal sink (at the output of the system). The properties of system, considered as a collection of objects interconnected both between them and with the environment, depends on the properties of the components, the features of structure, and the way in which each of them transmits flow through a connection is constituted (interface level).

A physical system consists of real objects, while a mathematical model consists of abstract objects. The mathematical modelling of a physical system means to associate to it mathematical relations. The model works only with abstract objects and replaces the real flows in physical system with a special kind of information flow, e.g. a flow of functions transmitted and processed through mathematical operators. The problem of separability is a specific problem concerning the interface level. Separability is the property of system that allows us to consider for each interfacing levels of the system the elements placed before the interfacing level as ideal sources and the elements placed after the interfacing level as ideal sinks. This means that the transfer of information is performed without losses. Since the real connections are with losses the separability is always an idealization. Consequently we will speak about the separability hypothesis.

From the point of view of system structure we usually admit the existence of three types of basic connections. These are: the series connection (figure 1a), the parallel (derivation) connection (figure 1b) and the feedback connection (figure 1c).

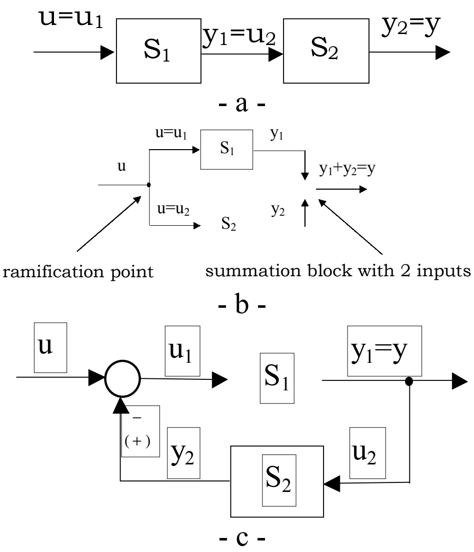


Fig. 1. Fundamental connections.

Within this context, the first aspect of separability is revealed at the interaction between the real systems and the environment. The interaction takes place generally in both directions: from the environment to system and from the system to environment. None of these is ideal source or ideal sink. Assuming the separability hypothesis, the connection between the real system and environment is put in the ideal form of system definition and all the systems are interconnected with the environment via the variables u and y ($u_1 = u$, $y = y_2$ in figure 1a, $u_1 = u$, $u_2 = u$, $y = y_1 + y_2$ in figure 1b, $u_1 \underline{+} y_2 = u$, $y = y_1$ in figure 1c).

The second aspect of separability is revealed concerning connections inside the structure. Assuming the separability hypothesis, in figure 1 the subsystems S_1 and S_2 are considered ideal, i.e. without losses or consumption at the interface level ($y_1 = u_2$ in figure 1a, y_1 and y_2 are mutual independent in figure 1b, $u_2 = y_1$ and u1 $= u \mp y_2$ in figure 1c).

Based on separability hypothesis the mathematical models (MM) associated to these structures can be obtained using the collection of the mathematical models of the subsystems and the connection relations. So we hold the following primary form of mathematical models:

- for the series connection

$$\begin{cases} \text{MM of subsystem } S_1 \\ \text{MM of subsystem } S_2 \\ u = u_1\,,\ y_1 = u_2\ ,\ \ y_2 = y \quad (*) \end{cases} \tag{1}$$

- for the parallel connection:

$$\begin{cases} \text{MM of subsystem } S_1 \\ \text{MM of subsystem } S_2 \\ u = u_1\,,\, u = u_2\,,\, y = y_1 + y_2 \quad (*) \end{cases} \qquad (2)$$

- for the feedback connection:

$$\begin{cases} \text{MM of subsystem } S_1 \\ \text{MM of subsystem } S_2 \\ u_1 = u - y_2 \text{ or } u_1 = u + y_2 \quad (*) \\ y = y_1 = u_2 \quad (*) \end{cases} \qquad (3)$$

The separability hypothesis has allowed us to use the form denoted with (*) for the connection relations and consequently to use the mathematical models of subsystem S_1 and S_2 in the form they are used in connection with an ideal environment.

In order to obtain from the *primary* models (1), (2) and (3) of the three types of fundamental connections considered above some *canonical* forms we have to make reductions and substitutions, representing operator composition. Also, the separability hypothesis makes possible the composition of the operators representing the mathematical models of the subsystems.

Naturally, in many situations the separability hypothesis is not a plausible one. What is to be done? The simple answer is that for a non-separable ensemble we need to build a new model. The composition of operators is not available anymore. Practically this means a new modelling effort that in general is not negligible.

As it was underlined before, the fundamental hypothesis that considers valid the primary models (1), (2) or (3) are called separability hypothesis. From a systemic point of view, the context is as following: in any system consisting of two physical systems implemented in order to transmit a command in a defined way, interactions appear both in the direction of command transmission, which is the reason for the connection, and in the opposite direction. We can say that the physical systems are separable related to the connection when the interaction in the opposite direction of the transmitted command is conceptually negligible. As well we can say that a system is separable related to the environment when the systemic feedback associated to the transfer of the input u and the output y are negligible or doesn't exist. In this case the connection relations having the form (1), (2) and (3) are accepted and they represent the basic models for the fundamental connections.

First it can be seen that in the above assessment we admit, implicitly, that in any system consisting of two physical systems there is, from a systemic point of view, only one connection type and this is the feedback connection. Second we admit, explicitly, the feedback negligibility as the separability hypothesis for the real system modelling, both for the connections inside the structure and for the connection system-environment.

Obviously, can appear a situation when the separability hypothesis is unrealistic, and consequently the models (1), (2) and (3) are not valid. The valid models have to include the above neglected feedback.

In the next section are presented some examples in order to consider thoroughly the concepts described above and what happens when the separability hypothesis is adopted. In a given case the user decides the adopting of separability hypothesis.

Examples on applying the separability hypothesis.

The first example refers to a case where we consider the concept of input variable. Like previous mentioning, a system may be considered as a set of objects interacting with the environment using two categories of variables: input variables (u) and output variables (y). From the point of view of input variable the behaviour of environment versus the systems in this interaction is like of that of an ideal generator. In the same time, from the point of view of output variable the behaviour of system versus the environment is like of that of an ideal generator. Using these assumptions we can separate our system from the environment using representations similar to the one presented in figure 2.

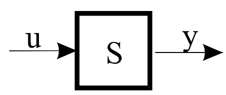


Fig. 2. Interface variables between the system and the environment.

In real cases the environment has not an infinite inertia and there is a two-directional interaction regarding input and output variables.

In order to illustrate this problem concerning the input variable, we consider the very simple electrical diagram from figure 3a consisting of a voltage generator controlling a circuit consisting of a process that is not detailed (the proceeds details being irrelevant for this case). The generator is modelled using two ideal components: a power supply u_e and an internal resistance R_i. The control of the consumer is made using the terminal voltage u_l:

$$u_1(t) = u_e(t) - R_i i_1(t)\,. \tag{4}$$

In equation (4) i_l represents the current in the loop issued by the generator and the circuit. The representation in (figure 3b) shows the feedback character of this current. In order to consider in the systemic mode the voltage u_l as the input of the circuit we use at least one of the approximations:

$$R_i \cong 0 \text{ or } i_l \cong 0 \tag{5}$$

that means the concepts of ideal voltage generator ($R_i \cong 0$) or ideal sink ($i_l \cong 0$) . In this case the circuit may be considered as a system that interacts with the environment through $u = u_l = u_e$. From the point of view of energy flow this means the neglecting the energy dissipated to transmit the input signal u.

Analogous hypothesis are used to introduce the input variable u (or the output variable y) of the models (1), (2) and (3) in the most of practical situations. However, sometimes it is very difficult to highlight the way to introduce such simplifying hypothesis, e.g. the servo-problems in remote domain.

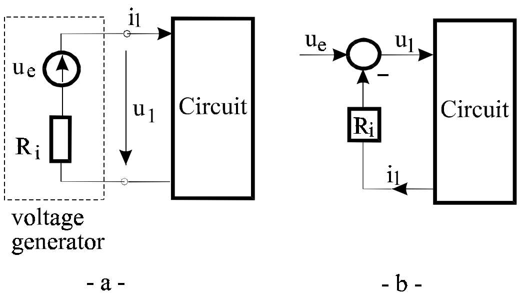


Fig. 3. Regarding the separability using ideal voltage generator and / or ideal sink assumptions.

The second example refers to the circuit in figure 4. It is used in order to discuss some aspects regarding the series connection. The two quadripole considered as independent systems with the orientation $u_1 \rightarrow y_1$ and $u_2 \rightarrow y_2$ have the following correspondent mathematical models:

$$R_1C_1 \cdot \dot{y}_1(t) + y_1(t) = u_1(t) \tag{6}$$

$$R_2C_2 \cdot \dot{y}_2(t) + y_2(t) = u_2(t)\,. \tag{7}$$

We are interested in two aspects.

First we would like to demonstrate that in the frame of connection the dependency (6) between u_1 and y_1 is not valid anymore. Also, from the systemic point of view we cannot assimilate the connection with a series connection. Secondly, it follows the identification of the reduction we have to make from the systemic point of view in the system consisting of two quadripoles in order to associate it with a series connection.Considering null initial conditions, the ensemble has the following primary mathematical model:

$$\begin{cases} R_1 i(t) + \dfrac{1}{C_1}\displaystyle\int_0^t i_1(t)\cdot dt = u_1(t) \\ y_1(t) = \dfrac{1}{C_1}\displaystyle\int_0^t i_1(t)\cdot dt \\ i(t) = i_1(t) + i_2(t) \\ R_2 i_2(t) + \dfrac{1}{C_2}\displaystyle\int_0^t i_2(t)\cdot dt = y_1(t) \\ y_2(t) = \dfrac{1}{C_2}\displaystyle\int_0^t i_2(t)\cdot dt \end{cases} \tag{8}$$

In order to obtain the mathematical model for the channel $u_1 \rightarrow y_1$ from the whole circuit we have to remove the intermediate variables i(t), $i_1(t)$ and $i_2(t)$ from equations (8). We notice that $i(t) = \dfrac{1}{R_1}\cdot[u_1(t) - y_1(t)]$ and $i_1(t) = C_1 \cdot \dot{y}_1(t)$. So

$$i_2(t) = \frac{1}{R_1}\cdot[u_1(t) - y_1(t)] - C_1 \cdot \dot{y}_1(t)\,. \tag{9}$$

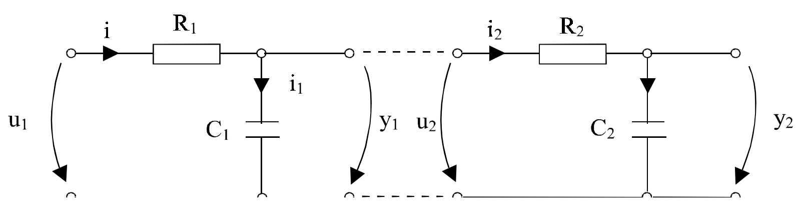


Fig. 4. Regarding to the connection of two passive quadripoles.

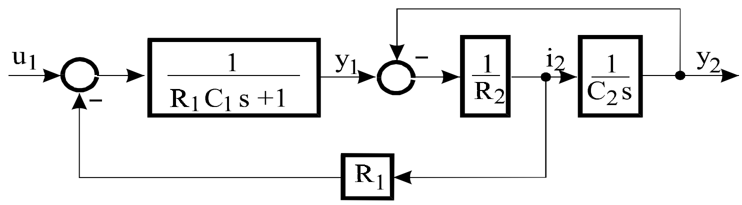


Fig. 5. The block diagram, which expresses the interactions between the quadripoles in fig. 4.

Considering the forth equation from (8) we obtain $C_2R_2(i_2(t))^{\bullet} + i_2(t) = C_2\dot{y}_1(t)$ and replacing in it $i_2(t)$ from (9) finally yields:

$$R_1C_1R_2C_2\ddot{y}_1(t)+R_1C_1(1+\frac{C_2}{C_1}+\frac{R_2C_2}{R_1C_1})\dot{y}_1(t)+$$
$$+y_1(t)=R_2C_2\dot{u}_1(t)+u_1(t)$$

It can be seen that, as consequence of connection with the second quadripole, for the channel $u_1 \to y_1$ the mathematical model (6) isn't valid. So, connecting the quadripoles in a cascade connection they cannot be considered as separable subsystems.

In order to explain what happens, we eliminate the current i from equation (8). Then we separate from resulted equations the first two:

$$\begin{cases} R_1i_1(t)+\dfrac{1}{C_1}\displaystyle\int_0^t i_1(t)\cdot dt=u_1(t)-R_1i_2(t) \\ y_1(t)=\dfrac{1}{C_1}\displaystyle\int_0^t i_1(t)\cdot dt \end{cases}, \quad (10)$$

from the last two:

$$\begin{cases} R_2i_2(t)+y_2(t)=y_1(t) \\ y_2(t)=\dfrac{1}{C_2}\displaystyle\int_0^t i_2(t)\cdot dt \end{cases}. \quad (11)$$

Removing i_1 from equations (10), further we obtain:

$$R_1C_1\cdot\dot{y}_1(t)+y_1(t)=u_1(t)-R_1i_2(t). \quad (12)$$

Removing i_2 from equation (11) and observing that $y_1 = u_2$, we obtain exactly the equation from (7). Also the second quadripole behaves as an independent subsystem in the frame of connection. According with equation (11) and (12) the system with two quadripoles has the equivalent block diagram shown in figure 5. Excepting the input variable, the first block corresponds to equation (6) and the $y_1 \to y_2$ channel to equation (7).

The presence of i_2 current in the block scheme is imposed by equations (12).

Using this equation we can reach the equation form (6) only if we neglect the current i_2 from (12). Hence the highlighting of i_2 is not required any more in the new block diagram (figure 6). It can be noticed that the scheme is similar with the one presented in figure 1a where the equations corresponding to the first and the second block are exactly the equations (6) and (7).

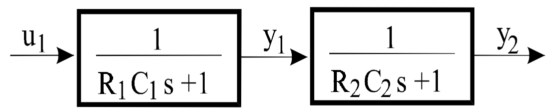


Fig. 6. Simplifying the structure in figure 5 via separability hypothesis (13).

For this case the approximation

$$R_1i_2(t) = 0 \quad (13)$$

represents the separability hypothesis. The systemic result interpretation is that the feedback considered at the input of the first subsystem is neglected. From energetic point of view the interpretation is that the energy transmitted from the first quadripole to the second one is neglected.

The third example is focused on a parallel connection. The system in figure 7 has three circuits and a voltage generator. For the sake of simplicity at the interface level the circuits 1 and 2 are considered as voltage generators and circuit 3 is an input resistor. We would like to establish the conditions for the circuits 1 and 2 in order to be assimilated with two systems connected in parallel.

The whole system (assembly) is characterized by the following equations:

$$u_1(t) = u_e(t) - R_i i_1(t) - R_i i_2(t), \quad (14)$$

for the interface between circuits 1 and 2 and the voltage generator, and for the interface between circuits 1 and 2 and the circuit 3. In Figure 7, i_e is current of output interface circuit.

$$\begin{cases} y(t) = R_s i_e(t) \\ y_1(t) = u_{e_1}(t) - R_{i_1} i_e(t) \\ y_2(t) = u_{e_2}(t) - R_{i_2} i_e(t) \\ y(t) = y_1(t) + y_2(t) \end{cases} \qquad (15)$$

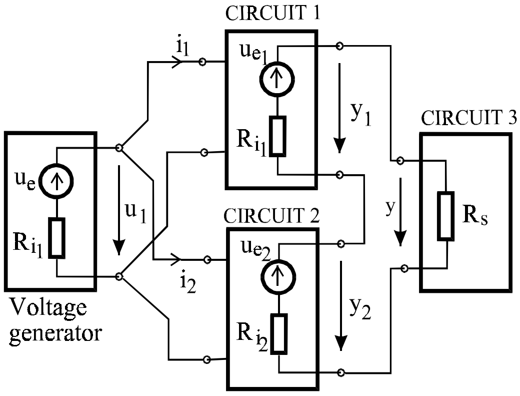


Fig. 7. Electrical diagram used as example for the parallel connection.

From (15) is obtained

$$\begin{cases} y_1(t) = u_{e_1}(t) - R_{i_1} \dfrac{u_{e_1}(t) + u_{e_2}(t)}{R_{i_1} + R_{i_2} + R_s} \\ y_2(t) = u_{e_2}(t) - R_{i_2} \dfrac{u_{e_1}(t) + u_{e_2}(t)}{R_{i_1} + R_{i_2} + R_s} \\ y(t) = y_1(t) + y_2(\mathrm{t}) \end{cases} . \qquad (16)$$

The aspects connected with the equation (14) are quite similar with those considered in the first example. We may consider that the circuits 1 and 2 have the voltage u_1 as common entry, independent of the two circuits considered, only if $R_{i1} \cong 0$ or $i_1 \cong 0$ and $i_2 \cong 0$ (as in equation (5)) are valid. Otherwise we have to take into account that u_1 is not constant and that both circuit 1 and 2 are influencing each other.

The last equation in (15) and (16) is apparently corresponding to the parallel connection. This is based on the fact that the equations $y = y_1 + y_2$ from figure 1b and equations (2) were written by omitting that y_1 and y_2 are mutually dependent. They are also input variables with respect to y. This is possible only when R_s has a great value and / or R_{i1} and R_{i2} have very small values:

$$R_s \to \infty \text{ and/or } \{R_{i1} \text{ and } R_{i2}\} \to 0. \qquad (17)$$

This is equivalent with the approximation $i_e(t) \cong 0$. In this case the separability conditions are (5) and (17). The relations (17) can be interpreted like in figure 8 that corresponds to equations (15): in order to model the equality $y = y_1 + y_2$, the specific feedback of output circuit connections are neglected. From the energetic point of view, on the one side, the energy dissipated in circuits 1 and 2 used to generate the output voltages y_1 and y_2 is neglected and, on the other side, the energy transmitted from the circuits 1 and 2 to circuit 3 is neglected too.

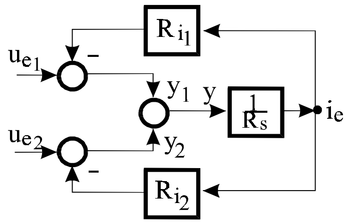


Fig. 8. Systemic structure associated to connection of circuits 1, 2 and 3 in fig. 7.

Further there are underlined some aspects about the load disturbance concept. It is important for any connection between an energy generator device and a consumer. A two-directional interaction takes place as for any other connection. To represent the action of generator on consumer, named control, is used from informational point of view a control variable u. To describe the influence of consumer on the generator is used a second variable, the so-called load disturbance variable v. In fact, the presence of the load or of the load disturbance is just the reason of being of generator and of connection. The generator and the consumer cannot be considered in such a case as separable subsystems. Nevertheless, in the design of control systems the load disturbances are often considered as external inputs of generator – consumer system. From the sake of simplicity, the load disturbance is taken as results of the direct interaction of the process with environment. The representation in figure 9 suggests the idea of last part of above discussion.

The electrical drive shown in figure 10 illustrates such a situation. The motor M drives the load machine LM. The behaviour can be described by the equality:

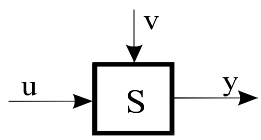


Fig. 9. Representation of a system with a control input and a last disturbance input.

$$(J + J_\ell)\dot{\Omega}(t) = T_a(t) - T_\ell(t)\,, \tag{18}$$

where Ω is angular velocity of the shaft, T_a is the active torque developed by the motor, $T_\ell(t)$ is the load torque oppose by the load machine, J is the inertia moment of drive motor and J_ℓ the inertia moment of load machine.

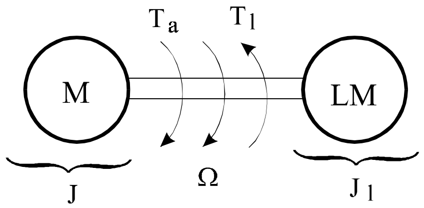


Fig. 10.

The block diagram in figure 11 corresponds to the model (18). It is of the same type as that from figure 9.

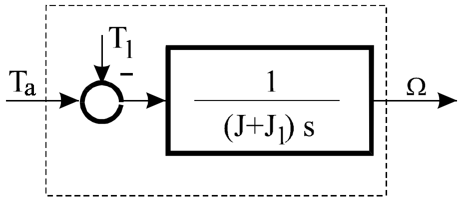


Fig. 11. A simplified structure without feedback for a drive motor – load machine ensemble.

In fact, the problem is more complicated: the load torque is not an independent variable, but a variable in correlation with other variables from the system. Mostly T_ℓ depends on Ω [2], also

$$(J + J_\ell)\dot{\Omega}(t) = T_a(t) - T_\ell(\Omega(t))\,. \tag{19}$$

If we take in account that the moment of inertia of load machine is sometimes a function of velocity then the model (19) is replaced by

$$[J + J_\ell(\Omega)]\dot{\Omega}(t) = T_a(t) - T_\ell(\Omega(t))\,. \tag{20}$$

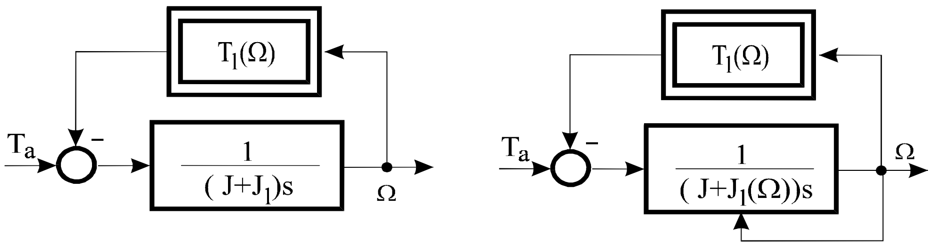
The block diagrams in figures 12a and 12b correspond respectively to the models (19) and (20). Both structures are more complicated as those of figure 11. This is obtained from these two structures by replacing the feedbacks with an equivalent and independent load torque T_ℓ. Such a replacement enables to typify the load torque variations. In this case the energetic aspect has a new image: the energy consumed to cover the resistance of load machine appears as a dissipated energy in the external environment.

The separability hypothesis is expressed by the relations:

$T_\ell = T_\ell(t)$, respectively $T_\ell = T_\ell(t)$, $J_\ell =$ constant.

Sometimes the load disturbances are consequences of some external actions on the ensemble through the load machine ensemble. In such cases the interactions are more complex and this justify the usage of separability hypothesis.

The last example refers to the translational mechanical systems shown in figures 13a and 13b.



- a - - b -

Fig. 12. Closed-loop structure for the drive motor – load machine ensemble.

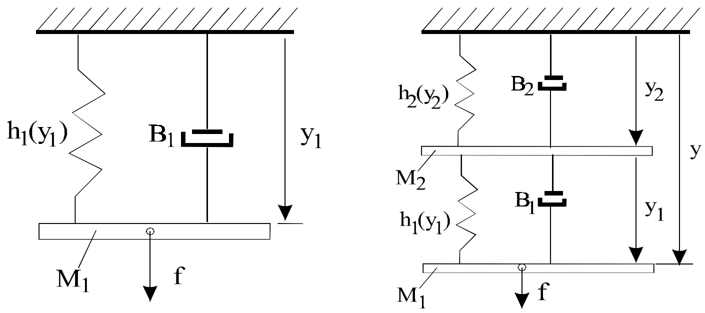


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Fig. 13. Spring – damper systems.

The translation result as an effect to the action of the external force f and the reaction of spring – damper devices. B_1 and B_2 are the viscous damping constant and $h_1(y_1)$ and $h_2(y_2)$ are the non-linear characteristic of the springs.

The mathematical model of the translational system in figure 13a that described the movement in the vicinity of equilibrium point y_s, obtained as solution of equation $h_1(y_{1s}) = M_1 g$, is ([3]).

$$M_1(\delta y_1)^{(2)} + B_1(\delta y_1)^{(1)} + h_1^{'}(y_{1s})(\delta y_1) = f(t)\,.$$

Here, M_1, g, h_1' and δy_1 are, respectively, the mass, the normal acceleration, the derivative and the incremental displacement in the vicinity of y_{1s}.

The equations

$$\begin{aligned}&M_1\ddot{y}_1(t) = -B_1\dot{y}_1 - h_1(y_1) + f(t) + M_1 g\\ &M_2\ddot{y}_2(t) = -B_2\dot{y}_2 - h_2(y_2) + B_1\dot{y}_1 + h_1(y_1) + M_2 g\,.\\ &y = y_1 + y_2\end{aligned}$$

describes the second translational system, from figure 13b. The linearization around the equilibrium values y_{1s} and y_{2s}, given by equalities $h_1(y_{1s}) = M_1 g$ and $h_2(y_{2s}) = (M_1+M_2)g$, yields the model

$$\begin{aligned}&M_1(\delta y_1)^{(2)} + B_1(\delta y_1)^{(1)} + h_1^{'}(y_{1s})(\delta y_1) =\\ &= \underbrace{f - \widetilde{f}}_{u_1}\\ &M_2(\delta y_2)^{(2)} = -B_2(\delta y_2)^{(1)} -\\ &- h_2^{'}(y_{2s})(\delta y_2) + B_1(\delta y_1)^{(1)} + h_1^{'}(y_{1s})(\delta y_1) \quad (21)\\ &\widetilde{F} = \frac{M_1}{M_2}[-B_2(\delta y_2)^{(1)} - h_2^{'}(y_{2s})(\delta y_2) +\\ &+ B_1(\delta y_1)^{(1)} + h_1^{'}(y_{1s})(\delta y_1)]\\ &\delta y = \delta y_1 + \delta y_2\end{aligned}$$

The correspondent block diagram is given in figure 14. To the first equation (21) correspond block A, to the second the bloc B, and block C to the last equation.

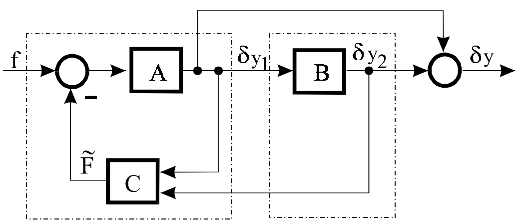

Fig. 14. Bloc diagram associated to the model (21).

It is to observe that the situation is quit similar with those of example 2º. The separability condition consist is the neglecting of reaction: $\widetilde{F}(t) \cong 0$.

That means:

> M_2 very big and / or {$h_1'(y_{1s})$ and $h_2'(y_{2s})$ very little, B_2 very little}.

From the energetic point of view the separability appears as when the mechanical work of the force f don't covers the energy necessary for the mass - spring – damper subsystem M_2-B_2-h_2.

3. PROPORTIONAL ELEMENTS AND PROPORTIONAL FEEDBACKS

3.1. *Proportional Feedback*

The proportional models are frequently models of physical systems. With notations from figure 2, these models are characterized by the equation

$$y(t) = ku(t)\,,\ \text{k = constant.} \tag{22}$$

The concept of dynamic system as a model of a physical system is that of a strict causal system. Limiting the presentation to linear, continuous time and non-variable systems, they have the expression

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t)\\ y(t) = Cx(t)\end{cases}. \tag{23}$$

None of the particularizations of model (23) will lead to (22). Therefore (22) doesn't belong to the class of systems (23).

In control system theory there is frequently encountered the example of state feedback (figure 15)

$$u = Fx + w \tag{24}$$

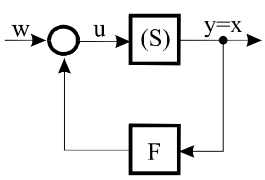

Fig. 15. Stabilizing of a system (S) using a proportional feedback compensator F.

If we consider that (S) has the equations (23), then the ensemble has the model:

$$\begin{cases}\dot{x}(t) = (A + BF)x(t) + Bw(t)\\ y(t) = Cx(t)\end{cases}. \tag{25}$$

The result is important because it is of form (23). The proportional element (22) doesn't introduce new state variables but provokes the change of the transition characteristics in state space by changing the intensity of the linkage between state variables. This means a modification inside the structure of system (23) and not it's interaction with a second system, as it is commonly considered. Considering the previous section of the paper, this modification emphasizes the fact that element (22) is assimilated as inseparable part of the new structure. In other words, the structure in figure 15 is not physically realizable; it is only a mathematical decomposition of the model of a physical system.

The proportional element (22) can be considered as a particular case of the system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}, \tag{26}$$

situated at rand of causality: for A=0, B=0, C=0, D≠0, model (22) is obtained.

To the model (26) we can associate the open loop structure in figure 16. It has the output $y(t) = y_1(t) + y_2(t)$, where y_1 belongs to the linear time invariant and causal system (S)

$$(S)\ \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y_1(t) = Cx(t) \end{cases} \tag{27}$$

and

$$y_2(t) = Du(t) \tag{28}$$

is a parallel proportional feed-forward channel. While the proportional element (28) is an inseparable subsystem of system (26), its presence is allowable only together with the system (27)

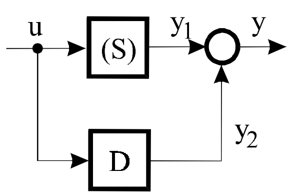

Fig.16. Block diagram of system (26) composed from a causal subsystems (S) and a proportional element D.

Operating with models of form (26) in open loop (serial or parallel) structures, doesn't generate any phenomenological or mathematical interpretation difficulties. Contrarily, operating with such models in closed loop can cause essential difficulties. In contrar with it, the models of type (23) can be used without any restrictions.

In the following section two case studies are considered. The first case shows a situation when a closed loop system modeled by (26) is not a valid one. In the second case, the system obtained from the first through a gain modification leads to a valid system that can be applied as model in practice.

3.2. *Case - studies*

1°. Let the structure from figure 17, where (S) is a SISO system of type (23) with the transfer function

$$H(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0} \tag{29}$$

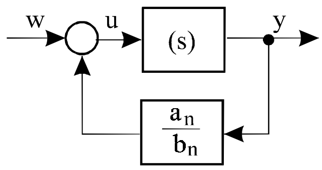

Fig. 17. A SISO continuous system with a proportional feedback connection and a non-causal effect [6].

where $a_n \neq 0, b_n \neq 0, a_0 \neq 0$ and the feedback is of the form: $u(t) = \frac{a_n}{b_n} y(t) + w(t)$.

The close-loop transfer function is:

$$H(s) =$$

$$= \frac{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_n}{\left(a_{n-1} - b_{n-1}\frac{a_n}{b_n}\right)s^{n-1} + \ldots + \left(a_1 - b_1\frac{a_n}{b_n}\right)s + \left(a_0 - b_0\frac{a_n}{b_n}\right)} =$$

$$= \beta_1 s + \beta_0 + R(s)$$

where R(s) is a rational fraction, which is strictly proper. It shows that the structure has a strong derivative character. In the same time, has resulted a reduction of the system order with one. In this case the feedback connection changes the system by inhibiting one of the state variables. This means an anti-causal effect. To underline some features of the problem in [4] are considered two examples.

Trying to model the system from figure 17 in Simulink we will observe that this is not possible and that a message of the form "*Trouble solving algebraic loop containing block.... Stopping simulation. There may be a singularity in the solution.*" appears. The situation is identical also in the case when the structure from figure 17 is included in a more complex system.

The result can be generalized by the structure from figure 18. If $H_1(s)$ and $H_2(s)$ are proper and $\lim_{s\to\infty} H_1(s)H_2(s) = 1$ the ensemble will have a derivative character, and the Simulink model will not work.

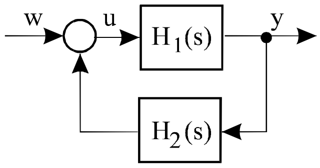

Fig. 18. A closed-loop connection.

2°. For the SISO linear system (29) is considered the feedback connection:

$$u(t) = \frac{a_0}{b_0} y(t) + w(t).$$

Results the structure from figure 19, with the transfer function

$$H(s) = \frac{b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}{a_n s^n + \left(a_{n-1} - \frac{a_0}{b_0} b_{n-1}\right)s^{n-1} + \ldots + \left(a_1 - \frac{a_0}{b_0} b_1\right)s} \qquad (30)$$

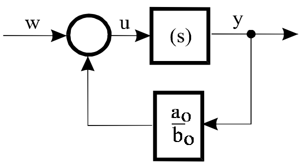

Fig. 19. A SISO continuous system with a proportional feedback connection and a causal effect.

It can be noticed that the ensemble behaves like an integrator system. This is due both to the positive feedback and to the fact that the gain a_0/b_0 is equal to the inverse of the transfer coefficient of the system (S). The system order is not modified. We are in the situations of the models (23), (24) and (25).

More than that, the situation has a consequence that is usable in practice. Due to the integrator character, a necessary condition for the system from figure19 to be in a steady-state regime is

$$w = 0. \qquad (31)$$

By including the system (30) in a stable structure like the structure from figure 20, the equality (31) is forced. If w has more components, $w(t) = \sum \alpha_i w_i(t)$, the structure imposes in steady-state regime the condition: $\sum \alpha_i w_i = 0$.

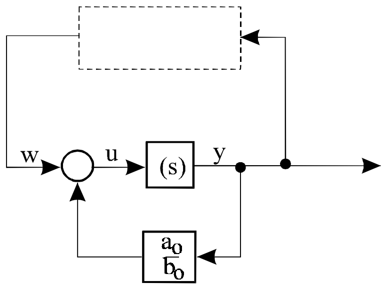

Fig. 20. A closed-loop realised with an integrator system.

This property was used in [5] in relation with the achievement of a speed controller for a medium power hydro-generator. Using relations of the form (31) the proper droop of the system was provided.

It is obvious that, from practical point of view, the achievement of a gain on the feedback channel, equal to the inverse of the transfer coefficient of the system (S) is possible only theoretically. Practically, the implemented value will be a little bit larger or smaller than the inverse of the value of the transfer coefficient. The structure from figure 19 will force a condition of the form

$$w = \varepsilon, \text{ with } \varepsilon \text{ very small,}$$

and the system from figure 18 will have a negative or a positive gain of very large absolute value. An example for these case is discused in [4].

The result can be generalised in the sense that a structure like the one in figure 18 with $H_1(0)\cdot H_2(0) = 1$ behaves in steady-state regime as the structure from figure 18, and in dynamic regime it has also an integrator character.

4. CONCLUSION

The paper considers two fundamental problems of system theory: the problem of mathematical model separability and the problem of proportional systems.

From the informational point of view, the connections between two physical systems are feedback connections. Obtaining series, parallel or feedback, as basic connections in system theory, is the consequence of a simplifying of the real model considering that the systems are separable. The separability hypothesis permits the maintaining of system matemathical models when they are interconnected. The separability hypothesis takes different aspects depending both on the interconnected physical systems and the way of connecting.

Proportional systems are idealized models of real systems. Generally, this idealization is an advatageous one. There are also many situations when operating with proportional elements leads to block of computing by the appearance of algebraic loops.

5. REFERENCES

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